

Article

Frequency Response Modeling of Transformer Windings Utilizing the Equivalent Parameters of a Laminated Core

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Abstract: The aim of the article is to present the method for modeling transformer winding inductance, taking into account the complex magnetic permeability and equivalent electric conductivity of the magnetic core. In the first stage of the research, a physical model of a 24-turn coil wound on the distribution transformer core was prepared. The Frequency Response Analysis (FRA) measurements of the coil were taken; then, the inductance of the coil as a function of frequency was calculated from the received frequency response curves. In the second stage, two-dimensional (2D) and three-dimensional (3D) computer models of the coil based on the finite element method (FEM) were established. In order to obtain the equivalent inductance characteristics of the winding modeled in 2D and 3D in a wide frequency range, the equality of the reluctance of the limbs and yokes in both models was assured. In the next stage of the research, utilization of the equivalent properties for the laminated magnetic material simulations was proposed. This outcome can be used to calculate the frequency response of the winding of the power transformer. The other obtained result is the method for modeling the resonance slope, which is visible on the inductance curve received from the FRA measurement.

Keywords: transformer windings; Frequency Response Analysis (FRA); frequency-dependent parameters; equivalent conductivity; core lamination

1. Introduction

Power transformers are among the fundamental elements of the energy system. Reliable and trouble-free operation of transformers determines a stable supply of energy to consumers, which is why the constant monitoring of the internal condition of these units is necessary. The increasing age of the transformer population has the effect of reducing the reliability and availability of the energy system and, as a consequence, increasing distribution costs and reducing the value of energy sales. Hence, a reliable assessment of their technical condition is becoming more and more important.

There are several generally accepted and standardized technical condition assessment techniques for transformers [1]. Online diagnostic is commonly used, because this allows detection of various types of defects and stresses without disconnecting the units from the electrical grid. Technological developments enable the continuous onsite monitoring of the transformers' condition, which provides automatic evaluation, trending and early detection of oncoming failures. Major failures of transformers, leading to their removal from service, generates high costs related to necessary remedial work (sometimes away from the installation site, e.g., back in the factory) and decreases the reliability of the energy system.

Not less important for maintaining the good technical condition of transformers are offline diagnostic methods. In contrast to online methods, offline ones require disconnecting the transformer

from the electrical grid; thus, they are performed during planned inspections once every few years and, ultimately, are more expensive.

The evaluation of the technical condition of power transformers is carried out using many technical methods, both online and offline, which in combination with economic methods give a complete picture of the most efficient maintenance strategy.

Accurate knowledge about the major failures of power transformers and their equipment is essential not only for manufacturers to improve the quality of their products, but also for electric utilities when planning purchases, organizing maintenance and benchmarking their performance. The worldwide transformer reliability survey was performed by The International Council on Large Electric Systems (CIGRE) work group A2.37. The working group collected 964 major failures that occurred in the period from 1996 to 2010 in transformers with a voltage class of 69kV and above. The survey has shown that the main failure modes across all voltage classes are dielectrical (36.6%), mechanical (20%), electrical (16.5%) and thermal (10.9%). Taking into consideration the damage location, the main contributors were winding (36.6%), tap changer (23.2%) and bushing (14.4%) related failures. Failures initiated in the core and magnetic circuit represent about 4% [2].

Mechanical damage to the active parts of transformers occur mainly as a result of short circuits, but also during overvoltages, seismic events or other shocks (e.g., when transporting the unit) and result in bending, breaking, displacement, loosening and vibration in a winding and in a magnetic circuit.

Frequency Response Analysis (FRA) is used to assess the mechanical condition of the active part of a transformer, mainly the mechanical damage to the windings. The FRA test is currently one of the standard post-production and periodic power transformers measurements. It is recommended to test the frequency response particularly after the shipment of transformers and after events where high currents have occurred. Several frequency response analyzers are available on the market. Furthermore, the FRA diagnostic method has been standardized and described in IEC 60076-18 [3] and other documents [4,5], which provide the baseline for FRA practice.

FRA is a comparative method and involves the comparison of two measured frequency response curves. Consequently, it needs the reference measurement of the same unit, the so-called fingerprint, measured at the factory on a newly built transformer or in its healthy state (i.e., after installation on the worksite). Differences between the reference curve and the one obtained after an incident may indicate the occurrence of mechanical distortion of the active part of the tested transformer.

FRA measurement is typically based on applying a low voltage sinusoidal signal to the one of the transformer terminals and measuring the response signal on the impedance connected to the other terminal. Measurement of the frequency response can be performed in several configurations, but according to the standard [3], the main one is an end-to-end open configuration, which consists of injecting the signal at the beginning of the winding and measuring it at the end. This test is simple, takes only the tested winding into consideration, with some influence of the capacitances of other windings, and allows each winding to be examined separately. The remaining windings stay open during the measurement, which results in a visible influence of the core characteristics on FRA diagrams, especially at low frequencies. Therefore, this makes it suitable for magnetic core assessment [4,6].

The results of FRA measurements are usually presented as Bode plots, where the magnitude is the scalar ratio of the output (U_2) and input (U_1) signal evaluated as a signal damping in dB. The magnitude is usually denoted as FRA. The phase angle shift of the frequency response in degrees is also evaluated:

$$\text{FRA} = 20 \log \frac{U_2}{U_1} [\text{dB}], \quad (1)$$

$$\varphi = \text{tg}^{-1}(\angle U_2 - \angle U_1). \quad (2)$$

The frequency response (FR) curves are usually presented on a logarithmic scale, which allows analysis of the results in all frequency ranges, from low to high.

The example of a frequency response amplitude curve is shown in Figure 1. The shape of this curve is determined mainly by winding inductances and capacitances, which in turn are dependent on the mechanical condition of the active part of the transformer [7]. Frequency response is usually divided into three frequency subranges: low, medium and high. The division is not strict, depending on the transformer's size and power rate. Moreover, the division is related to the source of the deformations. The behavior of the FR curve in the low frequency (LF) range is related to the magnetizing inductance and bulk capacitance of the measured winding. The curve characteristically decreases at the low frequencies due to the domination of the winding inductance. The FR curve in the LF range is vulnerable to short circuits between coils and wires. This frequency range ends at the inflection point after the first parallel resonance [8]. In the medium subrange (MF), the influence of the magnetic circuit disappears and the behavior of the FR is determined by the interaction of local inductances and capacitances. Therefore, MF allows the detection of deformations in the transformer winding and is the most important in the interpretation of results. Above approximately 200 kHz, the high frequency (HF) range is influenced by the test setup, the connection quality, the wave phenomena in the windings, etc. [4,8–10].

Currently, research related to FRA focuses on the correct interpretation of the measured frequency responses of the windings. When performing an analysis of the results, one should take into consideration several factors, such as the connection system, the winding geometry or the history of failures and repairs. In practice, this means that only a very experienced diagnostician can perform the correct analysis of the FRA results. Therefore, researchers and specialists strive to develop tools that support the correct interpretation of results and automatize the analysis process itself by developing a database of a unit's defects and the corresponding changes in FRA curves [11]. Furthermore, not every difference in compared curves indicates damage or deformations in the tested winding. Even between phases of the same transformer there are always visible differences in the low frequency range, which are related with a magnetic flux distribution in the ferromagnetic core [12]. In addition, small differences are also easy to see between two phases in the middle frequency range [6]. The correct comparison of two curves requires not only measurements on the same (or sister/twin) unit, but also under the same operating conditions. A curve shift caused by the position of the tap changer, core magnetization (changes in inductance parameters), bushing replacement (changes in capacitive parameters), the type of FRA configuration, etc. can lead to misinterpretation of the FR results [4,11–13]. Numerous experiments and modeling techniques for finding the correlation between the shift of the frequency response curve and the mechanical (or the electrical) windings' faults have been described in literature (see for instance [10] and [13–16]).

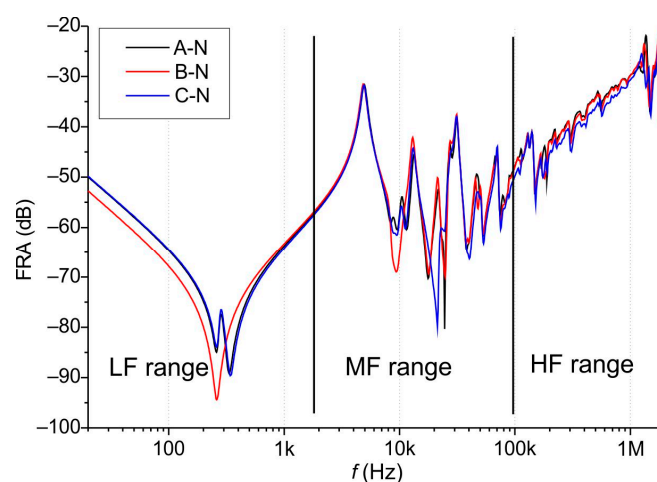


Figure 1. Example of frequency response (FR) amplitude curves of a three-phase transformer, measured in an end-to-end open test setup, presented in a logarithmic scale. Three frequency ranges are marked (Reproduced with permission from [13]).

The active part of the transformer, like every other electrical machine, can be represented by an equivalent *RLC* circuit. Even small physical changes in the active part of the transformer have a great influence on its *RLC* network. Precise knowledge about the resistance (*R*) of the winding, the inductance (*L*) of the coils and the capacitance (*C*) of the insulation layers, between the wires, to the ground, to the tank, etc. allows analysis of the mechanical condition of the transformer, especially of the transformer winding. Changes in the value of the *RLC* parameters caused by faults occurring in the winding results in changes in its frequency response [7]. Furthermore, the reversal of this approach leads to development of recognition techniques for the various transformer faults corresponding to changes in the values of particular parameters of the *RLC* networks [10].

Different models have been developed to simulate the behavior of transformer windings [10,15–17]; however, models based on lumped *RLC* parameters together with electromagnetic field studies allow reproduction of the same frequency response as the actual tested unit. The simulation of mechanical faults, such as axial displacement, a short circuit fault, the loss of clamping pressure, an inter-turn fault, etc., helps to understand and identify the problem inside the transformer without the need to make expensive experiments on real units. Eventually, simulation of the influence of the transformer winding faults on its frequency response will lead to the establishment of a standard code for FRA results identification [14].

The main aim of this paper is to present the method for obtaining the equivalent parameters of the laminated core material in order to determine the frequency-dependent inductance of the transformer winding. The analysis of the influence of the core properties on the inductance is performed with computer models created using FEM software and with reference to FRA measurement. The values of the winding inductance obtained during the simulations are compared to the inductances obtained from the FRA measurements of the physical coil model. The article provides a way to equalize the results obtained from two-dimensional (2D) and three-dimensional (3D) models, which is significant for simplifying and reducing the computation time of the electromagnetic field models. In addition, the novel approach to simulation of the resonance caused by the influence of mutual inductances and capacitances associated with the windings remaining on the other columns of the physical model [13] is presented.

2. Physical and Computer Models

2.1. Physical Coil Model

The physical model of the tested winding is a 24-turn helical coil placed on a laminated distribution transformer core. As shown in Figure 2, the coil is wound on a pressboard tube mounted on the outer column of the core. On the other two columns, the factory windings are mounted: an LV (Low Voltage) winding on the middle column and both LV and HV (High Voltage) windings on the outer column. The 24-turn coil was wound using the rectangular cross-section wire originating from the LV winding. The ferromagnetic core used for the survey came from a 800 kVA, 15/0.4 kV distribution transformer and its dimensions are 961.2×1000 mm. The active part of the transformer was pulled out of the tank and the windings were unplugged from the bushings.

The physical model was measured with an Omicron FRAnalyzer meter, i.e., the winding frequency response, was measured in the range from 20 Hz to 20 MHz. The measurement signal was applied at the beginning of the coil and registered at its end. Based on the frequency response of the coil, its inductance as a function of frequency was determined.

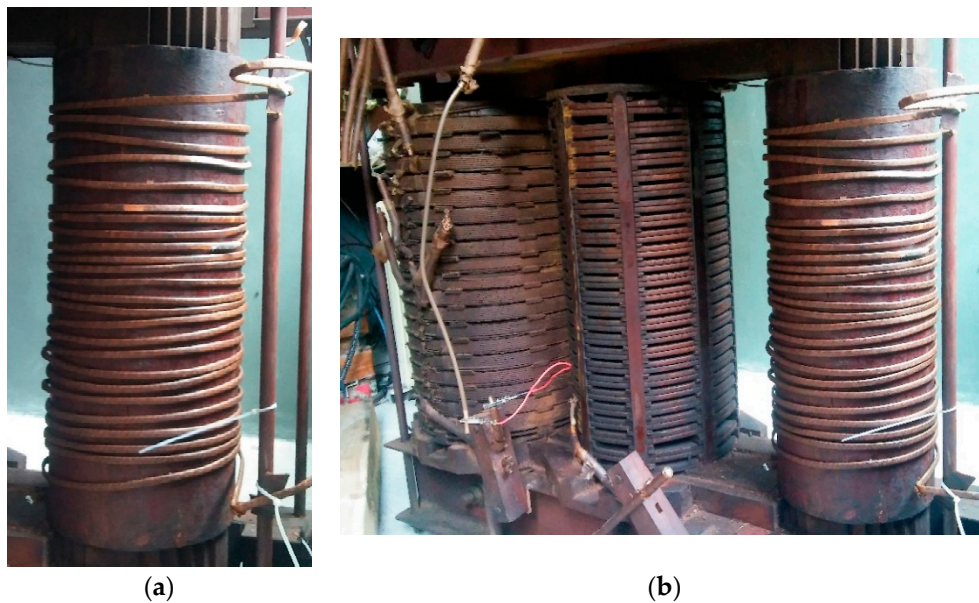


Figure 2. Physical model of 24-turn coil: (a) Coil wound on the first column of laminated core; (b) Active part of distribution transformer with the factory windings on the other two columns.

2.2. Computer Model of the Coil in FEM Software

The computer models utilize the Finite Elements Method (FEM). The geometry of the 3D coil is shown in Figure 3 and it was prepared while maintaining the original dimensions of the physical model. Similar to the physical model, the turns of the coil were evenly distributed over the entire available limb height. The computational domain was reduced to half of the model due to the symmetry of the object. The boundary conditions for the problem are set as a Dirichlet boundary at the symmetry plane (the magnetic field is tangential to the boundary and flux cannot cross it) and at the outer region boundary the conditions of the problem are set as a Neumann boundary.

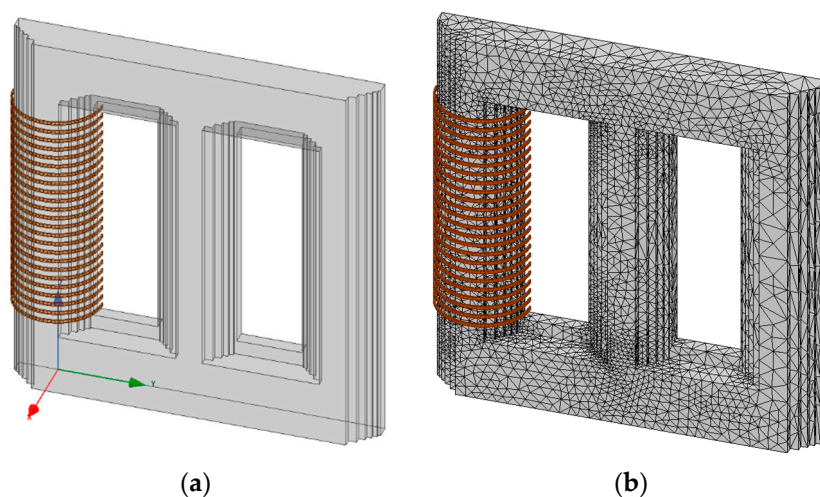


Figure 3. Three-dimensional (3D) coil model in finite element method (FEM) software: (a) computational domain (1/2 of the model due to the symmetry of the object); (b) finite element mesh.

The 2D model was made in a cylindrical symmetry. The geometry and FEM mesh of the 2D model is shown in Figure 4. During the analysis, the solver assumed that the 2D model sweeps around the z-axis of a cylindrical coordinate system. On the outer boundary of the model, a so-called “ballooning” boundary condition has been established.

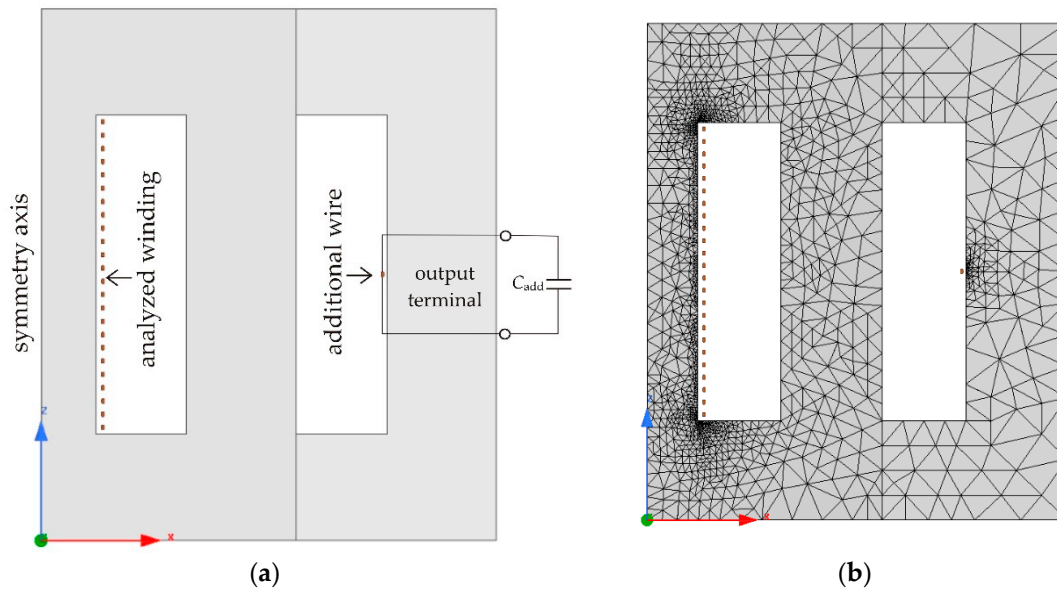


Figure 4. Two-dimensional (2D) coil model in FEM software: (a) computational domain (cylindrical symmetry of the object); (b) finite element mesh.

This model allows the field simulation to be performed, showing the influence of the C and L parameters with satisfactory accuracy. The additional column visible on the right side of the model (Figure 4) allows the influence of the other windings mounted on the core to be taken into account, in particular their inter-turn capacitances and capacitances to the ground, on the frequency response of the tested winding. The value of the magnetic coupling between the analyzed winding and the additional wire can be moderated by changing magnetic permeability value of the column on the right side.

There is no need to simulate the complete windings mounted on the other limbs, because during the FRA measurement the opposite windings are seen as a concentrated inductance and capacitance and can be modeled as a single wire.

The software used for the simulations was ANSYS Maxwell v. 19.2, especially the 2D and 3D eddy current solver, which computes steady-state, time-varying (AC) magnetic fields at a given frequency. Equations (3)–(5) were chosen from the Maxwell calculation package.

The eddy current field solver calculates the eddy currents by solving the magnetic vector potential and the electric scalar potential in the field equation:

$$\nabla \times \frac{(\nabla \times \mathbf{A})}{\mu} = (\gamma + j\omega\epsilon)(-j\omega\mathbf{A} - \nabla\phi), \quad (3)$$

where $\mathbf{A}$ —magnetic vector potential, ϕ —electric scalar potential, μ —magnetic permeability, ω —angular frequency at which all quantities are oscillating, γ —conductivity and ϵ —permittivity.

The inductances of the coil, both self and mutual, have been computed using the energy of the electromagnetic field delivered by the FEM:

$$W_{AV} = \frac{1}{8} \int \mathbf{B} \cdot \mathbf{H}^* d\Omega, \quad (4)$$

$$L = \frac{8W_{AV}}{I_{\max}^2} = \int \mathbf{B} \cdot \mathbf{H}^* d\Omega, \quad (5)$$

where W_{AV} —average energy of magnetic field, L —inductance, $\mathbf{B}$ —magnetic flux density, $\mathbf{H}^*$ —magnetic field intensity conjugate and $I_{\max}$ —peak current.

The results were obtained in the form of inductance matrices and determined as a frequency function according to above formulas. The inductance values lying between the simulated frequency points were obtained by linear interpolation.

3. Core Modeling

In order to simulate the correct behavior of the ferromagnetic core in a wide frequency range, the laminated core material is modeled using an equivalent frequency-dependent magnetic permeability and an equivalent conductivity.

3.1. Equivalent Magnetic Permeability

Assuming a one-dimensional propagation of the electromagnetic wave in the laminated ferromagnetic core with a sheet thickness of $2D$, the y -component of the magnetic intensity vector can be written as [18]:

$$H_y(z) = H_{y0} \frac{\cosh kz}{\cosh kD}, \quad (6)$$

where k equals:

$$k = \sqrt{j\omega\mu\gamma} = \frac{(1+j)}{\delta}, \quad (7)$$

and δ is the skin-depth:

$$\delta = \sqrt{\frac{2}{\omega\mu\gamma}}. \quad (8)$$

The complex permeability in the laminated core is given by the equation [19]:

$$\underline{\mu} = \frac{1}{H_{y0}} \cdot \bar{B} = \frac{1}{H_{y0} \cdot 2D} \int_{-D}^D (\mu H_y) dz. \quad (9)$$

After substituting Equations (6–8) into Equation (9), the permeability can be written as:

$$\underline{\mu} = \mu_0 \mu_r \cdot \frac{\sinh kD}{kD \cdot \cosh kD} = \mu_0 (\mu' + j\mu''), \quad (10)$$

where $\underline{\mu}$ —complex permeability, H_{y0} —magnetic field intensity on the surface, $\bar{B}$ —average magnetic flux density, μ_0 —vacuum permeability, μ_r —relative permeability.

After splitting into real and imaginary parts, it takes the form of:

$$\mu' = \frac{\operatorname{Re}(\underline{\mu})}{\mu_0} = \frac{\mu_r \delta}{2D} \left(\frac{\sinh\left(\frac{2D}{\delta}\right) + \sin\left(\frac{2D}{\delta}\right)}{\cosh\left(\frac{2D}{\delta}\right) + \cos\left(\frac{2D}{\delta}\right)} \right), \quad (11)$$

$$\mu'' = \frac{-\operatorname{Im}(\underline{\mu})}{\mu_0} = \frac{\mu_r \delta}{2D} \left(\frac{\sinh\left(\frac{2D}{\delta}\right) - \sin\left(\frac{2D}{\delta}\right)}{\cosh\left(\frac{2D}{\delta}\right) + \cos\left(\frac{2D}{\delta}\right)} \right). \quad (12)$$

The real part of the complex permeability represents the ability of the core material to conduct the magnetic flux, while the imaginary part represents the losses generated in the core due to eddy currents circulating inside the laminations.

The complex permeability is calculated taking into account the maximum permeability value of the ferromagnetic material, the conductivity and the thickness of the steel sheet. Figure 5 presents the frequency-dependent complex permeability calculated according to Equations (9) and (10) using the following parameters: $D = 0.23$ mm, $\mu_r = 510$, $\gamma = 1.2$ MS/m.

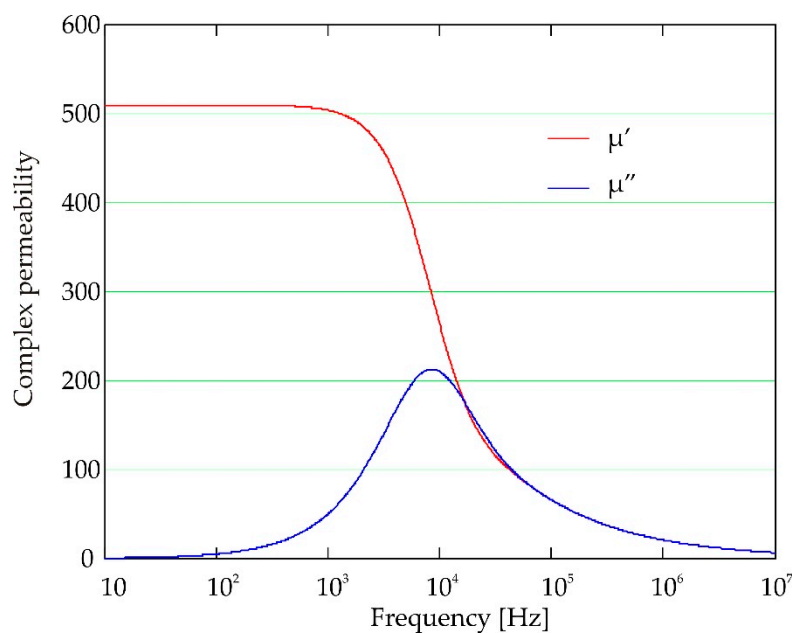


Figure 5. The complex permeability of the core material as a function of frequency.

In the computer analysis, it was assumed that the permeability of the material varies with the frequency according to the real part of the complex permeability.

3.2. Equivalent Conductivity

Transformer cores are made of stacked layers of thin steel sheets in order to reduce the power losses generated by the eddy currents induced by a time-varying magnetic flux. The sheets are insulated from each other by a non-conducting layer of insulation. The technical parameters of the sheets from which the cores are made are given by the manufacturer for typical values of magnetic flux and frequency. However, it should be noted that the operating conditions and geometry of the devices utilizing these cores are different from the conditions under which the measurements contained in the technical data were obtained. For this reason, the actual value of the conductivity of the laminated core is much smaller and can be assumed to be 60% of the conductivity of a single sheet.

Numerical calculations of the electromagnetic field of the laminated cores, which take into account the effect of the eddy currents, require very fine discretization of each sheet and space between the sheets (insulation layers). Such an operation is usually impossible or very impractical due to the limited computational capabilities of the computer hardware and the duration of the analysis. The problem of the complexity of a computational model with a highly detailed finite element mesh that takes into account the core lamination can be omitted by replacing the laminated core with a homogeneous material with a correspondingly changed conductivity value [20]. This conductivity is called the equivalent conductivity.

The main application of the equivalent conductivity is a numerical field calculation in the presence of hysteresis and eddy-current losses. In the literature there are several approaches to the determination of equivalent conductivity, however, in order to be comparable with the laminated core, every model with equivalent conductivity should fulfill the following requirements [20]:

- The dissipated power should not change,
- The magnetic field outside the core must be the same.

The often-used definition assumes that the homogeneous medium is anisotropic and the equivalent conductivity is equal to the electric conductivity of a single steel sheet in a direction parallel to the lamination, while in a direction orthogonal to the lamination it is determined with an equivalent

value [21]. Numerous publications (see for instance [21–23]) describe and develop the anisotropic equivalent conductivity models, because by using this approach the core losses can be obtained with more satisfactory accuracy than using isotropic models.

The research carried out in this paper indicates that the isotropic equivalent conductivity can be successfully used to recreate the core behavior in frequency response modeling. Contrary to the cases described above, where the equivalent conductivity was obtained for the eddy-current losses calculations on the basis of the dissipated power equality, in the simulations below, an attempt was made to obtain the value of the equivalent conductivity on the basis of the energy accumulated in the electromagnetic field of the coil model. Such an approach seems to be more appropriate, while the aim is the calculation of the coil inductance.

Figure 6 shows a model of a laminated core of infinite length in x and z directions.

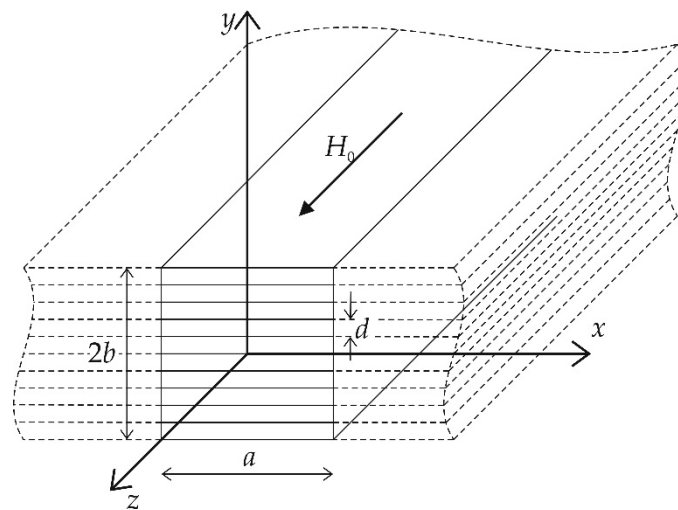


Figure 6. Laminated magnetic core with infinite x, z dimensions.

For a single lamination, whose thickness d in y direction is much smaller than its length and width in x and z directions, the field analysis can be reduced to a one-dimensional (1D) problem. Hence, the differential equation for the z -component of the magnetic intensity vector in one steel sheet of thickness $2b$ can be described as [24]:

$$\frac{d^2 \underline{H}_z(y)}{dy^2} = j\omega\mu\gamma \underline{H}_z(y). \quad (13)$$

The boundary condition for the above problem is:

$$\underline{H}_z(y)|_{y=\pm b} = \underline{H}_0. \quad (14)$$

The propagation of the electromagnetic wave into the ferromagnetic homogeneous block is then given by:

$$\underline{H}_z = \underline{H}_0 \frac{\cosh \alpha y}{\cosh \alpha b}, \quad (15)$$

where:

$$j\omega\mu\gamma_e = \alpha^2, \omega\mu\gamma_e = k^2, \delta_e = \sqrt{\frac{2}{\omega\mu\gamma_e}}. \quad (16)$$

After substituting α into Equation (13), the equation takes the following form:

$$\underline{H}_z = H_0 \frac{\cosh \frac{y}{\delta_e} \cos \frac{y}{\delta_e} + j \sinh \frac{y}{\delta_e} \sin \frac{y}{\delta_e}}{\cosh \frac{b}{\delta_e} \cos \frac{b}{\delta_e} + j \sinh \frac{b}{\delta_e} \sin \frac{b}{\delta_e}}. \quad (17)$$

The energy stored in the electromagnetic field is defined by:

$$W_m = \frac{1}{2} \operatorname{Re} \int_{-b}^b \mu \cdot \underline{H}_z \cdot \underline{H}_z^* \cdot dy = \frac{1}{2} \int_{-b}^b \mu \cdot |\underline{H}_z|^2 \cdot dy. \quad (18)$$

By applying Equations (15) to (16) and next, calculating the above integral, the energy stored in the homogenous ferromagnetic block with equivalent conductivity γ_e is:

$$W_{me} = \frac{\mu \cdot H_0 \cdot \delta_e}{4} \frac{(\sinh 2(\frac{b}{\delta_e}) + \sin 2(\frac{b}{\delta_e}))}{(\cosh \frac{b}{\delta_e} \cos \frac{b}{\delta_e})^2 + (\sinh \frac{b}{\delta_e} \sin \frac{b}{\delta_e})^2}, \quad (19)$$

The corresponding derivation can be carried out for a laminated core consisting of n sheets and thickness d . In order to compare the electromagnetic energy in Equation (17), the substitution $2b = d$ was made. The energy stored in the package of n steel sheets with conductivity γ and thickness d takes the form of:

$$W_m = \frac{n \cdot \mu \cdot H_0 \cdot \delta}{4} \frac{(\sinh(\frac{d}{\delta}) + \sin(\frac{d}{\delta}))}{(\cosh \frac{d}{2\delta} \cos \frac{d}{2\delta})^2 + (\sinh \frac{d}{2\delta} \sin \frac{d}{2\delta})^2} \text{ with } \delta = \sqrt{\frac{2}{\omega \mu \gamma}}, \quad (20)$$

W_{me} and W_m have the same value when:

$$\frac{d}{\delta} = \frac{2b}{\delta_e} = \frac{n \cdot d}{\delta_e}, \quad n \cdot \delta = \delta_e, \quad n \sqrt{\frac{2}{\omega \mu \gamma}} = \sqrt{\frac{2}{\omega \mu \gamma_e}}. \quad (21)$$

The above assumption leads to a simple expression for the equivalent conductivity:

$$\gamma_e = \frac{\gamma}{n^2}. \quad (22)$$

The equivalent conductivity in the homogeneous isotropic medium depends only on the number of steel sheets in the transformer core and the conductivity of a single ferromagnetic sheet. The obtained formula coincides with the formula derived in [22]. Despite the fact that it does not take into account the core stacking factor, it also corresponds to the formula from [23].

4. Results

Taking into account the substitute values γ_e and μ' for the core material, the parameters presented in the above paragraph allowed two practical results to be obtained.

4.1. Equivalence of 2D and 3D Models

Because of a cylindrical symmetry, the 2D model contains much more ferromagnetic material than the 3D model. In order that the value of the magnetic flux was the same as in the 3D model, the value of the magnetic permeability in the 2D model was corrected with the assumption that the value of the total reluctance $\mathcal{R}$ should be the same in both cases. The reluctance networks of the 2D and 3D models are shown in Figure 7. A similar approach can be found in [19]; however, without modeling the outer column. The 2D reluctance calculation take into consideration the geometry of the model

with an additional outer column, which provides a proper value for the magnetic coupling coefficient of the analyzed coil and the additional turn.

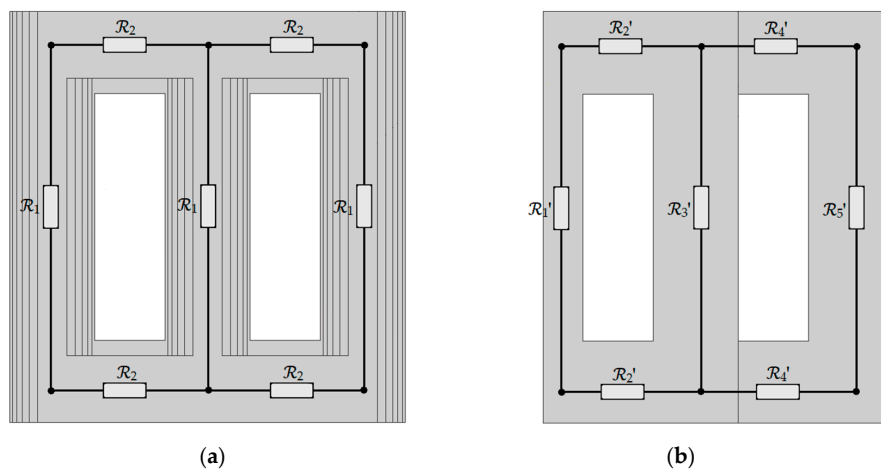


Figure 7. The reluctance network of: (a) 3D model and (b) 2D model.

For the abovementioned reason, the initial value of the relative permeability $\mu_r = 510$ in the 3D model, was reduced to $\mu_r = 214$ in the 2D model. Further values of the permeability in the 2D model at higher frequencies were assumed according to Equation (11), with attention paid to the skin depth δ , which should have the same values as the 3D model.

The development of the 2D model, which provides results equivalent to the 3D model, is significant, because it allows simplification of the model of the transformer in FEM software. Modeling of the active part of the transformer in 2D is often necessary, since the complex design of a real transformer makes 3D simulation computationally costly because of the large number of the winding turns. Numerical calculations which take many days make it very difficult to verify the various parameter configurations of the simulated model, thus simplifying the model is very important for conducting trial simulations. The inductance curves determined from the 2D and 3D simulations together with the inductance calculated from the FRA measurements are shown in Figure 8.

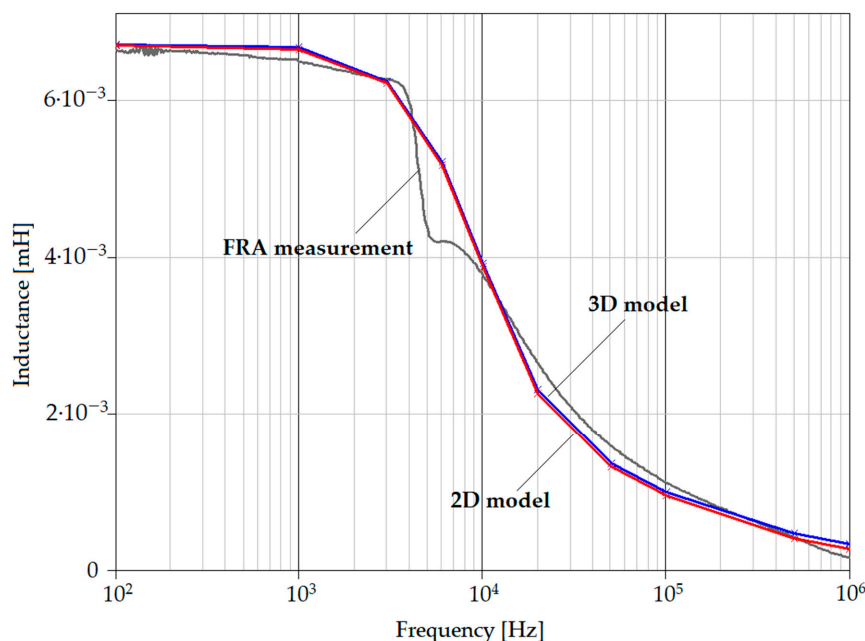


Figure 8. Inductance of a 24-turn coil determined from the Frequency Response Analysis (FRA) measurement and from the 2D and 3D FEM analysis as a function of frequency.

At this stage of the research, calculations were carried out without the presence of an additional column with an additional wire, which models the presence of the other windings. For the reason mentioned above, the inductances obtained on the basis of the 2D and 3D computer simulations correspond, however in the range from 3 kHz to 10 kHz they diverge from the measured curve (Figure 8). In this range, the inductance obtained from real coil measurements is apparently decreasing, deviating from the theoretical dependence of the gradual decrease in inductivity along with the frequency increase. This effect, in FRA diagnostics it is often referred to as a first resonance, is caused by the influence of the mutual inductance and ground capacitances associated with the windings remaining on the other columns of the physical model [13].

4.2. Resonance Simulation for 2D Model

Creating the 2D model in the manner shown in Figure 4a enables simulation of the resonance visible on the inductance characteristic obtained from the FRA measurements. Including the additional turn representing the influence of the windings from the other transformer columns combined with the capacitance associated with these windings made it possible to obtain inductance values coinciding with the physical coil model in a wide frequency range. The results of this simulation are shown in Figure 9.

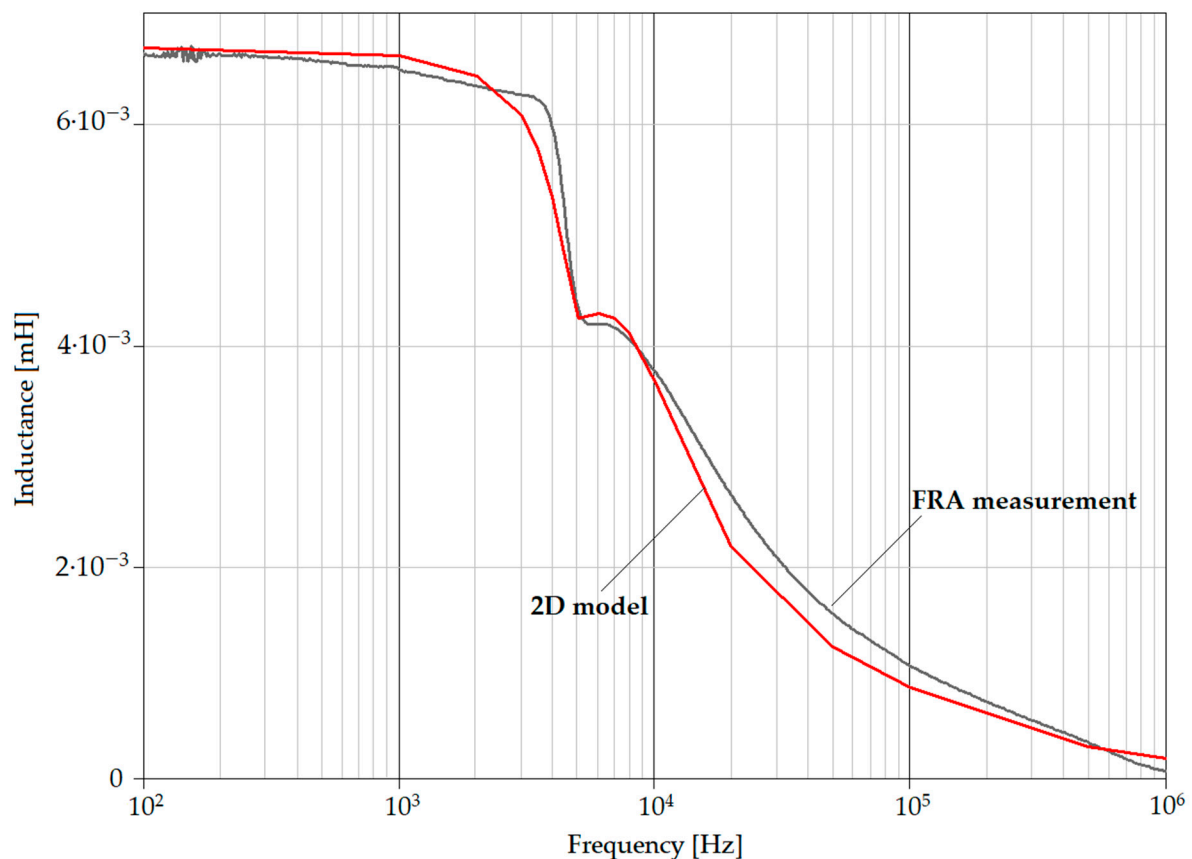


Figure 9. Inductance of the 24-turn coil determined from 2D FEM model with an additional wire, which provides the simulation of the resonance slope.

The simulation of the resonance slope was accomplished by the ANSYS Maxwell software by creating the output terminal in the 2D model. The output was connected with the capacitance C_w resulting from the capacitance of the HV-winding. The value of this capacitance was assumed to be

5pF and was used to recalculate the capacitance of the one-turn additional wire C_{add} according to the following formula:

$$C_{add} = \vartheta^2 \cdot N_{LV}^2 \cdot C_w = \left(\frac{N_{HV}}{N_{LV}} \right)^2 \cdot N_{LV}^2 \cdot C_w, \quad (23)$$

where ϑ is the turns ratio of the tested transformer and N_{HV} and N_{LV} are number of turns in the HV and LV windings. The capacitance C_{add} is then equal to:

$$C_{add} = \left(\frac{1520}{288} \right)^2 \cdot 288^2 \cdot 5\text{pF} = 12\mu\text{F}. \quad (24)$$

5. Conclusions

The survey conducted in this paper explains utilizing the equivalent parameters for the laminated core in order to simulate the variations of the inductance of the transformer winding in a wide frequency bandwidth. The knowledge put across this paper can be helpful for simulating the simulated frequency response on the basis of the *RLC* parameters obtained from the electromagnetic field model of the transformer, which will be exactly the same as the frequency response of the real transformer winding. Nowadays, the FRA method is one of the standard post-production and periodic transformer diagnostic measurements. The main problem of this comparative method is the interpretation of the frequency response curves obtained. The modeling of the various mechanical transformer faults by changing the values of particular parameters of the *RLC* transformer network helps to understand and identify the problem inside the transformer without the need to make expensive measurements on real units and, consequently, provides standardization for the classification of FRA signatures.

The first aim of the paper was to provide the equivalent parameters of the transformer core, especially the complex magnetic permeability and equivalent conductivity and to use these quantities in the 2D and 3D field models for the active part of the transformer. In order to balance the values of the magnetic flux in the 2D and 3D models, equality of the total reluctance of both models has been assumed. As a result, equivalent 2D and 3D models in a wide frequency range were obtained (Figure 8). At this stage, the simulated models did not take into account the influence of other windings on the inductance curve; thus, in the range from 3 kHz to 10 kHz, the simulated curves diverge from the measured one.

Another aim of the article was to explain the occurrence of the so-called first resonance, which is easy to see on the frequency response of the tested winding. The resonance, also visible on the inductance curve, is caused by the influence of the mutual inductance and capacitances associated with the windings remaining on the other columns of the physical model (Figure 2). This influence was simulated by adding an additional wire to the 2D model, whose output was connected to the external capacitance C_{add} (Figure 4). This simple approach allowed simulation of the resonance with satisfactory accuracy (Figure 9) and the construction of a simplified numerical field model of the actual coil in two-dimension cylindrical geometry.

The research in this article can be used in assessment of the FRA signature, especially for simulations of various windings faults and, consequently, for FRA measurement standardization.

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