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A Three-Dimensional Radiation Transfer Model to Evaluate Performance of Compound Parabolic Concentrator-Based Photovoltaic Systems

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Abstract: In the past, two-dimensional radiation transfer models (2-D models) were widely used to investigate the optical performance of linear compound parabolic concentrators (CPCs), in which the radiation transfer on the cross-section of CPC troughs is considered. However, the photovoltaic efficiency of solar cells depends on the real incidence angle instead of the projection incidence angle, thus 2-D models can't reasonably evaluate the photovoltaic performance of CPC-based photovoltaic systems (CPVs). In this work, three-dimensional radiation transfer (3-D model) within CPC- θ_a/θ_e , the CPC with a maximum exit angle θ_e for radiation within its acceptance angle (θ_a), is investigated by means of vector algebra, solar geometry and imaging principle of plane mirror, and effects of geometry of CPV- θ_a/θ_e on its annual electricity generation are studied. Analysis shows that, as compared to similar photovoltaic (PV) panels, the use of CPCs makes the incident angle of solar rays on solar cells increase thus lowers the photovoltaic conversion efficiency of solar cells. Calculations show that, 2-D models can reasonably predict the optical performance of CPVs, but such models always overestimate the photovoltaic performance of CPVs, and even can't predict the variation trend of annual power output of CPV- θ_a/θ_e with θ_e . Results show that, for full CPV- θ_a/θ_e with a given θ_a , the annual power output increases with θ_e first and then comes to a halt as $\theta_e > 83^\circ$, whereas for truncated CPV- θ_a/θ_e with a given geometric concentration (C_t), the annual power output decreases with θ_e .

Keywords: compound parabolic concentrator; concentrating photovoltaic system; three-dimensional radiation transfer; photovoltaic performance; theoretical analysis

1. Introduction

In recent years, solar photovoltaic systems have attracted much attention for electricity generation due to the gradual depletion of traditional fossil fuels and severe environmental issues caused by their use [1]. However, the high cost of the electricity produced by PV systems still limits their broader worldwide application. In China, the cost of electricity generated by PV systems was about 0.12–0.15 \$/kWh in 2016, two times of the cost of electricity generated by conventional electricity generation technologies, and the enterprises have to rely on subsidies from governments to survive. Concentrating solar radiation onto a photovoltaic can reduce the solar area per unit output, thus allowing the cost of the total system to be reduced per unit of energy delivered. Concentrated PV systems are divided into high concentration PV systems (HCPVs) and low concentration PV systems (LCPVs). HCPVs are usually equipped with sophisticated devices for tracking the Sun and cooling the solar cells [2,3], but LCPVs are simple in the structure and easy to control and hence more attractive. A study by Mallick and Eames showed that, compared to similar PV panels, a LCPV can reduce the

cost of electricity by up to 40% [4]. Due to the advantages of non-tracking, high reliability and low-cost, various types of low solar concentrators were developed and tested for the use with solar cells in the past two decades. Among these, the compound parabolic concentrator (CPC) was regarded as to be most suitable and efficient low concentrator.

Mallick and Eames developed an asymmetric CPV with a geometric concentration of 2.01 for building integration in Europe, and a 62% increase in power output was measured as compared to similar PV panel [5,6]. Brogren et al. experimentally tested a CPC with a geometric concentration ratio of 3 for concentrating solar radiation on Cu(In, Ga)Se₂ solar cells, and found that the use of CPC increased the maximum power output of the solar cells by 90% [7]. A comparative study by Yousef et al. under hot and arid climatic conditions showed that, in comparison with similar solar panels, the electricity generated by CPC (2.4×)-based CPV system with and without cooling of solar cells was 52% and 33% higher, respectively [8]. Bahaidarah et al. experimentally investigated the performance of CPC-based PV/T systems with and without glazing and concluded that the use of glass reduced the power output and thus was recommended for higher thermal gain, and the unglazed system was suitable for greater electric power output [9]. Stationary CPCs are commonly oriented in the east-west direction with a yearly fixed tilt angle for efficient solar radiation concentration [10], therefore, the increase in electricity from reflective CPVs is limited due to limited geometric concentration. In order to increase the geometric concentration of CPCs, dielectric totally internally reflecting compound parabolic concentrators (DCPCs) were suggested by Rabl and Winston [11]. Muhammad-Sukki et al. experimentally investigated the performance of a mirror symmetrical dielectric totally internally reflecting CPC (4.9×)-based photovoltaic system for building applications, and found that the use of DCPC made the maximum electrical power output point increase by a factor of 4.2 [12]. Theoretically, the use of a CPC should make the power output of solar cells increase by the factor of its geometric concentration, but the actual increase is much less than this value, a result of the optical loss due to multiple reflections of solar rays on the way to the solar cells and electricity loss mainly resulting from lower photovoltaic conversion efficiency due to higher cell temperature, non-uniform irradiation and increased solar incidence angle [13]. To make the distribution of solar radiation on solar cells more uniform, Su and Li et al. proposed a lens-walled CPC, and such CPC shares the advantages of high optical efficiency, large acceptance angle, and low cost as compared to reflective CPCs and dielectric CPCs [14,15]. A CPC-based PV system with rolling mark on the reflectors was tested by Hatwaambo et al., and found that the use of semi-diffuse reflectors slightly but insignificantly improved the performance of the PV system [16,17]. However, a study by Yu and Tang showed that the use of semi-diffuse reflectors in CPVs can't noticeably improve the uniformity of solar flux distribution except when reflectors with perfect diffuse reflection are employed, which in turn results in a great reduction of annually collected radiation [17]. To simplify the fabrication of CPC-based PV systems and make radiation distribution on solar cells more uniform, a compound plane mirror concentrator (CPC-A) was investigated as an alternative to CPC by Tang et al., who found that the annually radiation collected by optimized CPC-As and CPCs is almost identical [18].

It is well known that the incidence angle and cell temperature have a significant effect on the photovoltaic conversion efficiency of solar cells [19,20]. To ensure LCPVs operate efficiently, their cooling is required and the heat removed from solar cells can be used for other thermal applications; such CPC-based PV/T systems were widely investigated in the past [21–23]. Recently, Baig et al. theoretically and experimentally investigated the performance of a crossed compound parabolic concentrator-based PV system (CCPC) with a thermal extraction unit, and found that such a system had an average electrical efficiency of 10–16% when evaluated at five different locations [24]. However, the effect of solar incidence angle on the photovoltaic performance of CPVs was rarely investigated. The incidence angle of solar rays reflecting from the parabolic reflectors of CPVs gradually increases from the upper tip downwards to the lower end, and radiation reflecting from the parabolic reflector near solar cells is very large, hence can't be efficiently converted into electricity output [25]. The earlier work by Rabl and Winston indicated that the use of CPCs with a restricted exit angle ($\text{CPC-}\theta_a/\theta_e$) can

improve the photovoltaic performance of CPVs thanks to the limited solar incident/exit angle [26]. For CPC- θ_a/θ_e , the solar incidence angle on the absorber is limited within θ_e for the radiation within its acceptance angle (θ_a), and common CPC, the one without restriction of exit angle, is a special case of CPC- θ_a/θ_e for $\theta_e = 90^\circ$ [27]. An comparative study conducted by Yu et al. showed that, in comparison with CPV-20/90, power output increases of 2.1%, 5.4% and 8.17% from CPV-20/65 were measured for projected incident angle $\theta_p = 0^\circ, 10^\circ$ and 16° , respectively. Yu also found that, for a CPV- θ_a/θ_e , the solar flux distribution on solar cells had an insignificant effect on the power output, but the solar incident angle had a significant effect [28].

For linear CPCs, the projected angles of incident and reflected rays (all projected angles mentioned in this work is the angle of projection of solar rays on the cross-section of the CPC-trough relative to the normal of the CPC's aperture) are identical [29], therefore theoretical analysis on the optical performance based on radiation transfer on cross-section of CPCs (2-D model) is reasonable, hence its wide use in the past. Recently, Yu et al. proposed a 2-D model to investigate the annual power output of CPV- θ_a/θ_e based on the angular distribution of annual collectible radiation, and found that, with increase of θ_e , the annual electricity generated by E-W CPV- θ_a/θ_e decreased, except full CPV- θ_a/θ_e with the aperture's tilt-angle being yearly adjusted four times at three tilts [27]. However, the solar incident angle mentioned in the paper is the projection angle, but the efficiency of solar cells is dependent on the real solar incidence angle [28,30]. Therefore, to reasonably evaluate the photovoltaic performance of CPVs, a three-dimensional radiation transfer model is required. Theoretically, using three-dimensional ray tracing analysis to investigate the long-term performance of CPVs is feasible, but not practical due to the numerous ray-tracing analyses necessary to find the angular dependence of the photovoltaic efficiency of solar cells on both real and projected incident angles because of time variation of the sun's position over the sky dome in a day and daily variation of the Sun's declination [31]. In this work, a trial is first made to suggest a mathematical procedure to find the real solar incidence angle of all radiation on solar cells of CPV- θ_a/θ_e for solar radiation from any direction of the sky based on three-dimensional radiation transfer within CPVs (3-D model) by means of vector algebra, convenient but powerful, solar geometry as well imaging principle of plane mirror. The objective of this work is to study the effect of θ_e on the annual electricity generation from CPV- θ_a/θ_e as well the reasonability of the 2-D model proposed by Yu [27] for evaluating the performance of CPVs.

2. Optical and Photovoltaic Conversion Efficiency of CPVs

2.1. Equation of Reflectors of CPC- θ_a/θ_e

As shown in Figure 1, CPC- θ_a/θ_e consists of an upper parabola and lower plane mirrors. To make analysis easier and convenient, the width of the absorber/solar cell of the CPVs is set to be 1, and the right parabola in the Cartesian coordinates system shown in Figure 1 is expressed by Equation (1):

$$\begin{cases} z = \frac{(\sin \theta_e + \sin \theta_a) \sin \phi}{1 - \cos(\phi + \theta_a)} - 0.5 \\ x = \frac{(\sin \theta_e + \sin \theta_a) \cos \phi}{1 - \cos(\phi + \theta_a)} \end{cases} \quad (\theta_t \leq \phi \leq \theta_e) \quad (1)$$

where ϕ is the polar angle of any point on the parabolic reflector relative to the x-axis; θ_a the acceptance half-angle, θ_t the edge-ray angle of CPC- θ_a/θ_e after being truncated with a geometric concentration ratio of C_t . For full CPC- θ_a/θ_e , $\theta_t = \theta_a$ and $C_t = \sin \theta_e / \sin \theta_a$; whereas for truncated CPCs with a given C_t , the θ_t is calculated based on Equation (1) [25]. Obviously, the CPC- $\theta_a/90$, the one without restriction of exit angle, is a special case of CPC- θ_a/θ_e for $\theta_e = 90^\circ$. The plane mirror of CPC- θ_a/θ_e is expressed by:

$$x = c \tan \gamma_{pl} (z - 0.5) (0.5 \leq z \leq z_D) \quad (2)$$

and γ_{pl} , the tilt-angle of plane mirrors relative to the x-axis, is given by Equation (3) [27]:

$$\gamma_{pl} = 0.5(\theta_e - \theta_a) \quad (3)$$

The terms z_D and x_D are the z- and x- component of the lower end (D) of the right parabola, respectively, and calculated from Equation (1) by setting $\phi = \theta_e$.

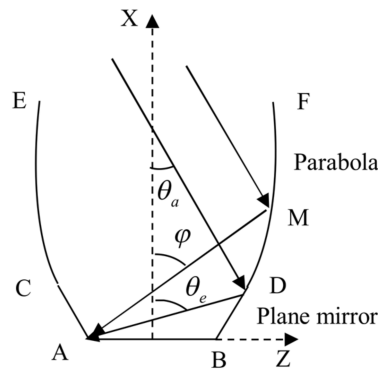


Figure 1. Geometry of CPC- θ_a/θ_e .

2.2. Coordinate System to Determine the Vector of Solar Rays

CPC-troughs are usually oriented in the east-west direction with the aperture tilted at β from the horizon. For analysis convenience, a coordinate system with the x-axis normal to the aperture, y-axis pointing to east and z-axis pointing to northern sky is introduced (see Figure 2). The unit vector of incident solar ray at any time of a day in this coordinate system is expressed [31,32]:

$$n_s = (n_x, n_y, n_z) \quad (4)$$

where:

$$n_x = \cos \delta \cos \omega \cos(\lambda - \beta) + \sin \delta \sin(\lambda - \beta) \quad (5a)$$

$$n_y = -\cos \delta \sin \omega \quad (5b)$$

$$n_z = -\cos \delta \cos \omega \sin(\lambda - \beta) + \sin \delta \cos(\lambda - \beta) \quad (5c)$$

where λ is the site latitude, ω the solar hour angle, and δ the declination of the Sun which vary with day number counted from the first day of a year [31]. The irradiation situation within linear CPCs is uniquely determined by the projected angle of incident rays (θ_p), and is given by Equation (6):

$$\tan \theta_p = \left| \frac{n_z}{n_x} \right| \quad (6)$$

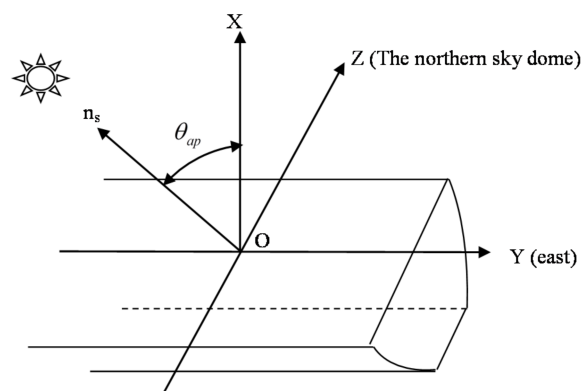


Figure 2. Unit vector of incident solar rays in the suggested coordinate system.

For symmetric CPCs, the optical and photovoltaic performance for $n_s = (n_x, n_y, \pm n_z)$ are identical. Thus, to make the analysis convenient and simple, the vector of the incident solar radiation at any time is set to be $n_s = (n_x, n_y, -|n_z|)$, i.e., in this work solar rays are assumed to be always incident towards onto the right reflector of CPCs (see Figure 3).

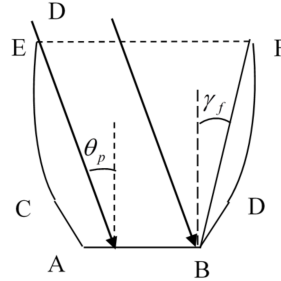


Figure 3. Radiation directly irradiating on solar cells.

2.3. Photovoltaic Conversion Efficiency of Solar Cells

In general, the electricity from a CPV is affected by many factors such as cell temperature and electric load [33]. To simplify the analysis but make our comparative study of the electricity from different CPVs meaningful, it is assumed that, except for the solar incidence angle (θ_{in}), the effects of all other factors on the photovoltaic efficiency of CPVs with different geometry are identical, and the photovoltaic efficiency of solar cells is subjected to the following correlation, suggested based on experimental measurements of PV panel under outdoor conditions by Yu et al. [28]:

$$\eta_{pv} = 15.5494 + 0.02325\theta_{in} - 0.00301\theta_{in}^2 + 9.4685 \times 10^{-5}\theta_{in}^3 - 1.134 \times 10^{-6}\theta_{in}^4 \quad (0 < \theta_{in} < 65^\circ) \quad (7a)$$

$$\eta_{pv} = 41.52 - 0.4784\theta_{in} \quad (65^\circ < \theta_{in} < 90^\circ) \quad (7b)$$

2.4. Optical and Photovoltaic Conversion Efficiency of CPVs

For CPV- θ_a/θ_e , the incidence angle of solar rays reflecting from different parts, even different points of reflectors, is different. For convenience of the analysis, the radiation on solar cells is divided into six parts: radiation directly irradiating on solar cells (I_1), radiation incident on right plane mirror and arriving on solar cells after reflection (I_2), radiation incident on right parabolic reflector and arriving on solar cells after multiple reflections (I_3), radiation on upper right parabola at $\theta_p > \theta_a$ and arriving on solar cells after reflection from parabola first then from left mirror (I_4), radiation on left mirror and arriving on solar cells after reflection (I_5) and that incident on left parabola and arriving on solar cells after multiple reflections (I_6). Hence, the optical efficiency factor of CPV- θ_a/θ_e is given by:

$$f = \frac{I_1 + I_2 + I_3 + I_4 + I_5 + I_6}{I_{ap}} = f_1 + f_2 + f_3 + f_4 + f_5 + f_6 \quad (8)$$

where I_{ap} is the radiation on the aperture of CPVs; f_i ($i = 1, 2, 3, 4, 5, 6$) is the energy fraction of the radiation on solar cells contributed by I_i . Similarly, the photovoltaic conversion efficiency of CPV- θ_a/θ_e can be expressed by:

$$\eta = \frac{P_1 + P_2 + P_3 + P_4 + P_5 + P_6}{I_{ap}} = \eta_1 + \eta_2 + \eta_3 + \eta_4 + \eta_5 + \eta_6 \quad (9)$$

where P_i ($i = 1, 2, 3, 4, 5, 6$) is the power output from CPVs due to the contribution of I_i , and the η_i ($i = 1, 2, 3, 4, 5, 6$) is the photovoltaic conversion efficiency of CPVs contributed by P_i .

2.4.1. Calculation of f_1 and η_1

As seen from Figure 3, the solar cell of CPVs is fully directly irradiated as $\theta_p \leq \gamma_f$ and partially irradiated as $\gamma_f < \theta_p \leq \theta_t$, thus, the energy fraction of the radiation directly irradiating on solar cells of CPVs is simply expressed by $f_1 = I_1/I_{ap} = \Delta z_1/C_t$ as follows:

$$f_1 = \begin{cases} 1/C_t & (\theta_p \leq \gamma_f) \\ 0.5(1 + C_t)(1 - \tan \theta_p / \tan \theta_t)/C_t & (\gamma_f < \theta_p \leq \theta_t) \\ 0 & \text{else} \end{cases} \quad (10)$$

where γ_f , the angle that the solar cells of CPVs is fully irradiated for $\theta_p \leq \gamma_f$, is calculated by Equation (11):

$$\tan \gamma_f = (C_t - 1) \tan \theta_t / (C_t + 1) \quad (11)$$

$\theta_{in,1}$, the solar incident angle on solar cells, is given by:

$$\cos \theta_{in,1} = n_s \cdot n_{abs} = n_s \cdot (1, 0, 0) = n_x \quad (12)$$

where $n_{abs} = (1, 0, 0)$ is the vector of normal to solar cells of CPVs. Therefore, η_1 is given by:

$$\eta_1 = \Delta z_1 \eta_{pv}(\theta_{in,1}) / C_t = f_1 \eta_{pv}(\theta_{in,1}) \quad (13)$$

where $\eta_{pv}(\theta_{in,1})$, the photovoltaic efficiency of solar cells as the function of $\theta_{in,1}$, is estimated by Equation (7).

2.4.2. Calculation of f_2 and η_2

In this work, CPV- θ_a/θ_e with $\theta_a < 2\theta_e - 0.5\pi$ is investigated, and for such CPVs, radiation irradiating on plane mirrors will arrive on solar cells after one reflection as indicated by Yu et al. [27]. The imaging principle of plane mirrors indicates that solar rays pointing to solar cells' image formed by a plane mirror will hit on solar cells after reflection, therefore, for $\theta_p \leq \theta_a$, all radiation irradiating on the entire right mirror (BD) will strike on the solar cells after reflection (see Figure 4a); for $\theta_a < \theta_p \leq \theta_t$, the radiation irradiating on lower part of mirror (MB) (see Figure 4b), will redirect onto the solar cells, and for $\theta_t < \theta_p < \theta_{p,c1}$, the plane mirror is partially irradiated, thus only radiation irradiating on the middle part of the mirror will redirect onto the solar cells (see Figure 4c), whereas for $\theta_p > \theta_{p,c1}$, no radiation arrives on solar cells from the plane mirror (see Figure 4d). Hence, the energy fraction of radiation irradiating on the right mirror and arriving on solar cells after one reflection is given by:

$$f_2 = \Delta z_2 \rho / C_t \quad (14a)$$

and:

$$\Delta z_2 = \begin{cases} z_D - 0.5 + x_D \tan \theta_p & \theta_p \leq \theta_a \\ \cos \psi - \sin \psi \tan \theta_p & \theta_a < \theta_p \leq \theta_t \\ (h_t + \sin \psi)(\tan \varphi_{ap} - \tan \theta_p) & \theta_t < \theta_p \leq \theta_{p,c1} \\ 0 & \theta_p > \theta_{p,c1} \end{cases} \quad (14b)$$

where ρ is the reflectivity of reflectors; $h_t = 0.5(C_t + 1)/\tan \theta_t$ the height of CPVs; $\psi = 2\gamma_{pl}(\theta_e - \theta_a)$ the opening angle of V-trough formed by two plane mirrors of CPVs; $\theta_{p,c1} = \text{Min}(\varphi_{ap}, 0.5\pi - \psi)$ the critical angle, and for radiation at $\theta_p \leq \theta_{p,c1}$, no radiation arrives on solar cells from plane mirrors. As shown in Figure 4d, the φ_{ap} in Equation (14b) is calculated by:

$$\tan \varphi_{ap} = \frac{0.5 + \cos \psi + 0.5C_t}{h_t + \sin \psi} \quad (15)$$

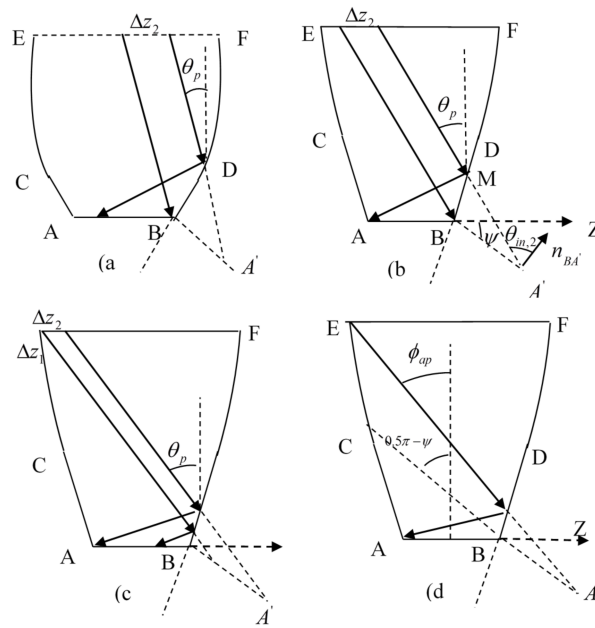


Figure 4. Radiation incident on right mirror and arriving on solar cells after reflection for (a) $\theta_p \leq \theta_a$; (b) $\theta_a < \theta_p \leq \theta_t$; (c) $\theta_t < \theta_p \leq \theta_{p,c1}$; (d) Critical angle $\theta_{p,c1} = \text{Min}(\phi_{ap}, 0.5\pi - \psi)$.

The photovoltaic efficiency of CPVs due to contribution of I_2 is given by:

$$\eta_2 = \Delta z_2 \rho \eta_{pv}(\theta_{in,2}) / C_t = f_2 \eta_{pv}(\theta_{in,2}) \quad (16)$$

where $\theta_{in,2}$ is the incidence angle of solar rays reflecting from the right plane mirror, and can be directly calculated based on n_s and $n_{BA'} = (\cos\psi, 0, \sin\psi)$, the vector of normal to solar cells' image BA' (see Figure 4b), as:

$$\cos \theta_{in,2} = n_s \cdot n_{BA'} = n_x \cos \psi + n_z \sin \psi \quad (17)$$

2.4.3. Calculation of f_3 and η_3

As well known, for ideal solar concentration, all solar rays within its acceptance angle (i.e., $\theta_p \leq \theta_a$) will arrive on solar cells after more than one reflection, but the solar incidence angle on solar cells differs for radiation incident on different point of the parabolic reflectors. Hence, to find the power output of CPVs contributed by I_3 , the finite element method must be employed. As shown in Figure 5, the radiation incident on a finite element around M of parabola and arriving on solar cells after more than one reflection is given by:

$$\Delta I_3 = \delta z_3 \eta_{opt,3} \quad (18)$$

and δz_3 is given by:

$$\delta z_3 = -(dz + dx \tan \theta_p) \quad (19)$$

The “-” in Equation (19) is a result of the fact that x- and z-components of point M decrease with ϕ_M . The $\eta_{opt,3}$ in Equation (18), the optical efficiency of CPVs for radiation irradiating on finite element around M, is dependent on reflection number of solar ray on way to solar cells and can be calculated by the mathematical procedure presented in Appendix A. The power output due to contribution of ΔI_3 is as:

$$\Delta P_3 = \Delta I_3 \eta_{pv}(\theta_{in,3}) = \delta z_3 \eta_{opt,3} \eta_{pv}(\theta_{in,3}) \quad (20)$$

where $\theta_{in,3}$, the solar incidence angle on cells, is determined by the method in Appendix A. The total radiation received by solar cells is calculated by integrating $\Delta I_3 = \delta z_3 \eta_{opt,3}$ from $\phi = \theta_t$ to θ_e , thus f_3 is expressed by:

$$f_3 = - \int_{\phi=\theta_t}^{\phi=\theta_e} (dz + dx \tan \theta_p) \eta_{opt,3} / C_t \quad (21)$$

Similarly, one has:

$$\eta_3 = - \int_{\phi=\theta_t}^{\phi=\theta_e} (dz + dx \tan \theta_p) \eta_{opt,3} \eta_{pv}(\theta_{in,3}) / C_t \quad (22)$$

It must be noted that f_3 and η_3 are zero for $\theta_p > \theta_a$ because no radiation reflected from the parabola arrives on the solar cells in this case, except for radiation irradiating on the upper (right) parabolic reflector which is included in I_4 as discussed in the next section.

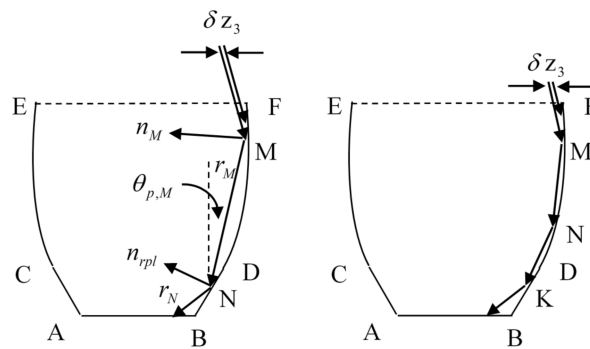


Figure 5. Radiation irradiating on a finite element of right parabola at $\theta_p \leq \theta_a$ and arriving on solar cells after two reflections (left) and three reflections (right).

2.4.4. Calculation of f_4 and η_4

As shown in Figure 6, the solar ray striking at tip (F) of right parabola at $\theta_p = \theta_{p,c2}$ just redirects to the end (B') of the solar cells' image first, then hits at end (B) of the solar cells after two reflections. Hence, solar rays irradiating on upper part (FK) of right parabola at $\theta_a < \theta_p < \theta_{p,c2}$ will arrive on solar cells after two reflections [27]. According to the principle that projected angles of incident and reflective rays on the cross-section of CPVs are identical [27], one has:

$$\theta_{p,c2} = \phi_{ap} - 2\gamma_{ap} \quad (23)$$

where γ_{ap} , the tilt-angle of tangent line to parabolic reflectors at the upper tip, is determined by Equation (A2) in Appendix A. As shown in Figure 6, the ϕ_K of point K, from which solar ray reflecting just points to end B' of solar cells' image for $\theta_a < \theta_p < \theta_{p,c2}$, is subjected to following equation group:

$$\begin{cases} \theta_{p,K} = 2\gamma_K + \theta_p \\ \tan \gamma_K = (\cos \theta_a - \cos \phi_K) / (\sin \theta_a + \sin \phi_K) \\ \tan \theta_{p,K} = (z_K + 0.5 + \cos \psi)(x_K + \sin \psi) \end{cases} \quad (24)$$

Given θ_p and the geometry of CPVs, ϕ_K can be obtained by iterative calculations from Equation (24). As shown in Figure 6, the radiation incident on the finite element around point M of right parabola and arriving on solar cells after two reflections is calculated by:

$$\Delta I_4 = -(dz + dx \tan \theta_p) \rho^2 \quad (25)$$

The power output due to contribution of ΔI_4 is given by:

$$\Delta P_4 = \Delta I_4 \eta_{pv}(\theta_{in,4}) = -(dz + dx \tan \theta_p) \rho^2 \eta_{pv}(\theta_{in,4}) \quad (26)$$

where $\theta_{in,A}$, the incident angle on solar cells, is calculated based on r_M and $n_{AB'} = (\cos\psi, 0, -\sin\psi)$, the vector of normal to solar cells' image (AB'), as:

$$\cos \theta_{inA} = -r_M \cdot (\cos \psi, 0, -\sin \psi) = \cos \theta_{r,M} \cos \psi - (n_z + 2 \cos \theta_{i,M} \cos \gamma_M) \sin \psi \quad (27)$$

Substituting Equations (A1) and (A4) in Appendix A into above obtains:

$$\cos \theta_{in,4} = n_x \cos(\psi + 2\gamma_M) + n_z \sin(\psi + 2\gamma_M) \quad (28)$$

The total radiation that incident on the upper parabola at $\theta_a < \theta_p < \theta_{p,c2}$ and arrive on solar cells, I_4 , is calculated by integrating Equation (25) from $\varphi = \theta_t$ to φ_K , hence f_4 is given by:

$$f_4 = \begin{cases} -\int_{\theta_t}^{\phi_K} (dz + dx \tan \theta_p) \rho^2 / C_t & (\theta_a < \theta_p < \theta_{p,c2}) \\ 0 & else \end{cases} \quad (29)$$

Similarly, one has:

$$\eta_4 = \begin{cases} -\int_{\theta_t}^{\phi_K} (dz + dx \tan \theta_p) \rho^2 \eta_{pv}(\theta_{in,4}) / C_t & (\theta_a < \theta_p < \theta_{p,c2}) \\ 0 & else \end{cases} \quad (30)$$

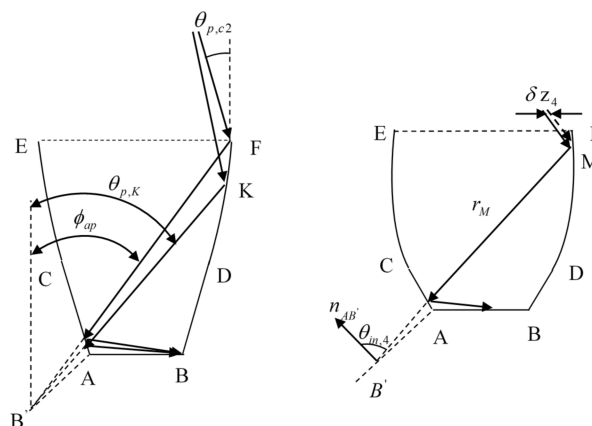


Figure 6. (Left) Scheme of critical angle $\theta_{p,c2}$ and critical point K; (Right) Radiation incident at $\theta_a < \theta_p < \theta_{p,c2}$ on a finite element of right upper parabola and arriving on solar cells after reflection from parabolic reflector first then from left mirror.

2.4.5. Calculation of f_5 and η_5

As seen in Figure 7, the left mirror of CPVs is fully irradiated as $\theta_p \leq \gamma_{fd}$ and partially irradiated as $\gamma_{fd} < \theta_p \leq \gamma_f$, hence, energy fraction of radiation irradiating on left plane mirror and arriving on solar cells after reflection is as follows:

$$f_5 = \Delta z_5 \rho / C_t \quad (31a)$$

and:

$$\Delta z_5 = \begin{cases} z_D - 0.5 - x_D \tan \theta_p & \theta_p \leq \gamma_{fd} \\ 0.5(C_t - 1) - h \tan \theta_p & \gamma_{fd} < \theta_p \leq \gamma_f \\ 0 & \theta_p > \gamma_f \end{cases} \quad (31b)$$

$$\tan \gamma_{fd} = (0.5C_t - z_D)/(h_t - x_D) \quad (31c)$$

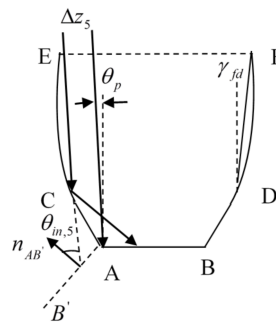


Figure 7. Transfer of radiation irradiating on left mirror for $\theta_p \leq \gamma_{fd}$.

The power output of CPVs due to contribution of $I_5 = \Delta z_5 \rho$ is calculated by:

$$P_5 = \Delta z_5 \rho \eta_{pv}(\theta_{in,5}) \quad (32)$$

Therefore, the photovoltaic efficiency of CPVs due to contribution of P_5 is given by:

$$\eta_5 = P_5 / C_t = f_5 \eta_{pv}(\theta_{in,5}) \quad (33)$$

As seen in Figure 7, the solar incident angle, $\theta_{in,5}$, is calculated based on n_s and $n_{AB'} = (\cos\psi, 0, -\sin\psi)$, the vector of normal to solar cells' image AB' , as:

$$\cos \theta_{in,2} = n_x \cos \psi = n_z \sin \psi \quad (34)$$

2.4.6. Calculation of f_6 and η_6

As seen in Figure 8, the left parabola is fully irradiated as $\theta_p \leq \gamma_{ap}$, partially irradiated as $\gamma_{ap} < \theta_p < \gamma_{fd}$, and fully shaded by itself for $\theta_p \geq \gamma_{fd}$. For radiation incident at $\gamma_{ap} < \theta_p < \gamma_{fd}$, the lower left parabola is irradiated, and critical point V is subjected to following equation:

$$\tan \theta_p = (0.5C_t - z_V) / (h_t - x_V) \quad (35)$$

and z_V and x_V as the function of φ_V are subjected to Equation (1). Given θ_p and geometry of CPVs, for $\gamma_{ap} < \theta_p < \gamma_{fd}$, φ_V can be obtained from Equation (35), whereas for $\theta_p \leq \gamma_{ap}$, $\varphi_V = \theta_t$. For symmetric CPVs, the optical and photovoltaic performance for $n_s = (n_x, n_y, \pm n_z)$ are identical. As seen from Figure 9, given φ_M , the radiation incident on a finite element around M of left parabola at θ_p and arriving on solar cells after more than one reflections is equal to that incident on a finite element around M of right parabola at $-\theta_p$ and arriving on solar cells after more than one reflection. Therefore, f_6 and η_6 can be simply obtained based on the method proposed to find f_3 and η_3 by setting $\theta_p = -\theta_p$ and $n_z = |n_z|$ as follows:

$$f_6 = - \int_{\phi=\phi_V}^{\phi=\theta_e} (dz - dx \tan \theta_p) \eta_{opt,6} / C_t \quad (36)$$

$$\eta_6 = - \int_{\phi=\phi_V}^{\phi=\theta_e} (dz - dx \tan \theta_p) \eta_{opt,6} \eta_{pv}(\theta_{in,6}) / C_t \quad (37)$$

where $\theta_{in,6}$ and $\eta_{opt,6}$ are the solar incident angle and optical efficiency of CPVs for solar rays incident on a finite element around M of left parabola, respectively, and calculated based on the method to find $\theta_{in,3}$ and $\eta_{opt,3}$ in Appendix A by setting $\theta_p = -\theta_p$ and $n_z = |n_z|$ in relevant expressions. It must be noted that f_6 and η_6 are zero for the case of $\theta_p \geq \gamma_{fd}$.

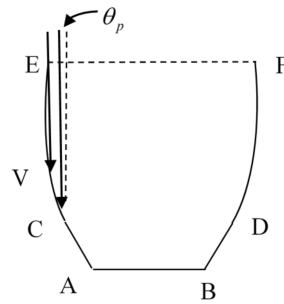


Figure 8. Irradiation situation of left parabolic reflector for $\theta_p > \gamma_{fd}$.

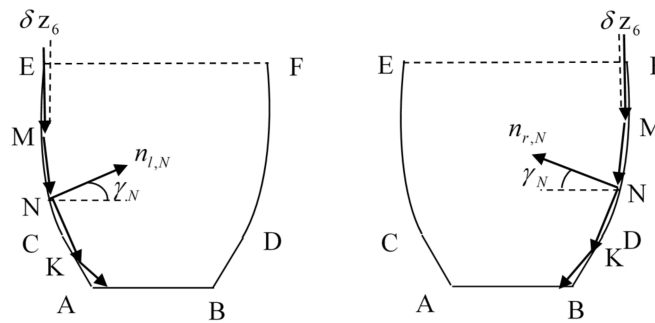


Figure 9. (Left) Transfer of radiation incident on a finite element around M of left parabola at θ_p ; (Right) Transfer of radiation incident on a finite element around M of right parabola at $-\theta_p$.

The analysis in the above indicates that, given the geometry of CPVs, the optical efficiency of CPVs only depends on θ_p because the irradiation situation within CPVs and reflection number of solar rays on their way to the solar cells are uniquely determined by θ_p [13], but the photovoltaic efficiency of CPVs is dependent on both θ_p and θ_{ap} as the solar incidence angle is dependent on n_x and n_z . The analysis in the Appendix A shows that the use of CPCs makes the incidence angle on solar cells increase, leading the photovoltaic efficiency of CPVs to decrease in comparison to similar PV panels.

3. Annual Optical and Photovoltaic Performance of CPVs

It is assumed that the CPV- θ_a/θ_e is oriented in the east-west direction, the length is considered to be infinite, and radiation from the ground is neglected, hence, radiation received by unit area of solar cells at any time of a day is given by:

$$I = C_t I_b g(\theta_{ap}) f \cos \theta_{ap} + I_{abs,d} = C_t I_b g(\theta_{ap}) n_x f + I_{abs,d} \quad (38)$$

The electricity generated by unit area of solar cells of CPVs at any time in a day is expressed by:

$$P = C_t I_b g(\theta_{ap}) n_x \eta + P_d \quad (39)$$

where I_b is the intensity of beam radiation, θ_{ap} , subjected to $\cos \theta_{ap} = n_x$, is the solar incident angle on aperture of CPVs, $g(\theta_{ap})$, being 1 for $n_x > 0$ otherwise zero, a control function. The $I_{abs,d}$ in Equation (38) is the sky diffuse radiation collected by unit area of solar cells, and P_d in Equation (39) is the power output generated by $I_{abs,d}$.

To calculate $I_{abs,d}$ and P_d , isotropic sky diffuse radiation is assumed, thus the sky diffuse radiation from a finite element on the sky dome and received by unit area of solar cells (see Figure 10) is:

$$dI_{abs,d} = C_t i_d \cos \theta f d\Omega \quad (40)$$

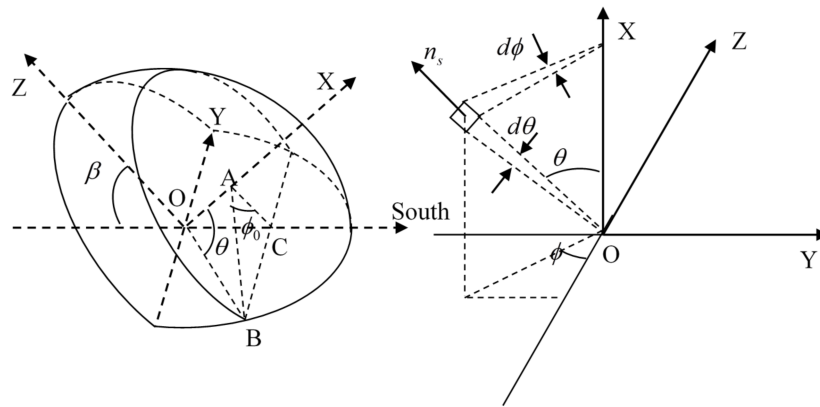


Figure 10. Sky dome viewed from the aperture of CPVs (Left) and vector of diffuse radiation from a finite element on the sky dome (Right).

The power output generated by $dI_{abs,d}$ is expressed by:

$$dP_d = C_t i_d \cos \theta \eta d\Omega \quad (41)$$

where $d\Omega = \sin \theta d\theta d\phi$ is the solid angle covered by a finite element on the sky dome, and the vector of sky diffuse radiation from the finite element (see Figure 10) is expressed by:

$$n_s = (\cos \theta, -\sin \theta \sin \phi, -|\sin \theta \cos \phi|) \quad (42)$$

The i_d in Equation (41) is the directional intensity of sky diffuse radiation, and can be determined based on the horizontal sky diffuse radiation I_d as:

$$I_d = \int_0^{2\pi} d\phi_s \int_0^{0.5\pi} i_d \cos \theta_i \sin \theta_i d\theta_i = \pi i_d \quad (43)$$

Thus, one has $i_d = I_d / \pi$, and $I_{abs,d}$ and P_d can be respectively calculated by integrating $dI_{abs,d}$ and dP_d over the sky dome viewed from the aperture of CPVs as:

$$I_{abs,d} = C_t i_d \iint \sin \theta \cos \theta f d\theta d\phi \quad (44)$$

$$P_d = C_t i_d \iint \sin \theta \cos \theta \eta d\theta d\phi \quad (45)$$

As shown in Figure 10, for the sky dome over the aperture, θ varies from 0 to 0.5π , and ϕ varies from ϕ_0 to $2\pi - \phi_0$. For $\theta \leq 0.5\pi - \beta$, $\phi_0 = 0$; whereas for $0.5\pi - \beta < \theta \leq 0.5\pi$, it is given by:

$$\cos \phi_0 = \frac{\tan(0.5\pi - \beta)}{\tan \theta} = \cot \theta \tan \beta \quad (0.5\pi - \beta < \theta \leq \pi) \quad (46)$$

Therefore, Equations (44) and (45) can be rewritten as:

$$I_{abs,d} = \frac{2C_t I_d}{\pi} \int_{\phi_0}^{\pi} d\phi \int_0^{0.5\pi} \sin \theta \cos \theta f d\theta = C_d I_d \quad (47)$$

$$P_d = \frac{2C_t I_d}{\pi} \int_{\phi_0}^{\pi} d\phi \int_0^{0.5\pi} \sin \theta \cos \theta \eta d\theta = C_{d,pv} I_d \quad (48)$$

where:

$$C_d = \frac{2C_t}{\pi} \int_{\phi_0}^{\pi} d\phi \int_0^{0.5\pi} \sin \theta \cos \theta f d\theta \quad (49)$$

$$C_{d,pv} = \frac{2C_t}{\pi} \int_{\varphi_0}^{\pi} d\varphi \int_0^{0.5\pi} \sin \theta \cos \theta \eta d\theta \quad (50)$$

Given β , C_d and $C_{d,pv}$ are constants, and can be calculated by numerical calculations. For similar solar panels without using CPCs, the power output at any time of a day is given by:

$$P_0 = I_b g(\theta_{ap}) \eta_{pv}(\theta_{ap}) n_x + C_{d0,pv} I_d \quad (51a)$$

$$C_{d0,pv} = \frac{2}{\pi} \int_{\varphi_0}^{\pi} d\varphi \int_0^{0.5\pi} \sin \theta \cos \theta \eta_{pv}(\theta) d\theta \quad (51b)$$

If two-dimensional sky diffuse radiation is used to calculate $I_{abs,d}$ and P_d , the vector of diffuse radiation from a finite element of sky vault is $n_s = (\cos \theta_p, 0, |\sin \theta_p|)$ as seen in Figure 11, and $I_{abs,d}$ is given by:

$$I_{abs,d} = C_t \left[\int_0^{0.5\pi} i_p \cos \theta_p f d\theta_p + \int_0^{0.5\pi-\beta} i_p \cos \theta_p f d\theta_p \right] = C_d I_d \quad (52)$$

where $i_p = 0.5I_d$ [13], and C_d is given by:

$$C_d = 0.5 \left[\int_0^{0.5\pi} \cos \theta_p f d\theta_p + \int_0^{0.5\pi-\beta} \cos \theta_p f d\theta_p \right] \quad (53)$$

Similarly:

$$P_d = C_{d,pv} I_d \quad (54)$$

$$C_{d,pv} = 0.5 \left[\int_0^{0.5\pi} \cos \theta_p \eta d\theta_p + \int_0^{0.5\pi-\beta} \cos \theta_p \eta d\theta_p \right] \quad (55)$$

Calculations show that C_d calculated based on two- and three-dimensional isotropic sky diffuse radiation is completely identical, but $C_{d,pv}$ calculated based on two-dimensional sky diffuse radiation is about 9% higher than that calculated based on three-dimensional sky diffuse radiation as seen in Figure 12. This means that, two-dimensional sky diffuse radiation can be used to estimate the sky diffuse radiation received by solar cells of CPVs, but not suitable for the estimation of P_d .

The daily radiation collected by unit area of solar cells of CPVs is estimated by integrating Equation (38) over the daytime as:

$$H_{day} = C_t \int_{-t_0}^{t_0} I_b g(\theta_{ap}) n_x f dt + C_d H_d \quad (56)$$

The daily electricity output of CPVs is given by integrating Equation (39) over the daytime:

$$P_{day} = C_t \int_{-t_0}^{t_0} I_b g(\theta_{ap}) n_x \eta dt + C_{d,pv} H_d \quad (57)$$

For similar PV panels without using CPCs, the daily electricity generation is given by:

$$P_{day,0} = \int_{-t_0}^{t_0} I_b g(\theta_{ap}) n_x \eta_{pv}(\theta_{ap}) dt + C_{d0,pv} H_d \quad (58)$$

The H_d in the above expressions is the daily sky diffuse radiation on the horizon, t_0 is the sunset time on the horizon in the day. At any time of a day, the position of the sun in terms of n_s can be determined, then f and η can be calculated. Therefore, given H_d and the time variation of I_b in a day, H_{day} , P_{day} and $P_{day,0}$ can be numerically obtained, then summing H_{day} , P_{day} and $P_{day,0}$ in all days of a year yields annual radiation on solar cells (S_a) and annual power output P_a and $P_{a,0}$.

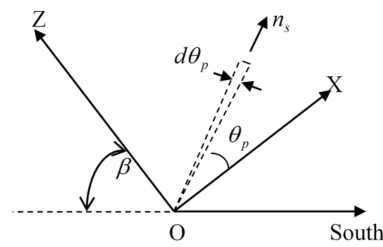


Figure 11. Vector of diffuse radiation from a finite element of two-dimensional sky.

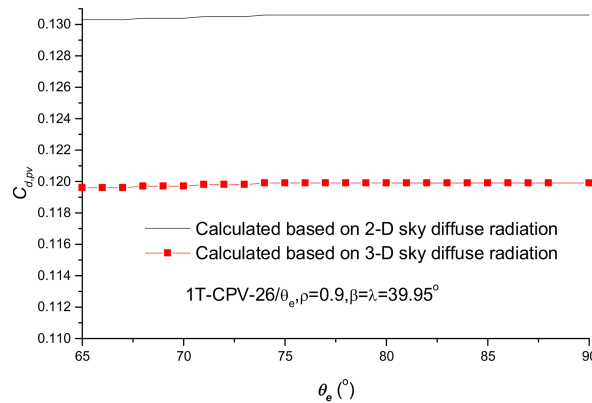


Figure 12. $C_{d,pv}$ of 1T-CPV-26/ θ_e calculated based 2-D and 3-D sky diffuse radiation.

4. Numerical Approach

In this work, monthly average daily global radiation on the horizon in Beijing ($\lambda = 39.95^\circ$) was used for calculations [34], and monthly average H_d and time variation of I_b in a day of the month are estimated based on correlations proposed by Collares-Pereira and Rabl [35]. The steps of φ , θ and ϕ for calculating f , η , C_d and $C_{d,pv}$ are set to be 0.1° ; the time interval for calculating the daily radiation on solar cells and daily power output is set to be 1 min. To fully study the effect of geometry of CPV- θ_a/θ_e on the performance, CPVs with aperture's tilt-angle being yearly fixed (1T-CPVs) and yearly adjusted four times at three tilts (3T-CPVs) are considered. For 1T-CPVs, the tilt-angle of CPVs' aperture (β) is taken to be site latitude (λ) [10]; whereas for 3T-CPVs, β is set to be λ during the period of 23 days before and after both equinoxes, and adjusted to be $\lambda - 22$ and $\lambda + 22$ in summers and winters, respectively [13,18]. All calculations are done using Visual Basic coding programmed by the authors based on the mathematical expressions presented in this work.

5. Results and Discussions

5.1. Annual Collectible Radiation

As seen from Figures 13 and 14, for full CPVs with a given θ_a , the annual radiation on solar cells of CPVs increases with the increase of θ_e due to the increase of geometric concentration factor; whereas for truncated CPVs with given C_t and θ_a , the annual collectible radiation is almost kept unchanged. It is seen from Figures 13 and 14 that the annual collectible radiation expected by the 3-D model is slightly lower than that expected by the 2-D model of Yu [27], and the deviation of S_a expected by 2-D model of Yu from that expected by 3-D model increases with θ_e . For full and truncated 1T-CPV-26/ θ_e , the maximum deviation is 0.35% and 0.18%, respectively; whereas for full and truncated 3T-CPV-26/ θ_e , the maximum deviation is about 0.95% and 0.24%, respectively. This is because the one-reflection model was employed in 2-D model of Yu, and optical loss due to multiple reflections of solar rays on way to solar cells of CPVs is not considered. These results show that 2-D model proposed by Yu et al. [27] can reasonably predict the optical performance of CPVs.

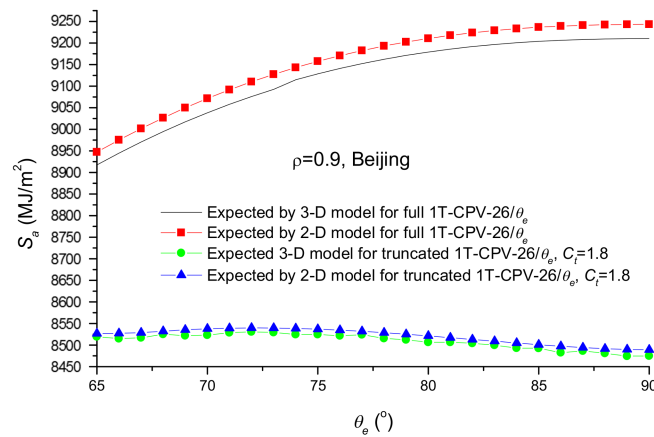


Figure 13. Effects of θ_e on annual radiation collection of 1T-CPV-26/ θ_e .

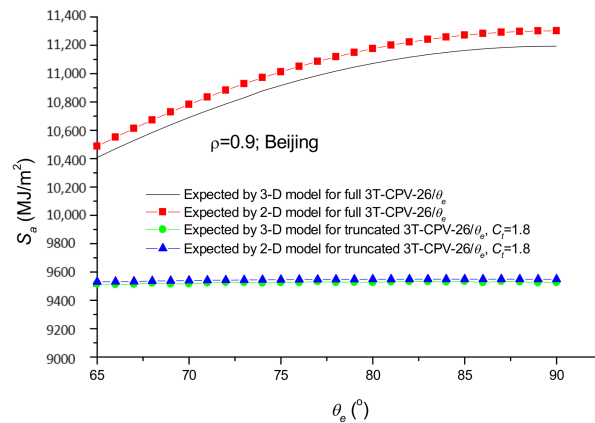


Figure 14. Effects of θ_e on annual collectible radiation of 3T-CPV-26/ θ_e .

Figure 15 presents comparisons of S_a and P_a of full 3T-CPV-20/ θ_e calculated based on the 3-D model of this work incorporated with two-reflection and three-reflection models. It is seen that the deviation of S_a and P_a estimated by two-reflection model from those estimated by three-reflection model are considerably small, and the maximum deviation of S_a and P_a is about 0.18% and 0.26%, respectively. This means that, 3-D model incorporated with two-reflection model can accurately predict the performance of CPVs. Consequently, this work the 3-D model incorporated with three-reflection model is used for calculations.

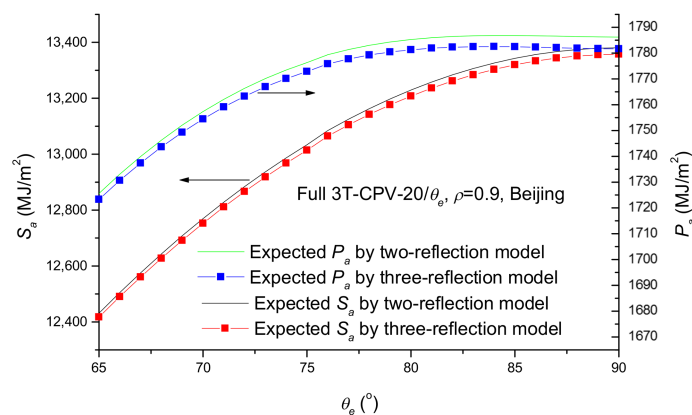


Figure 15. Comparisons of annual collectible radiation and power output of full 3T-CPV-20/ θ_e expected by 3-D model incorporated with two- and three-reflection models.

5.2. Annual Photovoltaic Performance of CPVs

Figure 16 presents the effect of θ_e on the annual power output of 1T-CPV-26/ θ_e in terms of $C_{pv} = P_a/P_{a,0}$, the annual power output increase factor of CPVs as compared to similar PV panel without using CPCs. It is seen that, for full CPV-26/ θ_e , the C_{pv} increases with the increase of θ_e first then comes to a halt as $\theta_e > 83^\circ$, a result of the fact that the geometric concentration factor of full CPC-26/ θ_e increases with θ_e but such an increase becomes slow as θ_e close to 90° ; whereas for truncated CPV-26/ θ_e with a given C_t , the C_{pv} decreases with θ_e , a result of the fact that, with the increase of θ_e , more radiation will arrive on solar cells at large angles as indicated by Yu [27]. The same situation is also observed for 3T-CPV-26/ θ_e as seen in Figure 17.

Figure 18 shows effects of θ_e on the annual average photovoltaic efficiency ($\eta_{a,pv}$) of 1T-CPV-26/ θ_e , a ratio of annual power output (P_a) of CPVs to the annual collectible radiation (S_a). As expected, the $\eta_{a,pv}$ decreases with the increase of θ_e because, with the increase of θ_e , more radiation arrives on solar cells at larger incident angles [27]. It is also shown that, given θ_e and θ_a , $\eta_{a,pv}$ increases with the decrease of C_t because, with the decrease of C_t , more radiation directly irradiates on solar cells, furthermore less radiation is lost due to reflections as multiple reflections usually takes place for radiation irradiating on the upper parabolic reflector. The same situation is also found for 3T-CPVs as shown in Figure 19. It is also seen from Figures 16 and 17 that the C_{pv} of CPVs is much less than C_t , a result of optical loss due to imperfect reflections and electrical loss due to lower photovoltaic efficiency resulting from increased solar incident angle.

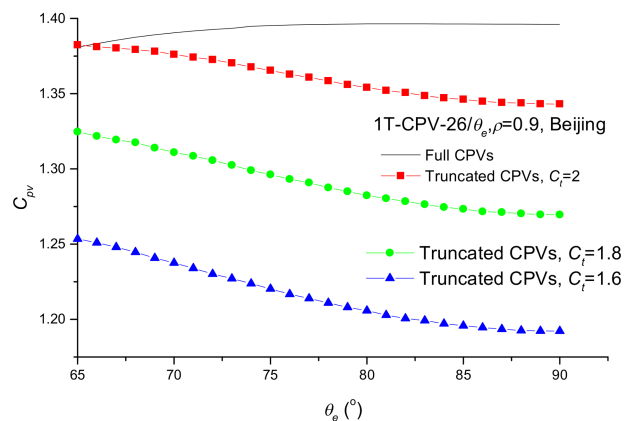


Figure 16. Effects of θ_e on the yearly electricity output of 1T-CPVs in terms of C_{pv} .

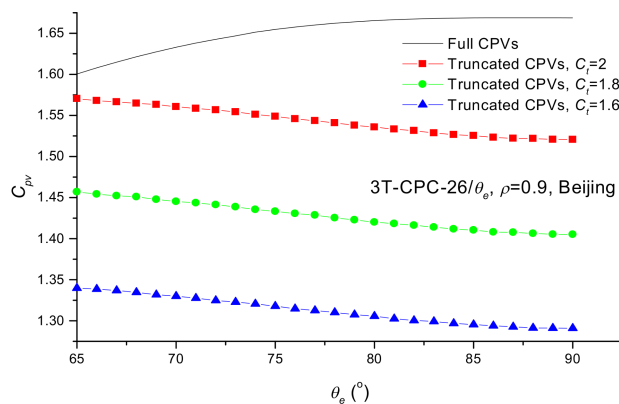


Figure 17. Effects of θ_e on the yearly electricity output of 3T-CPVs in terms of C_{pv} .

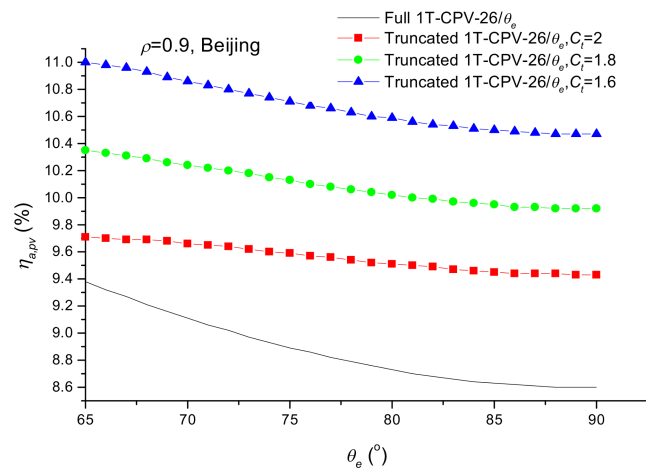


Figure 18. Effects of θ_e on the annually average photovoltaic conversion efficiency of 1T-CPVs.

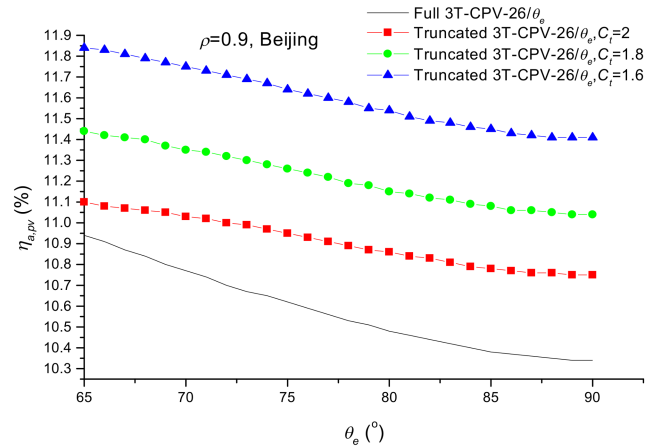


Figure 19. Effects of θ_e on the annually average photovoltaic conversion efficiency of 3T-CPVs.

Comparisons of the annual power output from 1T-CPV-26/ θ_e expected by 3-D model and the 2-D model of Yu are presented in Figures 20 and 21 in terms of P_a and $P_{a,CPV-26/\theta_e}/P_{a,CPV-26/90}$, the ratio of annual power output from CPV-26/ θ_e to that from CPV-26/90. As expected, the annual power output calculated based on the 2-D model are about 7.3–9% higher than those obtained by the 3-D model, and this is attributed to fact that the actual solar incidence angle on solar cells of CPVs is larger than the projected incidence angle which was used to determine the photovoltaic efficiency of solar cells in the 2-D model of Yu, leading the η_{pv} calculated based on the 2-D model to be overestimated. Figure 20 indicates that, for truncated 1T-CPV-26/ θ_e , the annual power output expected by both 2-D and 3-D models decreases with the increase of θ_e , indicating that the variation trends of P_a with θ_e expected by 2-D and 3-D models are in agreement as seen in Figure 21; whereas for full 1T-CPV-26/ θ_e , the P_a obtained based on 3-D model increases with θ_e , but the P_a expected based on 2-D model of Yu decreases with θ_e , indicating that variation trends of P_a with θ_e expected by 2-D model of Yu and 3-D model are not in agreement (see in Figure 21).

Comparisons of the annual power output from 3T-CPV-26/ θ_e expected by 2-D and 3-D models are presented in Figures 22 and 23. It is seen that, the 2-D model of Yu always overestimates (about 8.3%) the annual power output as compared to the 3-D model, but the variation trend of P_a with θ_e expected by 2-D model of Yu agrees with that obtained by the 3-D model for both full and truncated 3T-CPV-26/ θ_e . These results indicate that the 2-D model of Yu always overestimates the annual power output of CPVs in comparison to the 3-D model, and even can't predict the variation trend of P_a with θ_e .

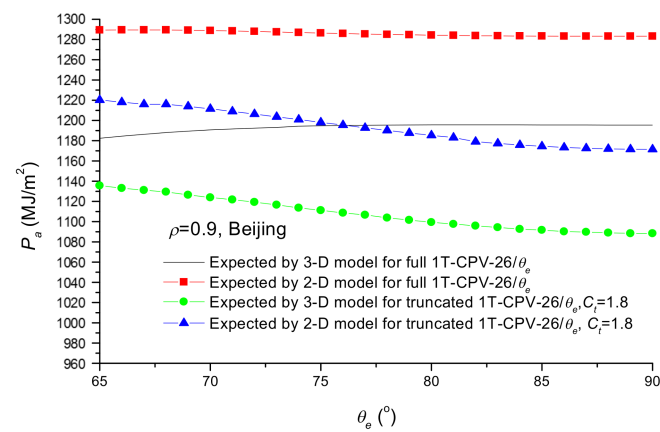


Figure 20. Comparisons of annual power output from 1T-CPV-26/ θ_e expected by 2-D and 3-D model.

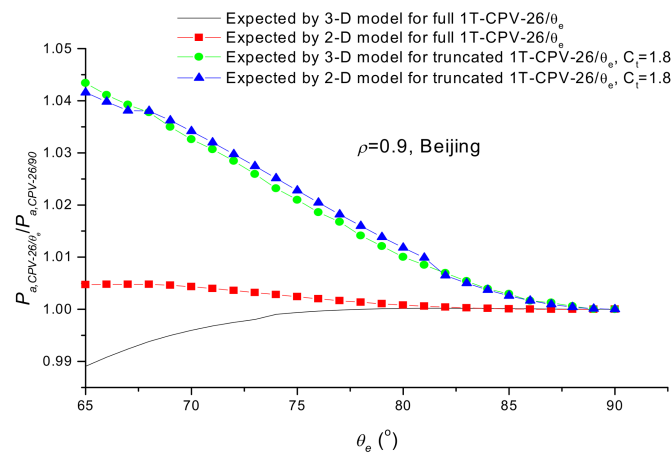


Figure 21. Comparisons of annual power output from 1T-CPVs expected by 2-D and 3-D model in terms of $P_{a,CPV-26/\theta_e}/P_{a,CPV-26/90}$.

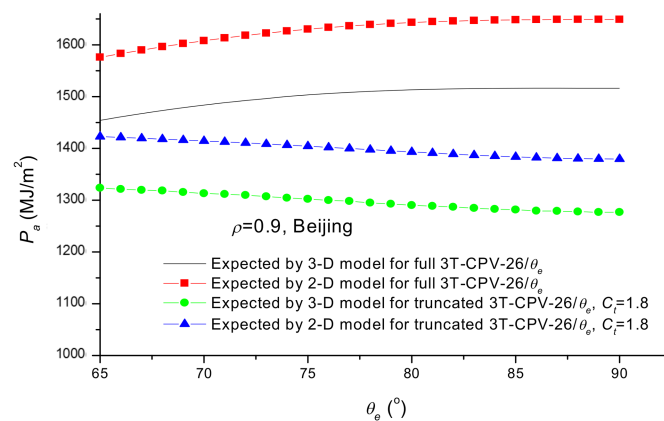


Figure 22. Comparisons of annual power output from 3T-CPV-26/ θ_e expected by 2-D and 3-D model.

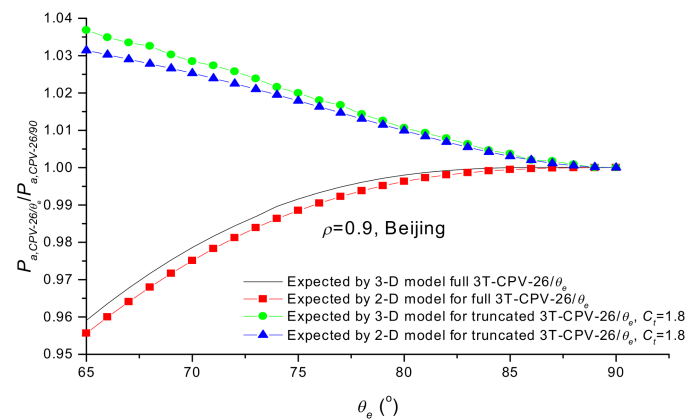


Figure 23. Comparisons of annual power output from 3T-CPVs expected by 2-D and 3-D model in terms of $P_{a,CPV-26/\theta_e}/P_{a,CPV-26/90}$.

6. Conclusions

In this work, a three-dimensional radiation transfer model (3-D model) to predict the performance of CPV- θ_a/θ_e by means of vector algebra, solar geometry and imaging principle of plan mirrors is suggested, and this model allows one to reasonably investigate the effect of geometry of CPVs on annual radiation collection and power output, thus it is helpful for the design of CPVs. The analysis shows that, for CPV- θ_a/θ_e with a given geometry, the optical efficiency uniquely depends on the projected angle of incident solar rays (θ_p), but the photovoltaic efficiency is dependent on θ_p and the real incidence angle on the aperture (θ_{ap}); and the use of CPCs makes solar incidence angle on solar cells increase thus lowering the photovoltaic efficiency of solar cells in comparison to similar PV panels. Calculations show that the two-dimensional radiation transfer model can reasonably predict the optical performance of CPVs, but such a model always overestimates (by about 7.3–9%) the photovoltaic performance of CPVs in comparison to the 3-D model, and even can't predict the variation trend of annual power output of CPV- θ_a/θ_e with θ_e . The results indicate that, for full CPV- θ_a/θ_e , the annual power output increases with θ_e first and then comes to a halt as $\theta_e > 83^\circ$; whereas for truncated CPV- θ_a/θ_e , the annual power output always decreases with θ_e . The method presented in this work also can be used to investigate the performance and design optimization of other linear concentrating PV systems.

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Author Contributions: Runsheng Tang was the supervisor of the work; Jingjing Tang, a Master program student, was responsible for the development of the 3-D mathematical model and the calculations using the 3-D model; Yamei Yu was responsible for the numerical calculations using the 2-D model.

Conflicts of Interest: The authors declare no conflict of interest.

Glossary

C_t	geometric concentration of truncated CPVs (dimensionless)
C_{pv}	annual power output increase factor of CPVs as compared to similar non-concentrating PV panel (dimensionless)
f	optical efficiency factor of CPVs (dimensionless)
H	daily radiation (J/m^2)
h_t	height of CPVs (m)
I	instantaneous radiation intensity (W/m^2)
i_M	Vector of solar rays incident on point M of reflectors;

n_M	Vector of the normal to reflectors at point M
n_s	Incident solar vector
r_M	reflective solar vector from point M of reflectors
P_a	annual power output from CPVs (MJ/m ²)
$P_{a,0}$	annual power output from similar PV panel without using CPCs (MJ/m ²)
S_a	annual radiation collected by CPVs (MJ/m ²)
t	solar time (s)

Greek Letters

β	tilt-angle of CPVs' aperture from the horizon (rad.)
δ	declination of the Sun (rad.)
φ_M	polar angle of point M on parabolic reflectors (rad.)
η	photovoltaic conversion efficiency of CPVs (dimensionless)
η_{pv}	photovoltaic conversion efficiency of solar cells as the function of θ_{in} (dimensionless)
γ	tilt-angle of any line relative to x-axis (rad.)
λ	site latitude (rad.)
θ_a	acceptance half-angle of CPCs (rad.)
θ_e	maximum exit angle of CPC- θ_a/θ_e for radiation over its acceptance angle (rad.)
θ_{ap}	solar incident angle on the aperture of CPVs (rad.)
θ_{in}	solar incident angle on solar cells of CPVs (rad.)
$\theta_{i,M}$	solar incident angle at point M of reflectors (rad.)
$\theta_{r,M}$	solar exit angle from point M of reflectors relative to the negative x-axis (rad.)
θ_p	projected angle of incident solar rays on the cross-section of CPC-troughs (rad.)
$\theta_{p,M}$	projected angle of solar ray reflecting from M of reflectors (rad.)
θ_t	edge-ray angle of truncated CPCs (rad.)
ρ	reflectivity of reflectors (set to be 0.9 in this work)
ψ	opening angle of V-trough formed by two plane mirrors of CPC- θ_a/θ_e (rad.)
τ_{day}	day length (s)
ω	hour angle (rad.)

Subscripts

0	sunset
a	annual
abs	absorber of CPCs; solar cells of CPVs
ap	aperture
b	beam radiation
c	critical value
i	incident solar ray
r	reflective solar ray; right reflector
pl	plane mirror

Appendix A

As shown in Figure 5, the incident angle of solar rays at M of the parabola is given by:

$$\cos \theta_{i,M} = n_M \cdot n_s = n_x \sin \gamma_M - n_z \cos \gamma_M \quad (A1)$$

where $n_M = (\sin \gamma_M, 0, -\cos \gamma_M)$ is the vector of normal to right parabola at point M, and γ_M , tilt-angle of tangent line to parabolic reflectors at M, can be calculated based on Equation (1) as:

$$\tan \gamma_M = \frac{dz}{dx} = \frac{\cos \theta_a - \cos \varphi_M}{\sin \theta_a + \sin \varphi_M} \quad (A2)$$

where φ_M is the polar angle of point M . The vector of solar ray reflecting from M of parabola can be calculated based on reflection law of light as [29]:

$$r_M = -n_s + 2(n_s \cdot n_M)n_M = \begin{pmatrix} -n_x + 2 \cos \theta_{i,M} \sin \gamma_M, -n_y, -n_z - 2 \cos \theta_{i,M} \cos \gamma_M \\ -\cos \theta_{r,M} - n_y, -n_z - 2 \cos \theta_{i,M} \cos \gamma_M \end{pmatrix} \quad (A3)$$

where $\theta_{r,M}$ is the exit angle of ray reflecting from M relative to negative x-axis, and given by:

$$\cos \theta_{r,M} = -r_M \cdot (1, 0, 0) = -n_x - 2 \cos \theta_{i,M} \sin \gamma_M = n_x \cos 2\gamma_M + n_z \sin 2\gamma_M \quad (A4)$$

As shown in Figure 5, the point (N), where r_M hits, is determined by projected angle ($\theta_{p,M}$) of r_M , and $\theta_{p,M}$ can be found based on the principle that projected angles of incident and reflective solar rays on cross-section of CPCs are identical [29] as:

$$\theta_{p,M} = \theta_p + 2\gamma_M \quad (A5)$$

The tilt-angles of lines MD and MB (see Figure 5) are calculated by:

$$\tan \gamma_{MD} = (z_M - z_D) / (x_M - x_D) \quad (A6)$$

$$\tan \gamma_{MB} = (z_M - 0.5) / x_M \quad (A7)$$

Appendix A.1. For the Case of $\theta_{p,M} \geq \gamma_{MB}$

In this case, the radiation reflecting from M directly arrives on solar cells of CPVs (see Figure 5), thus, one has:

$$\eta_{opt,3} = \rho \quad (A8)$$

and in this case, $\theta_{in,3}$, the solar incidence angle, is given by:

$$\cos \theta_{in,3} = -r_M \cdot (1, 0, 0) = \cos \theta_{r,M} = n_x \cos 2\gamma_M + n_z \sin 2\gamma_M \quad (A9)$$

It is known from Equation (A4) that, $\cos \theta_{r,M} = n_x - 2 \cos \theta_{i,M} \sin \gamma_M < n_x$ thus $\theta_{r,M} > \theta_{ap}$, indicating that the solar incidence angle on solar cells is increased after reflections from the reflectors of CPVs. This means that the use of CPCs makes the solar incidence angle increase thus lowering the photovoltaic conversion efficiency of solar cells in comparison to similar PV panels.

Appendix A.2. For the Case of $\gamma_{MD} \leq \theta_{p,M} < \gamma_{MB}$

In this case, the radiation reflecting from M is incident on the plane mirror then redirects onto solar cells (see Figure 5). The vector of radiation reflecting from N of right mirror is expressed by:

$$r_N = -i_N + 2(i_N \cdot n_{rpl})n_{rpl} \quad (A10)$$

where $i_N = -r_M$ is the vector of an incident solar ray at point N of right mirror, substituting $n_{rpl} = (\sin \gamma_{pl}, 0, -\cos \gamma_{pl})$ and Equation (A3) into the Equation (A10) obtains:

$$r_N = -\cos \theta_{r,M} + 2 \cos \theta_{i,N} \sin \gamma_{pl}, -n_y, -n_z - 2 \cos \theta_{i,M} \cos \gamma_M - 2 \cos \theta_{i,N} \cos \gamma_{pl} \quad (A11)$$

where $\theta_{i,N}$ is the solar incidence angle of r_M at N of the right mirror and given by:

$$\cos \theta_{i,N} = i_N \cdot n_{rpl} = -r_M \cdot n_{rpl} = \cos \theta_{r,M} \sin \gamma_{pl} - (n_z + 2 \cos \theta_{i,M} \cos \gamma_M) \cos \gamma_{pl} \quad (A12)$$

The incidence angle on solar cells in this case is given by:

$$\cos \theta_{in,3} = -r_N \cdot (1, 0, 0) = \cos \theta_{r,M} - 2 \cos \theta_{i,N} \sin \gamma_{pl} = n_x \cos 2(\gamma_{pl} - \gamma_M) - n_z \sin 2(\gamma_{pl} - \gamma_M) \quad (A13)$$

The optical efficiency $\eta_{opt,3}$ in this case due to two imperfect reflections is given by:

$$\eta_{opt,3} = \rho^2 \quad (A14)$$

Appendix A.3. For the Case of $\theta_{p,M} < \gamma_{MD}$

In this case, the radiation reflecting from M is incident on the lower part of the right parabola as shown in Figure 5 (right), and φ_N of point N is determined by:

$$\tan \theta_{p,M} = (x_M - x_N) / (z_M - z_N) \quad (A15)$$

Given φ_M of point M , x_M and z_M are calculated based on Equation (1), then φ_N can be found by substituting z_N and x_N as the function of φ_N into Equation (A15), and then solar vector reflecting from N of the parabola can be obtained based on Equations (A11) and (A12) by replacing γ_{pl} with γ_N as follows:

$$r_N = (-\cos \theta_{r,M} + 2 \cos \theta_{i,N} \sin \gamma_N, -n_y, -n_z - 2 \cos \theta_{i,M} \cos \gamma_M - 2 \cos \theta_{i,N} \cos \gamma_N) \quad (A16)$$

The $\theta_{i,N}$, incidence angle of solar ray r_M at point N of parabola, is given by:

$$\cos \theta_{i,N} = -r_M \cdot n_N = \cos \theta_{r,M} \sin \gamma_N - (n_z + 2 \cos \theta_{i,M} \cos \gamma_M) \cos \gamma_N \quad (A17)$$

where γ_N is the tilt-angle of line tangent to the parabola at N and can be found based on Equation (A2) by setting $\varphi_M = \varphi_N$. The exit angle of solar ray from N relative to negative x-axis is given by:

$$\cos \theta_{r,N} = -r_N \cdot (1, 0, 0) = \cos \theta_{r,M} - 2 \cos \theta_{i,N} \sin \gamma_N = n_x \cos 2(\gamma_N - \gamma_M) - n_z \sin 2(\gamma_N - \gamma_M) \quad (A18)$$

Case A: Two-reflection model

In this case, any further reflection of solar ray from N of parabolic reflector is not considered, and the exit angle of r_N is approximately regarded as the solar incident angle on cells. Thus one has $\theta_{in,3} = \theta_{r,N}$ and $\eta_{opt,3} = \rho^2$.

Case B: Three-reflection model

The next incident point of solar ray (r_N) is determined by projected angle of r_N , $\theta_{p,N}$, and it can be easily obtained based on Equation (A5) by setting $\theta_p = -\theta_{p,M}$ and $\gamma_M = \gamma_N$ as:

$$\theta_{p,N} = -\theta_{p,M} + 2\gamma_N = -\theta_p - 2\gamma_M + 2\gamma_N \quad (A19)$$

Given θ_p and φ_M , φ_N and $\theta_{p,N}$ can be calculated, then tilt-angles of lines ND and NB (see Figure 5 right) can be calculated by:

$$\tan \gamma_{ND} = (z_N - z_D) / (x_N - x_D) \quad (A20)$$

$$\tan \gamma_{NB} = (z_N - 0.5) / x_N \quad (A21)$$

Case B1: $\theta_{p,N} \geq \gamma_{NB}$

In this case, the solar ray from point N directly arrives on solar cells, therefore $\eta_{opt,3} = \rho^2$ and $\theta_{in,3} = \theta_{r,N}$.

Case B2: $\gamma_{ND} \leq \theta_{p,N} < \gamma_{NB}$

In this case, the solar ray from N strikes at K of plane mirror (see Figure 5 right), the vector of solar ray from K can be obtained based on the vector algebra similar to find vector r_N as follows:

$$r_K = (-\cos \theta_{r,N} + 2 \cos \theta_{i,K} \sin \gamma_{pl}, -n_y, -n_z - 2 \cos \theta_{i,M} \cos \gamma_M - 2 \cos \theta_{i,N} \cos \gamma_N - 2 \cos \theta_{i,K} \cos \gamma_{pl}) \quad (A22)$$

where $\theta_{i,K}$ is the incidence angle of a solar ray (r_N) at K of right mirror and given by:

$$\cos \theta_{i,K} = -r_N \cdot n_{rpl} = \cos \theta_{r,N} \sin \gamma_{pl} - (n_z + 2 \cos \theta_{i,M} \cos \gamma_M + 2 \cos \theta_{i,N} \cos \gamma_N) \cos \gamma_{pl} \quad (A23)$$

The incidence angle on cells in this case is given by:

$$\cos \theta_{in,3} = -r_K \cdot (1, 0, 0) = \cos \theta_{r,N} - 2 \cos \theta_{i,K} \sin \gamma_{pl} \quad (A24)$$

Substituting Equations (A1), (A17), (A18) and (A23) into the above one obtains:

$$\cos \theta_{in,3} = n_x \cos 2(\gamma_{pl} - \gamma_N + \gamma_M) + n_z \sin 2(\gamma_{pl} - \gamma_N + \gamma_M) \quad (A25)$$

The optical efficiency $\eta_{opt,3}$ in this case due to three imperfect reflections is given by:

$$\eta_{opt,3} = \rho^3 \quad (A26)$$

Case B3: $\theta_{p,N} < \gamma_{ND}$

In this case, the solar ray from N strikes at point K of lower parabola (ND), and φ_K of point K is calculated based on following equation together with Equation (1):

$$\tan \theta_{p,N} = (z_N - z_K) / (x_N - x_K) \quad (A27)$$

On knowing φ_K , the tilt-angle of tangent line to the parabola at K , γ_K , can be obtained based on Equation (A2) by replacing φ_M with φ_K , then the solar vector from point K of the parabola, r_K , can be obtained based on Equations (A22) and (A23) by replacing γ_{pl} with γ_K . For a three-reflection model, any further reflection of solar ray from K on reflectors of CPCs is not considered because the fraction of radiation that arrive on solar cells of CPVs after more than three reflections is considerably small [13]. Therefore, the exit angle of r_K is regarded as the solar incident angle ($\theta_{in,3}$) on solar cells, thus $\theta_{in,3}$ can be obtained from Equation (A25) by replacing γ_{pl} with γ_K . The $\eta_{opt,3}$ in this case is $\eta_{opt,3} = \rho^3$.

Analysis in the above shows that, for the radiation irradiating on the right parabola at $\theta_p \leq \theta_a$, the reflection number of solar rays on their way to solar cells is dependent on θ_p and φ_M , thus the optical efficiency $\eta_{opt,3}$ is dependent on θ_p and φ_M , but the solar incidence angle on solar cells is not only dependent on θ_p and φ_M but also dependent on n_x , being $\cos \theta_{ap}$, of the incident solar vector (n_s), therefore the photovoltaic conversion efficiency η_{pv} is dependent on both θ_p and θ_{ap} .

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