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Application of Dynamic Non-Linear Programming Technique to Non-Convex Short-Term Hydrothermal Scheduling Problem

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Abstract: Short-term hydro-thermal scheduling aims to obtain optimal generation scheduling of hydro and thermal units for a one-day or a one-week scheduling time horizon. The main goal of the problem is to minimize total operational cost considering a series of equality and inequality constraints. The problem is considered as a non-linear and complex problem involving the valve-point loading effect of conventional thermal units, the water transport delay between connected reservoirs, and transmission loss with a set of equality and inequality constraints such as power balance, water dynamic balance, water discharge, initial and end reservoir storage volume, reservoir volume limits and the operation limits of hydro and thermal plants. A solution methodology to the short-term hydro-thermal scheduling problem with continuous and non-smooth/non-convex cost function is introduced in this research applying dynamic non-linear programming. In this study, the proposed approach is applied to two test systems with different characteristics. The simulation results obtained in this paper are compared with those reported in recent research studies, which show the effectiveness of the presented technique in terms of total operational cost. In addition, the obtained results ensure the capability of the proposed optimization procedure for solving short-term hydro-thermal scheduling problem with transmission losses and valve-point effects.

Keywords: dynamic non-linear programming; non-smooth/non-convex optimization problem; short term hydro-thermal scheduling; transmission losses; valve point loading effect

1. Introduction

Power systems are faced with a series of challenging issues taking into account the advances and improvements within them. Remarkable research is being carried out in various areas such as the application and analysis of micro-grids (MGs) and distributed generations (DGs) in the optimal operation of power systems [1], transient stability analysis in power systems [2], dynamic operation and control of the systems [3], connection decisions of distribution transformers [4], and fault current analysis of power systems [5,6]. The authors implemented a direct search method (DSM) in [1] for solving economic dispatch (ED) of a medium-voltage MG considering several kinds of DGs. A modified artificial bee colony (MABC) optimization technique is applied in [7] for obtaining the optimal solution of the ED problem, where a novel mutation strategy based on the differential evolution (DE) method is used for improving the capability of the method in providing the optimal solution.
The valve-point loading effect of conventional thermal plants is considered in this study. The authors proposed a three-stage technique in [8] to solve the ED problem of distribution-substation-level MGs, where the main power grid and MGs are studied as two key parts of the system. In this reference, the ED of the main grid and local MGs are solved using sensitive factors and an improved direct search method in stages I and II, respectively, and the optimal reschedules from the original dispatch solutions are provided in stage III. The authors have addressed the ED problem considering voltage magnitudes and reactive power flows in [9], where linear programming method is utilized for solving the problem. In this study, thermal capacities of transmission lines and line power transmission, and exponential loads are studied using piecewise linear models. Power system expansion planning is studied in [10], where costs associated with the fuel and buying emission allowances, and benefits from selling emission allowances are considered. A piecewise linear objective function is proposed for calculating the sensitivity of operation cost with respect to limitations of emission.

Short-term hydro-thermal scheduling (STHTS) is defined as one of the most important and challenging issues in power systems operation. Thermal power plants operational costs are high; however, the initial costs of such generation units are lower. On the other hand, the operational costs of hydro power plants are insignificant; however, the construction costs of such plants are high [11,12]. Accordingly, the combination of these two types of power plants can be considered as an appropriate choice considering economic viewpoints. The main goal of short-term scheduling of hydro-thermal system is determining the optimal power generation of the hydro and thermal plants. The optimal solution provides the minimum total operational cost of the thermal units, while satisfying load demand and a series of equality and inequality constraints of the hydraulic and thermal power system network. The STHTS problem is proposed as a complex non-linear, non-convex and non-smooth optimization problem considering the water transport delay between connected reservoirs, the valve-point loading effect related to the thermal units, transmission loss and many equality and inequality constraints [13,14].

Different optimization methods are employed to obtain optimal solution of generation planning of hydrothermal systems, including heuristic and classical methods. A modified dynamic neighborhood learning based particle swarm optimization (MDNLPPO) method is introduced in [15] to solve the STHTS problem. In this reference, the proposed approach is applied on two test systems with different characteristics. STHTS problem is solved in [16] by employing quadratic approximation based on differential evolution with valuable trade-off (QADEVT) that minimizes fuel cost and pollutant emission simultaneously. The predator prey optimization (PPO) procedure is used in [17] to obtain the optimal power production planning of hydro and thermal units. In [18], a hybrid method differential evolution with adaptive Cauchy mutation is utilized to obtain the optimal generation scheduling of hydro and thermal units, in which water transport delay between connected reservoirs and the effect of valve-point loading of thermal power plants is taken into account. Particle swarm optimization (PSO) is introduced in [19] to deal with STHTS problem with non-convex and non-smooth cost function. The real coded genetic algorithm (RCGA) is used for the solution of STHTS problem with a series of equality and inequality restrictions and non-smooth/non-convex cost function. The suggested algorithm in this reference is armed with a restriction-management approach which eliminates the requirement of penalty parameters. In [20], by using the Lagrangian Relaxation (LR) method, not only are the electrical and hydraulic constrains handled, but also the existing network constraints are considered by employing DC power flow. The lexicographic optimization and hybrid augmented-weighted ε-constraint method are applied in [21] to produce Pareto optimal solutions for STHTS problem. In this reference, mixed integer programming (MIP) is introduced to obtain the optimal power generation planning of hydrothermal system in a day-ahead joint energy and reserve market. In [22], an improved merit order (IMO) and augmented Lagrangian Hopfield network (ALHN) is proposed to solve short-term hydrothermal scheduling with pumped-storage hydro units. The proposed method in this reference considers thermal, hydro and pumped-storage unit commitment (UC). The STHTS problem is solved in [23] with the consideration of AC network constraints, which is implemented
a combination of the Benders decomposition method and Bacterial Foraging oriented by Particle Swarm Optimization (BFPSO) method. The application of chaotic maps in a particular game problem called the Parrondo Paradox is studied in [24]. The proposed approach was used in a three-game problem and a more general N-game problem in which non-linear optimization problem is considered to define the parameters for the studied game.

In this study, the STHTS problem is solved using dynamic non-linear programming (DNLP) using general algebraic modeling system (GAMS) software. The valve-point effect of conventional thermal plants, which increases the complexity of solving STHTS problem, is considered in the solution of the problem. In addition, the power transmission loss of the hydro-thermal system is studied in the proposed study. Different case studies are solved to evaluate the performance and ensure the effectiveness of the introduced method. The optimal solutions are compared with those reported in previous studies in terms of total operational cost, which demonstrates the capability of the proposed method to identify solutions having less operational cost. In addition, optimal solutions obtained in this paper ensures the capability of the proposed method to deal with valve-point loading effect of thermal units and system power transmission loss.

The rest of the paper is organized as follows: The mathematical formulation of the STHTS problem is provided in Section 2. Section 3 introduces the proposed solution method for STHTS problem. In Section 4, the proposed approach is implemented on two test systems and the obtained optimal solutions are compared with those reported in previous studies. Finally, the paper is concluded in Section 5.

2. Problem Formulation

The optimal scheduling of hydro-thermal plant includes a non-linear optimization problem involving objective function and a set of linear, non-linear and dynamic constraints. The objective function and equality and inequality constraints of the STHTS problem are explained in the following [25].

2.1. Objective Function

The main goal of short-term planning of hydro-thermal system is determining the optimal power generation of the hydro and thermal plants so as to minimize the total operation cost of the thermal units since the cost of hydro production is insignificant. It should be mentioned that various constraints on the hydraulic and thermal power system network should be considered in the solution of the problem. The objective function to be minimized can be represented as follows [26]:

$$C(P) = \sum_{t=1}^{24} \sum_{i=1}^{N_S} a_i + b_i P_{ti}^t + c_i (P_{ti}^t)^2 + |e_i \sin(f_i(P_{tmin}^i - P_{ti}^t))|$$

(1)

where $C(P)$ is the total fuel cost. $N_S$ is indicator used for the number of thermal plants. Moreover, $P_{ti}^t$ is power generated by the $i$th thermal plant at time $t$. $a_i$, $b_i$, and $c_i$ are the cost coefficients of $i$th thermal plant. Considering multiple steam valves in conventional thermal power plants, it is essential to model the effect of valve-points on fuel cost. Valve-points effect can be modeled by a sinusoidal term, which will be added to the quadratic cost function [27]. $P_{tmin}^i$ is minimum power generation of thermal unit $i$. Moreover, $e_i$ and $f_i$ are valve-point coefficients of cost function of thermal unit $i$.

2.2. Power Balance Constraint

The total power generated by hydro and thermal plants should be equal to the sum of total load demand and transmission line losses.

$$\sum_{i=1}^{N_h} P_{ti}^t + \sum_{j=1}^{N_t} P_{tj}^t = P_t^D + P_t^L$$

(2)
where \( N_h \) is the number of hydro units. \( P_t^i \) is the generation of hydro units in megawatts (MW). Moreover, \( P_t^D \) and \( P_t^L \) are load demand and total transmission loss in MW, respectively. \( P_t^L \) can be calculated using the Kron’s loss formula known as B-matrix coefficients [28]. Equation (3) calculates power transmission loss utilizing Kron’s loss formula, which is defined as B-matrix coefficients method in this paper as follows:

\[
P_t^L = \sum_{i=1}^{N_h+N_i} \sum_{j=1}^{N_s} B_{ij} P_t^i + \sum_{i=1}^{N_h+N_i} B_{io} P_t^i + B_{00} = 1, 2, \ldots, T
\]

(3)

The coefficients are Kron’s loss formulation used to calculate power transmission of the hydrothermal system. The power loss of the system taking into account \( N_s \) hydro plants and \( N_h \) thermal units can be calculated by using such formulation. B-matrix coefficients for calculating the power loss are shown by \( B_{ij} \), \( B_{io} \), and \( B_{00} \). In such formulation, \( B_{mn} \) is element of matrix B with dimension of \((N_s + N_h) \times (N_s + N_h)\). In addition, \( B_{00} \) is considered as a constant.

The hydro power generation, \( P_t^j \), is a function of water discharge and storage volume, which can be calculated as follows:

\[
P_t^j = C_{1,j} (V_t^j)^2 + C_{2,j} (Q_t^j)^2 + C_{3,j} V_t^j Q_t^j + C_{4,j} V_t^j + C_{5,j} Q_t^j + C_{6,j}
\]

(4)

where \( V_t^j \) is the storage volume of reservoir in m³, and \( C_{1,j}, C_{2,j}, C_{3,j}, C_{4,j}, C_{5,j}, \) and \( C_{6,j} \) represent hydro power generation coefficients. Moreover, \( Q_t^j \) is the water discharge amount in m³.

### 2.3. Limitations of Power Production

The generator capacity constraints are expressed as:

\[
\begin{align*}
    p_{i}^{\text{min}} & \leq P_t^i \leq p_{i}^{\text{max}} \\
    p_{j}^{\text{min}} & \leq P_t^j \leq p_{j}^{\text{max}}
\end{align*}
\]

(5)

where \( p_{i}^{\text{min}} \) and \( p_{i}^{\text{max}} \) are the respective lower and upper bounds of power generation of thermal units. In addition, the minimum and maximum amounts of power production of hydro units are indicated by \( p_{j}^{\text{min}} \) and \( p_{j}^{\text{max}} \), respectively.

### 2.4. Hydraulic Network Constraints

#### 2.4.1. Water Dynamic Balance

The reservoir storage of hydro unit is related to previous inflow and spillage, and storage of reservoir discharge from upstream reservoirs, which can be formulated as:

\[
V_t^j = V_{t-1}^j + I_t^j - Q_t^j - S_t^j + \sum_{m=1}^{\phi_j} \left[ Q_{m(t-\tau_m)}^j + S_{m(t-\tau_m)}^j \right], m \in \phi_j
\]

(6)

where \( I_t^j \) is the inflow rate of the reservoir, \( \phi_j \) is set of instant upstream hydro plants of the \( j \)th reservoir. Additionally, \( \tau \) is time delay of immediate downstream plants.

#### 2.4.2. Reservoir Storage Volume Limits

The operating volume of reservoir should be limited in interval between minimum and maximum values, which can be stated as:

\[
V_t^{\text{min}} \leq V_t^j \leq V_t^{\text{max}}
\]

(7)
where $V_{j}^{\text{min}}$ and $V_{j}^{\text{max}}$ are the respective lower and upper bounds of operating volume of the reservoir of $i$th hydro unit.

2.4.3. Water Release Limits

The water release of hydro units should be limited to minimum and maximum values, which can be considered as:

$$Q_{j}^{\text{min}} \leq Q_{j}^{t} \leq Q_{j}^{\text{max}}$$

(8)

where $Q_{j}^{\text{min}}$ and $Q_{j}^{\text{max}}$ are the minimum and maximum release of the water reservoir of the $i$th hydro plant.

2.4.4. Initial and Final Reservoir Storage Volume

Initial and final volumes of reservoir storage should be taken into account in the formulation of STHTS problem as:

$$V_{j}^{t} \big|_{t=0} = V_{j}^{\text{begin}}$$

$$V_{j}^{t} \big|_{t=\tau} = V_{j}^{\text{end}}$$

(9)

where $V_{j}^{\text{begin}}$ is the elementary volume of the reservoir and $V_{j}^{\text{end}}$ is the final volume of the reservoir.

3. Solution Methodology

GAMS is defined as a practical tool to handle general optimization problems, which consists of a proprietary language compiler and a variety of integrated high-performance solvers. GAMS is specifically designed for large and complex problems, which allows creating and maintaining models for a variety of applications. GAMS is able to formulate models in many different types of problem classes, such as linear programming (LP), nonlinear programming (NLP), mixed-integer linear programming (MILP), mixed-integer nonlinear programming (MINLP) and dynamic nonlinear programming (DNLP). Nonlinear models created in GAMS area should be solved by using an NLP algorithm. This paper offers a novel approach based on the NLP method to obtain optimal planning of hydrothermal systems. Accordingly, the STHTS is modeled as a NLP in this study, and is solved by implementing OptQuest/NLP (OQNLP) solver. The STHTS problem is formulated as a nonlinear problem, which can be solved by GAMS software [29] using OQNLP solver [30]. OQNLP is a multi-start heuristic technique, which calls an NLP solver from different starting points. All feasible solutions obtained by such solvers are kept, and the best solution is reported as the final optimal solution. Such a method is capable of finding global optimal solutions of smooth constrained NLPs. A scatter search implementation called OptQuest is employed by OQNLP to compute starting points [31]. OQNLP is able to obtain global optimal solutions of smooth NLPs and MINLPs. A simplified pseudo-code is provided in Figure 1 for introducing the application of OQNLP to find the optimal solution of the optimization problems, which is divided into two levels. The first level generates candidate starting points and selects the best starting point among all of the points. Then, in the second level, new points are generated and evaluated in order to obtain the best solution in terms of generation cost.
4. Case Studies and Simulation Results

In this paper, the performance of the proposed solution is evaluated in several test systems. A Pentium IV PC with 2.8 GHz CPU and 4 GB RAM PC is used to solve the problem in GAMS. The scheduling horizon is chosen as 24 h of a day.

4.1. Test System 1

First test system consists of four hydro plants and an equivalent thermal plant. The hydraulic communication among hydro units of this system is demonstrated in Figure 2. Transmission losses are not considered in this test system. Cost coefficients of thermal plants are \( a_i = 0.002 \), \( b_i = 19.2 \), and \( c_i = 5000 \). The lower and upper operation limits of this thermal plant are 500 and 2500 MW, respectively. Data of thermal unit and hydro plants are adopted from [25]. Two different cases including convex and non-convex cost function are studied for this test system.

![Figure 2. Hydro subsystem used in the all test systems.](image_url)
4.1.1. Test System 1 Case 1: Quadratic Cost without Valve-Point Loading Effect

In this case, optimal generation scheduling of test system 1 is solved without consideration of valve-point loading impact. The hourly water discharge of the hydro plants and hydro power production, which is calculated by employing Equation (7), are shown in Table 1. In addition, thermal power production for case 1 is provided in Table 1. According to Table 1, the sum of power generation by four hydro units and one thermal plant meets total demand of the system. Hourly hydro discharges of the optimal solution are demonstrated in Figure 3. Considering Figure 3, hydro plant 4 has the maximum discharge among four hydro units, which shows that the power generation of hydro plant 4 is more than the others. In addition, hourly hydro and thermal plant generations are illustrated in Figure 4. The thermal units participate in power demand supply more than the hydro plants according to Figure 4. Moreover, total load demand is satisfied by the power generation of four hydro units and the thermal plants, which is obvious in Figure 4.

Table 1. Hourly plant discharges, power outputs and total thermal generation (test system 1, case 1).

<table>
<thead>
<tr>
<th>Hour</th>
<th>Hydro Plant Discharges (10^4 m³)</th>
<th>Hydro Power Output (megawatts (MW))</th>
<th>Thermal Generation (MW)</th>
<th>Total Generation (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.254 6.000 11.632 15.344 61.528 45.316 56.480</td>
<td>224.231 982.445 1370 6499.797</td>
<td>64.272 59.355 1237 7432.445</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>6.488 6.000 11.914 16.919 64.339 46.576 55.928</td>
<td>219.694 1003.464 1390 7499.797</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>6.594 6.000 12.303 18.537 65.777 47.804 56.376</td>
<td>209.020 981.024 1360 7432.445</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>6.592 6.000 12.741 20.000 66.102 49.556 57.320</td>
<td>189.900 927.092 1290 7499.797</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>6.431 6.000 13.129 20.000 64.972 51.296 57.694</td>
<td>306.000 810.039 1290 7432.445</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>6.617 6.000 13.531 20.000 66.284 52.396 58.133</td>
<td>306.000 927.187 1410 7499.797</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>7.005 6.000 13.923 20.000 69.238 52.934 58.611</td>
<td>306.000 1163.217 1650 7432.445</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>7.545 6.000 14.254 20.000 73.259 52.934 58.728</td>
<td>306.000 1509.079 2000 7499.797</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>7.927 6.000 14.568 20.000 76.166 53.464 58.565</td>
<td>306.000 1745.805 2240 7432.445</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>8.109 6.000 14.894 20.000 77.851 54.500 58.136</td>
<td>306.000 1823.512 2320 7499.797</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>8.087 6.000 15.277 20.000 78.367 55.994 57.612</td>
<td>306.000 1732.027 2230 7499.797</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>8.272 6.000 15.621 20.000 80.353 57.416 57.056</td>
<td>306.000 1809.175 2310 7499.797</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>8.165 6.254 16.047 20.000 79.963 60.180 56.475</td>
<td>306.000 1727.382 2230 7499.797</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>8.124 6.613 16.494 20.000 80.131 63.512 56.112</td>
<td>306.000 1694.245 2200 7499.797</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>8.043 6.927 16.939 20.000 80.074 66.727 55.351</td>
<td>306.000 1621.848 2130 7499.797</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>7.930 7.272 17.137 20.000 79.565 69.947 54.948</td>
<td>306.000 1599.540 2070 7499.797</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>7.950 7.670 15.694 20.000 79.858 72.868 57.561</td>
<td>306.000 1613.712 2130 7499.797</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>7.768 7.950 14.281 20.000 78.597 74.372 59.277</td>
<td>306.000 1621.754 2140 7499.797</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>7.662 8.374 12.888 20.000 77.830 76.131 60.031</td>
<td>306.000 1720.008 2240 7499.797</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>7.452 8.751 18.733 20.000 76.216 77.728 51.940</td>
<td>306.000 1768.116 2280 7499.797</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>7.063 15.000 19.145 20.000 73.145 101.607 49.988</td>
<td>303.055 1712.205 2240 7499.797</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>11.991 15.000 19.676 20.000 101.750 98.082 47.637</td>
<td>298.534 1573.998 2120 7499.797</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>11.931 15.000 20.368 20.000 100.691 94.269 44.320</td>
<td>292.356 1318.364 1850 7499.797</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>15.000 15.000 13.133 20.000 107.020 80.950 59.005</td>
<td>284.400 1058.625 1590 7499.797</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 3. Hourly hydro discharges volumes of the optimal solution for test system 1, case 1.
which is obtained by applying Equation (7), are provided in this table. In addition, power production of the proposed method is $914,660.00, which is obtained by applying FAPSO [39]; however, the proposed method in this paper, which include QEA [25], DE [25], RCGA-AFSA [34], RQEA [25], DRQEA [25], CRQEA [25], MDE [33], MAPSO [36], MDNLPSO [15], IDE [41], TLBO [37], RCGA-AFSA [34], SPPSO [33], SOHPSO_TVAC [38], PSO [39], Improved DE [40], IDE [40], and FAPSO [39], and is shown in Table 2. As it can be observed from this table, the best reported cost for this case is equal to $914,660, which is related to FAPSO [39], while total operational cost of the solution obtained by the proposed method is $884,733.965. Accordingly, the proposed method is capable to find better solution in comparison with previous methods in terms of total operational cost.

4.1.2. Test System 1 Case 2: Quadratic Cost Function with Valve-Point Loading

In this case, optimal power scheduling of test system 1 is obtained with consideration of valve-point loading effect. The parameters of valve-point loading impact of thermal unit are $e_i = 700$ and $f_i = 0.085$. The simulations are provided for case 2 with non-convex fuel cost. The optimal planning of discharge of four hydro units are reported in Table 3. In addition, power generation of hydro units, which is obtained by applying Equation (7), are provided in this table. In addition, power production of thermal power plants are presented in Table 3. It can be observed from Table 3 that the power demand during 24-h scheduling time is satisfied by total power generation of four hydro units and one thermal unit.

The optimal solution obtained in this research study is compared with those reported in recent paper, which include QEA [25], DE [25], RCGA-AFSA [34], RQEA [25], DRQEA [25], CRQEA [25], RCCRO [41], ACDE [42], MAPSO [36], TLBO [37], RCGA [34], RQEA [25], DE [25], MDE [33], DRQEA [25], HCRO-DE [35], MAPSO [36], MDNLPSO [15], IDE [41], TLBO [37], RCGA-AFSA [34], SPPSO [33], SOHPSO_TVAC [38], PSO [39], Improved DE [40], IDE [40], and FAPSO [39], and is shown in Table 4. As it can be seen in this table, the minimum total operational cost reported for this case is $914,660.00, which is obtained by applying FAPSO [39]; however, the proposed method in this
paper obtained the minimum cost equal to $901,191.9735, which shows the capability of the proposed method in obtaining optimal solution of the STHS problem for test system 1, case 2 with respect to other optimization methods.

Table 2. Comparisons of simulation results for test system 1, case 1. Employing quantum-inspired evolutionary algorithm (QEA); quantum-inspired evolutionary algorithm (WDA); real-coded genetic algorithm (RCGA); real-coded quantum-inspired evolutionary algorithm (RQEA); modified differential evolution (MDE); differential real-coded quantum-inspired evolutionary algorithm (DRQEA); hybrid chemical reaction optimization-differential evolution (HCRO-DE); modified adaptive particle swarm optimization (MAPSO); real coded genetic algorithm and artificial fish swarm algorithm (RCGA-AFSA); teaching learning-based optimization (TLBO); SPPSO; self-organizing hierarchical particle swarm optimization technique with time-varying acceleration coefficients (SOHPSO_TVAC); particle swarm optimization (PSO); improved differential evolution (IDE); fuzzy adaptive particle swarm optimization (FAPSO); dynamic neighborhood learning based particle swarm optimization (DNLPSO) and; modified dynamic neighborhood learning based particle swarm optimization (MDNLPSO).

<table>
<thead>
<tr>
<th>Optimization Method</th>
<th>Min. Cost ($)</th>
<th>Max. Cost ($)</th>
<th>Ave. Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>QEA [25]</td>
<td>926,538.29</td>
<td>930,484.13</td>
<td>928,426.95</td>
</tr>
<tr>
<td>WDA [32]</td>
<td>925,618.5</td>
<td>-</td>
<td>928,219.8</td>
</tr>
<tr>
<td>SPSO [33]</td>
<td>925,308.86</td>
<td>923,083.48</td>
<td>926,185.32</td>
</tr>
<tr>
<td>RCGA [34]</td>
<td>923,966.285</td>
<td>924,108.731</td>
<td>924,232,072</td>
</tr>
<tr>
<td>RQEA [25]</td>
<td>923,634.53</td>
<td>926,957.39</td>
<td>925,992.46</td>
</tr>
<tr>
<td>DE [25]</td>
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<td>923,871.51</td>
<td>924,419.37</td>
</tr>
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<td>FAPSO [39]</td>
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</tr>
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Table 3. Hourly plant discharges, power outputs and total thermal generation (test system 1, case 2).

<table>
<thead>
<tr>
<th>Hour</th>
<th>Hydro Plant Discharges (10⁴ m³)</th>
<th>Hydro Power Output (MW)</th>
<th>Thermal Generation (MW)</th>
<th>Total Generation (MW)</th>
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<td>6.000</td>
<td>15.327</td>
<td>54.329</td>
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<td>6.000</td>
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<td>53.613</td>
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<td>6.000</td>
<td>20.000</td>
<td>54.126</td>
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<td>6.000</td>
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<td>56.689</td>
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<td>6.661</td>
<td>12.694</td>
<td>60.242</td>
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<td>13.476</td>
<td>13.306</td>
<td>13.157</td>
<td>104.423</td>
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<td>6.021</td>
<td>10.104</td>
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<td>11.916</td>
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<td>86.633</td>
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<td>6.137</td>
<td>27.943</td>
<td>19.921</td>
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<tr>
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<td>5.000</td>
<td>6.000</td>
<td>21.954</td>
<td>19.476</td>
</tr>
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<td>6.544</td>
<td>21.608</td>
<td>58.135</td>
</tr>
<tr>
<td>17</td>
<td>5.234</td>
<td>8.985</td>
<td>14.157</td>
<td>19.978</td>
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<tr>
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<td>7.821</td>
<td>6.640</td>
<td>14.810</td>
<td>79.151</td>
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<tr>
<td>20</td>
<td>10.474</td>
<td>11.904</td>
<td>24.476</td>
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<td>24</td>
<td>15.000</td>
<td>6.721</td>
<td>13.133</td>
<td>19.190</td>
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</table>
Table 4. Comparisons of simulation results for test system 1, case 2.

<table>
<thead>
<tr>
<th>Optimization Method</th>
<th>Min. Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>QEA [25]</td>
<td>930,647.96</td>
</tr>
<tr>
<td>DE [25]</td>
<td>928,662.84</td>
</tr>
<tr>
<td>RC&amp;G-AFSA [34]</td>
<td>927,899.82</td>
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<tr>
<td>RQEA [25]</td>
<td>926,088.33</td>
</tr>
<tr>
<td>DRQEA [25]</td>
<td>925,485.21</td>
</tr>
<tr>
<td>CRQEA [25]</td>
<td>925,403.1</td>
</tr>
<tr>
<td>RCCR [41]</td>
<td>925,214.20</td>
</tr>
<tr>
<td>ACDE [42]</td>
<td>924,661.53</td>
</tr>
<tr>
<td>MAPSO [36]</td>
<td>924,636</td>
</tr>
<tr>
<td>TLBO [37]</td>
<td>924,590.78</td>
</tr>
<tr>
<td>RCGA [34]</td>
<td>923,966.28</td>
</tr>
<tr>
<td>RQEA [25]</td>
<td>923,634.53</td>
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<td>DE [25]</td>
<td>923,234.56</td>
</tr>
<tr>
<td>MDE [33]</td>
<td>922,856.38</td>
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<tr>
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<td>922,526.73</td>
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<tr>
<td>HCR-DE [35]</td>
<td>922,444.79</td>
</tr>
<tr>
<td>MAPSO [36]</td>
<td>922,421.66</td>
</tr>
<tr>
<td>MELPSO [15]</td>
<td>923,961</td>
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<tr>
<td>IDE [40]</td>
<td>923,016.29</td>
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<tr>
<td>TLBO [37]</td>
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<tr>
<td>RCGA-AFSA [34]</td>
<td>922,339.62</td>
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<td>SPPO [33]</td>
<td>922,336.31</td>
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<tr>
<td>SPSO [36]</td>
<td>922,018.24</td>
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<tr>
<td>PSO [39]</td>
<td>921,920</td>
</tr>
<tr>
<td>Improved DE [40]</td>
<td>917,250.1</td>
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<tr>
<td>IDE [40]</td>
<td>917,237.7</td>
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<tr>
<td>FAPO [39]</td>
<td>914,660.00</td>
</tr>
<tr>
<td>Proposed method</td>
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</tr>
</tbody>
</table>

4.2. Test System 2

This test system consists of four cascaded hydro power plants and three thermal plants. Valve-point loading effect of thermal plants and transmission losses are considered in this test system. Data of hydro and thermal generation units are adopted from [42]. Coefficients of transmission loss for this system are given as the following:

$$B = \begin{bmatrix} 0.34 & 0.13 & 0.09 & -0.10 & -0.08 & -0.01 & -0.02 \\ 0.13 & 0.14 & 0.10 & 0.01 & -0.05 & -0.02 & -0.01 \\ 0.09 & 0.10 & 0.31 & 0.00 & -0.11 & -0.07 & -0.05 \\ -0.01 & 0.01 & 0.00 & 0.24 & -0.08 & -0.04 & -0.07 \\ -0.08 & -0.05 & -0.11 & -0.08 & 1.92 & 0.27 & -0.02 \\ -0.01 & -0.02 & -0.07 & -0.04 & 0.27 & 0.32 & 0.00 \\ -0.02 & -0.01 & -0.05 & -0.07 & -0.02 & 0.00 & 1.35 \end{bmatrix} \times 10^{-4} \text{MW}^{-1}$$ \hspace{1cm} (10)

$$B_0 = [-0.75, -0.06, 0.70, -0.03, 0.27, -0.77, -0.01] \times 10^{-6}$$

$$B_{00} = 0.55 \text{MW}$$

4.2.1. Test System 2, Case 1: Quadratic Cost without Valve-Point Loading Effect

This test system consists of four cascades hydro plants and three thermal plants considering valve-point loading effect for all thermal units. In this case, transmission loss is not considered. The optimal hydro discharges and hydro power generation of four hydro units are provided in Table 5. Moreover, power generations of three thermal plants are reported in this table. According to Table 5, the sum of power generation of four hydro units and three thermal plants meets the load demand during the scheduling time of the STHS problem.
Table 5. Optimal discharges and power output for test system 2 case 1.

<table>
<thead>
<tr>
<th>Hour</th>
<th>Hydro Plant Discharges (10^4 m³)</th>
<th>Hydro Power Output (MW)</th>
<th>Thermal Power Output (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Plant 1</td>
<td>Plant 2</td>
<td>Plant 3</td>
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<td>11.919</td>
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<td>6.699</td>
<td>7.766</td>
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</tr>
<tr>
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</tr>
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<td>5.000</td>
<td>6.000</td>
<td>18.081</td>
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<td>8.717</td>
<td>7.235</td>
<td>10.050</td>
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<td>6.793</td>
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</tr>
<tr>
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<td>9.980</td>
<td>7.651</td>
<td>10.000</td>
</tr>
<tr>
<td>11</td>
<td>11.526</td>
<td>9.682</td>
<td>12.521</td>
</tr>
<tr>
<td>12</td>
<td>8.830</td>
<td>8.827</td>
<td>19.991</td>
</tr>
<tr>
<td>13</td>
<td>13.639</td>
<td>13.611</td>
<td>26.991</td>
</tr>
</tbody>
</table>

Proposed method provided the minimum fuel cost of $41,101.738, which is compared with simulated annealing (SA) [25], DE [11], chaotic artificial bee colony (CABC) [26], adaptive differential evolution (ADE) [23], RCGA [13], DE [10], SPPSO [12], RQEA [10], PSO [27], chaotic differential evolution (CDE) [23], clonal selection algorithm (CSA) [28], TLBO [29], TLBO [18], improved quantum-behaved particle swarm optimization (IQPSO) [30], quasi-oppositional teaching learning based optimization (QTLBO) [29], Improved differential evolution (IDE) [21], adaptive chaotic differential evolution (ACDE) [23], real coded chemical reaction based optimization (RCCRO) [22], differential real-coded quantum-inspired evolutionary algorithm (DRQEA) [10], and adaptive chaotic artificial bee colony algorithm (ACABC) [26], quasi-oppositional group search optimization (QOGSO), as shown in Table 6. Results show that proposed method is better than previous methods used in the test system 2, case 1. As it can be seen, the minimum obtained cost is $41,274.42 which is related to ACABC [43] compare to $41,101.738 obtained by proposed method.

Table 6. Comparison of obtained optimal costs for test system 2 case 1.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SA [44]</td>
<td>45,466</td>
<td>-</td>
<td>-</td>
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<tr>
<td>DE [32]</td>
<td>44,526.11</td>
<td>-</td>
<td>-</td>
</tr>
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<td>CABC [43]</td>
<td>43,362.68</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>ADE [43]</td>
<td>43,222.41</td>
<td>-</td>
<td>-</td>
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<td>RCGA [34]</td>
<td>42,886.352</td>
<td>43,261.912</td>
<td>43,032.334</td>
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<td>DE [5]</td>
<td>42,801.04</td>
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<td>-</td>
</tr>
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<td>SPPSO [33]</td>
<td>42,740.23</td>
<td>43,622.14</td>
<td>44,346.97</td>
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<td>RQEA [25]</td>
<td>42,715.69</td>
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<td>PSO [45]</td>
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<td>-</td>
<td>-</td>
</tr>
<tr>
<td>CDE [42]</td>
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<td>-</td>
<td>-</td>
</tr>
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<td>CSA [46]</td>
<td>42,440.574</td>
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<td>-</td>
</tr>
<tr>
<td>TLBO [47]</td>
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<td>42,407.23</td>
<td>42,441.36</td>
</tr>
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<td>42,441.36</td>
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<td>IQPSO [48]</td>
<td>42,359.00</td>
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<td>-</td>
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<td>GSO [49]</td>
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<td>42,339.35</td>
<td>42,379.18</td>
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<td>42,193.46</td>
<td>42,202.75</td>
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<td>42,130.15</td>
<td>42,145.37</td>
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<td>IDE [40]</td>
<td>41,856.5</td>
<td>-</td>
<td>-</td>
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<tr>
<td>ACDE [42]</td>
<td>41,593.48</td>
<td>-</td>
<td>-</td>
</tr>
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<tr>
<td>DRQEA [25]</td>
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<td>-</td>
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<tr>
<td>ACABC [43]</td>
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<td>-</td>
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4.2.2. Test System 2 Case 2: Quadratic Cost Function with Valve-Point Loading

The valve-point effects and transmission losses are considered in this case, which make the problem more complex. The optimal result obtained by OQNLP is reported in Table 7. The hourly discharge of four hydro plants and the power generation of the hydro units are prepared in this table. In addition, power generation of three thermal plants are reported in Table 7. The power transmission loss of the hydrothermal system by applying Equation (13) during 24-h scheduling time interval is also reported in this table. In this case, considering Table 7, total generation of four hydro units and three thermal plants meets total load demand and power transmission loss of the system.

Table 7. Hourly plant discharges, power outputs and total thermal generation for test system 2 case 2.

<table>
<thead>
<tr>
<th>Hour</th>
<th>Hydro Plant Discharge, 10^6 m^3</th>
<th>Hydro Plant Generation (MW)</th>
<th>Thermal Plant Generation (MW)</th>
<th>Loss MW</th>
<th>Total Generation, MW</th>
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<td>7.280</td>
<td>8.237</td>
<td>13.107</td>
<td>12.74</td>
<td>229.518</td>
</tr>
<tr>
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<td>5.092</td>
<td>6.000</td>
<td>12.818</td>
<td>13.07</td>
<td>229.518</td>
</tr>
<tr>
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<td>6.000</td>
<td>12.991</td>
<td>13.07</td>
<td>229.518</td>
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<td>6.000</td>
<td>14.354</td>
<td>13.07</td>
<td>229.518</td>
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<td>14.501</td>
<td>13.07</td>
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<td>14.962</td>
<td>13.07</td>
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<td>16.133</td>
<td>13.07</td>
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<td>6.000</td>
<td>16.399</td>
<td>13.07</td>
<td>229.518</td>
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<td>6.224</td>
<td>7.064</td>
<td>7.097</td>
<td>13.07</td>
<td>229.518</td>
</tr>
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</table>

Comparisons of simulation results for this case study are accomplished in Table 8. It can be observed that the obtained result using the proposed method outperform the results of QEA [25], ABC [43], DE [32], SPPSO [50], RQEA [25], DNLPSO [15], PSO [51], CSA [44], TLBO [33], SA-MOCDE [37], GSA [52], QOTLBO [33], MOCA-PSO [53], IDE [40], RCGA-AFSA [34], QABDEVT [16], ACDE [54]. Taking into account transmission losses and valve-point effects, the best reported solution is related to ACDE [54], which obtained total cost of $41,593.48. However, the proposed method provided the optimal solution with the total operational cost of $41,350.5574 which is better than other methods.

Table 8. Comparisons of simulation results for test system 2 case 2.

<table>
<thead>
<tr>
<th>Optimization Method</th>
<th>Min. Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>QEA [25]</td>
<td>44,686.31</td>
</tr>
<tr>
<td>ABC [43]</td>
<td>43,362.02</td>
</tr>
<tr>
<td>QOOGSO [49]</td>
<td>43,560.35</td>
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<td>DE [32]</td>
<td>42,801.04</td>
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<td>SPPSO [50]</td>
<td>42,740.23</td>
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<td>RQEA [23]</td>
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<td>42,645.00</td>
</tr>
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<td>PSO [51]</td>
<td>42,474.00</td>
</tr>
<tr>
<td>CSA [44]</td>
<td>42,440.57</td>
</tr>
<tr>
<td>TLBO [37]</td>
<td>42,386.13</td>
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<td>SA-MOCDE [37]</td>
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<td>GSA [52]</td>
<td>42,032.35</td>
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<td>QOTLBO [33]</td>
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<td>ACDE [54]</td>
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<td>Proposed method</td>
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5. Conclusions

In this study, dynamic non-linear programming is introduced to obtain optimal scheduling of a hydrothermal system. The valve-point loading impact of conventional thermal units and system power transmission loss are considered in finding the optimal solution of the short-term hydro-thermal scheduling problem by studying two test systems. Optimal solutions are reported and analyzed, and are compared with those provided in recent papers. Results showed the capability of the proposed method to obtain better solutions in terms of total operational cost in comparison with other heuristic algorithms. Test system 1 includes four cascaded hydro units and one equivalent thermal plant, in which daily savings are $29,926.035 and $13,468.026 in comparison with previously reported solutions for both cases 1 and 2, respectively. In addition, for test system 2, which contains four cascaded hydro units and three thermal plants, daily savings are $172,682 and $242,9226 in comparison with reported solutions in previous studies for both cases 1 and 2, respectively. The optimal solutions show that the proposed method is an effective and high-performance technique to solve short-term hydro-thermal scheduling problem considering transmission losses and valve-point loading effects. The future research trends in the area of short-term hydro-thermal scheduling can be concentrated on consideration of limitations of AC network constraints. In addition, the unit commitment problem of hydrothermal systems, considering the start-up cost, minimum uptime, and minimum downtime of the generation units can be considered as another research topic in this area. Moreover, the unavailability of the generation units and consideration of renewable energy sources such as wind power are other exciting subjects to be investigated. Also, middle and long-term scheduling of hydro-thermal system, considering the installation and maintenance cost of hydro and thermal plants, may be introduced as interesting subject in the area of hydro-thermal systems.

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Conflicts of Interest: The authors declare no conflict of interest.

Nomenclature

Indexes
- $t$: Time interval of planning
- $N_s$: The number of thermal plants
- $N_h$: The number of hydro units

Constants
- $a_i$, $b_i$, and $c_i$: Cost coefficients of $i$th thermal plant
- $e_i$ and $f_i$: Valve-point coefficients of cost function of $i$th thermal unit
- $P_{min}$: Minimum power generation of thermal unit $i$
- $P_{max}$: Maximum power generation of thermal unit $i$
- $V_{min}$: Lower of operating volume of reservoir of $i$th hydro unit
- $V_{max}$: Upper bounds of operating volume of reservoir of $i$th hydro unit
- $Q_{min}$: Minimum release of water reservoir of the $i$th hydro plant
- $Q_{max}$: Maximum release of water reservoir of the $i$th hydro plant
- $V_{begin}$: Elementary volume of reservoir
- $V_{end}$: Final volume of reservoir
- $P_D$: Load demand at time $t$
- $C_{1,j}$, $C_{2,j}$, $C_{3,j}$, $C_{4,j}$, $C_{5,j}$, and $C_{6,j}$: Hydro power generation coefficients
- $\phi_j$: Set of instant upstream hydro plants of $j$th

Variables
- $P_t$: Power generated by the $i$th thermal plant at time $t$
- $P_j$: Generation of hydro units
- $P_{tL}$: Total transmission loss at time $t$
- $V_j$: The storage volume of reservoir
- $Q_j$: The water discharge amount
- $I_j$: The inflow rate of the reservoir

Acronyms
- STHTS: Short-term hydro-thermal scheduling
DNLP  Dynamic non-linear programming
MGS  Micro-grids
DGS  Distributed generations
DSM  Direct search method
ED  Economic dispatch
MABC  Modified artificial bee colony
DE  Differential evolution
MDNLPSO  Modified dynamic neighborhood learning based particle swarm optimization
QADEVT  Quadratic approximation based on differential evolution with valuable trade-off
PPO  Predator prey optimization
PSO  Particle swarm optimization
RCGA  Real coded genetic algorithm
LR  Lagrangian relaxation
MIP  Mixed integer programming
IMO  Improved merit order
ALHN  Augmented Lagrangian hopfield network
UC  Unit commitment
BFPSO  Bacterial foraging oriented by particle swarm optimization
DNLP  Dynamic non-linear programming
GAMS  General algebraic modeling system
LP  Linear programming
NLP  Nonlinear programming
MILP  Mixed-integer linear programming
MINLP  Mixed-integer nonlinear programming
DNLP  Dynamic nonlinear programming
QEA  Quantum-inspired evolutionary algorithm
WDA  Whole distribution algorithm
SPSO  Small population-based particle swarm optimization
RQEA  Real-coded quantum-inspired evolutionary algorithm
MDE  Modified differential evolution
DNLPSO  Dynamic neighborhood learning based particle swarm optimization
HCRO  Hybrid chemical reaction optimization
MAPSO  Modified adaptive particle swarm optimization
RCGA-AFSA  Real coded genetic algorithm and artificial fish swarm algorithm
TLBO  Teaching learning-based optimization
SOHPSO_TVAC  Self-organizing hierarchical particle swarm optimization technique with time-varying acceleration coefficients
IDE  Improved differential evolution
FAPSO  Fuzzy adaptive particle swarm optimization
ACDE  Adaptive chaotic differential evolution
CABC  Adaptive chaotic artificial bee colony
CSA  Clonal selection algorithm
IQPSO  Improved quantum-behaved particle swarm optimization
GSO  Group search optimization
ACDE  Adaptive chaotic differential evolution algorithm

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