Identification of Critical Transmission Lines in Complex Power Networks

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Abstract: Growing load demands, complex operating conditions, and the increased use of intermittent renewable energy pose great challenges to power systems. Serious consequences can occur when the system suffers various disturbances or attacks, especially those that might initiate cascading failures. Accurate and rapid identification of critical transmission lines is helpful in assessing the system vulnerability. This can realize rational planning and ensure reliable security pre-warning to avoid large-scale accidents. In this study, an integrated “betweenness” based identification method is introduced, considering the line’s role in power transmission and the impact when it is removed from a power system. At the same time, the sensitive regions of each line are located by a cyclic addition algorithm (CAA), which can reduce the calculation time and improve the engineering value of the betweenness, especially in large-scale power systems. The simulation result verifies the effectiveness and the feasibility of the identification method.

Keywords: critical line; vulnerability; power flow redistribution; integrated betweenness; cyclic addition algorithm

1. Introduction

Due to the expansion of power grids and increases in load demand, the power systems gradually approach their operating limits. This increases the uncertainty of the system dynamic behavior and the risk of large blackouts [1]. Many power outages have occurred in recent years [2,3], for instance, the western North America blackouts in 1996 [4], the North American blackouts in 2003 [5], and the India blackouts in 2012 [6]. Frequent accidents expose the potential problems of current power system analysis methods, especially concerning system vulnerability.

In a blackout, critical lines often trigger or promote the failure propagation. These lines usually play important roles in the network structure, and their disconnections greatly increase power system vulnerability and can even cause serious consequences. Accurately identifying the critical transmission lines can realize rational planning and provide reliable security pre-warning. It also helps to ensure control measures are carried out effectively to avoid the occurrence of blackouts.

At present, researches on the identification of critical or vulnerable transmission lines are usually based on three main factors: the network topology [7–16], the electrical characteristics [17–23], and the failure mechanism [24–27]. Power grids can be conceptually described as networks, so that topology-based theory provides a feasible way to study network vulnerability [7] and locate the critical transmission lines [8,9]. Some indices, such as betweenness [10–13], are proposed to describe the importance of different components. In topology-based methods, the betweenness of an edge is defined as the number of the shortest paths through it. Reference [14–16] indicated that the nodes...
and lines with high betweenness and high degrees play key roles in guaranteeing the connectivity and stability of the grid. However, pure topology-based methods ignore the electrical characteristics and often cannot completely reflect the complexity of operations [17]. Some researchers try to locate important substations and lines by using the electrical parameters [18–22]. An electrical betweenness was proposed in Reference [19] to quantify the role of each line in the entire grid by using the sum of absolute value of power in different directions. Reference [20] improved the electrical betweenness by considering the maximal load demand and the capacity of generators in power grids. A concept of network centrality for power systems based on system responses and topological features was presented in Reference [21]. It aims to identify critical components with an equivalent electrical model. Chen et al. [22] calculated electrical efficiency based on the admittance matrix, but assumed that power was transmitted through the most effective paths. In reality, power flow is not limited to transfer along the lines with the smallest impedance, but in all possible ways. In Reference [23], a hybrid flow betweenness (HFB) based identification method was proposed. The new betweenness covers the direction of power flow, the maximum transmission capacity, the electrical coupling degree between different lines, and outage transfer distribution factor with a more comprehensible physical background. From the perspective of the development mechanism of large-scale blackouts, Reference [24] searched for sensitive transmission lines on the basis of the criticality of system cascading collapses. Wang et al. [25] used the fault chain theory to perform cascading failures initiated by faults on different transmission lines. The number of disconnected lines is basically used to derive the vulnerability index for identifying the sensitive initial lines. This type of method [24–27] is usually based on numerous simulations, and cannot reflect the actual power flow properties and operational states well.

It is reasonable to assess network vulnerability and identify critical transmission lines of power systems with electrical and topological characteristics. However, the existing methods are mainly based on the inherent parameters of the whole power system, without enough ability to take real-time data into account. In this study, a new integrated betweenness is proposed to comprehensively consider the function of a line and the impact when it is removed from a power system. Compared with previous studies, the betweenness evaluates the role of the transmission line from the perspective of the power transmission distance, improves the impact analysis of fault flow distribution, and considers the system security with the tolerance of the remaining network. At the same time, a new searching algorithm, named cyclic addition algorithm (CAA), is proposed to locate the sensitive regions that are significantly affected by a removed line. Thus the calculation time of the betweenness can be reduced and the new identification method is more valuable in engineering applications.

The rest of the paper is structured as follows: Section 2 introduces the definition and the calculation method of the integrated betweenness of a transmission line, which consists of the indices of the power transmission, the fault flow distribution, and the influence on system security. Section 3 presents the detailed identification process with integrated betweenness and CAA. The simulation results with the IEEE 39-bus system are provided in Section 4, which prove the accuracy and effectiveness of the identification method. Finally, conclusions are given in Section 5.

2. Integrated Betweenness to Identify Critical Lines

“Vulnerability” or its opposite concept “robustness” is often used to evaluate the ability of a power grid to provide key services or functions in normal operations, with random failures or under intentional attacks [18]. A critical transmission line plays an important role in maintaining the security and stability of the power system. When it is removed due to external attacks or its own failure, the system may suffer from a serious impact, and even have a high probability of cascading trips. Therefore, the integrated betweenness proposed in this study comprehensively considers the role of a transmission line in a power system, the redistribution of power flow caused by a fault on the line, and the corresponding influence on system security.
Taking line \( i \) as an example, \( D_i, H_i \) and \( C_i \) are respectively the indices of the power transmission, the fault flow distribution, and the influence on system security. The integrated betweenness \( W_i \) of line \( i \) can be calculated by:

\[
\begin{align*}
W_i &= D'_i + H'_i + C'_i \\
D_i &= Dp_i - DP0 \\
H_i &= \frac{\sum Z_m^i}{B_i} \\
C_i &= \frac{1}{2} (C_i^g + C_i^{\text{max}})
\end{align*}
\]

(1)

where \( D'_i, H'_i, \) and \( C'_i \) are normalized absolute values of \( D_i, H_i, \) and \( C_i, \) as can be obtained by a linear normalization method. The values of \( D'_i, H'_i, \) and \( C'_i \) are all between 0 and 1. \( D_i \) is the index of power transmission, and is used to reflect the role of line \( i \) in the power transmission network. It can be calculated by the difference of the transmission distances of the whole power grid with and without line \( i \). \( DP0 \) is the transmission distance of active power in the grid inclusive of line \( i \), while \( Dp_i \) is the transmission distance after the removal of line \( i \); \( H_i \) is the index of fault flow distribution. It reflects the impact caused by a fault on line \( i \). \( H_i \) can be obtained by the ratio of the amount and the distribution entropy \( B_i \) of transferred power. \( Z_m^i \) is the power change on line \( m \) when a fault happens on line \( i \). \( \Psi \) is the set of lines with increased power. The greater the transferred power is, the larger the impacts on the other lines are. The smaller the distribution entropy \( B_i \), the more concentrated the power flow transfer is. A substantial increase in the transmission power can bring about harmful effects, such as the exposure of hidden failures. \( C_i \) is the index of influence on system security. \( C_i^g \) and \( C_i^{\text{max}} \) are respectively the global and the maximum influence considering the tolerance of the remaining network. The calculation method of \( D_i, H_i, \) and \( C_i^{\text{max}} \) is described in Sections 2.1–2.3.

2.1. The Index of Power Transmission

The importance of a line depends on its role in power transmission. The transmission distance of active power is defined as:

\[
D_p = \sum_{l \in T} \sum_{g \in G, d \in L} |x_l p_l(g, d)|
\]

(2)

where \( x_l \) is the electrical length of line \( l \). It can be expressed by the reactance value, \( x_l = x_{ab} \), where \( x_{ab} \) is the element at row \( a \) and column \( b \) in the reactance matrix of a power grid; \( a \) and \( b \) are the two ends of line \( l \); \( p_l(g, d) \) is the active power on line \( l \) when power is transmitted from generator node \( g \) to load node \( d \); \( G \) is the set of generators, \( L \) is the set of loads and \( T \) is the set of lines.

The above equation can be simplified to:

\[
D_p = \sum_{l \in T} |x_l| |p_l|
\]

(3)

where \( p_l \) is the total active power on line \( l \), as calculated by a DC power flow model [28]:

\[
p_l = p_{ab} = \frac{\theta(a) - \theta(b)}{x_{ab}}
\]

(4)

where \( \theta(a) \) and \( \theta(b) \) are the phase angles of \( a \) and \( b \). The index of power transmission of line \( i \) is [29]:

\[
D_i = Dp_i - DP0
\]

(5)

where \( DP0 \) and \( Dp_i \) are the transmission distances with and without transmission line \( i \). \( D_i \) can express the change of the transmission distance of active power directly. From Equations (3)–(5), \( D_i \) can be expressed as:
\[ D_i = \sum_{l \in T_i} |\Delta \theta_l| - \sum_{l \in T_0} |\Delta \theta_l| \]  

where \( \Delta \theta_l \) is the phase difference of the two ends of line \( l \); \( T_0 \) and \( T_i \) are the sets of lines in the system before and after the removal of line \( i \). Based on DC power flow model, \( \Delta \theta \) can be calculated by:

\[ \Delta \theta = A^T \theta = A^T B^{-1} P^g \]  

where \( \Delta \theta \) is the matrix of phase difference; \( A \) is the node-branch incident matrix; \( B \) is the electrical susceptance matrix; and, \( P^g \) is the vector of injected power. Equation (7) can be used to calculate \( \Delta \theta_0 \) and \( \Delta \theta_i \), the matrices of transmission distances with and without line \( i \). Equation (5) can then be expressed as:

\[ D_i = \sum_{a=1}^{r_i} |\Delta \theta_{ia}| - \sum_{a=1}^{r_0} |\Delta \theta_{0a}| \]  

where \( r_0 \) and \( r_i \) are the numbers of lines in the system before and after the removal of line \( i \); \( \Delta \theta_{0a} \) and \( \Delta \theta_{ia} \) are the elements at row \( a \) in matrix \( \Delta \theta_0 \) and \( \Delta \theta_i \).

2.2. The Index of Fault Flow Distribution

\( F^m_i(g, d) \) is defined as the change of power on line \( m \), as related to the power transferred from generator \( g \) to load \( d \), when a fault happens on line \( i \). It can reflect the impact on line \( m \) caused by the removal of faulty line \( i \). \( F^m_i(g, d) \) can be calculated by the line outage distribution factor (LODF) [30]:

\[ F^m_i(g, d) = P_{gd} \times \text{LODF}^m_i(g, d) \]  

where \( \text{LODF}^m_i(g, d) \) is the power increment of line \( m \) when unit power is transferred between generator \( g \) and load \( d \), and a fault happens on line \( i \); \( P_{gd} \) is the power flow from generator \( g \) to load \( d \), as calculated by the power flow tracing method [31].

Considering the reverse overload, the change of power can be calculated as:

\[ Z^m_i = \left| \sum_{g \in G, d \in L} P_{0m}(g, d) + \sum_{g \in G, d \in L} F^m_i(g, d) \right| - \sum_{g \in G, d \in L} P_{0m}(g, d) \]  

where \( P_{0m}(g, d) \) is the power flow on line \( m \) in normal operating condition, and \( Z^m_i \) is the change of power on line \( m \) when line \( i \) is disconnected. In the DC power flow model, \( Z^m_i \) can also be obtained using the calculation result of Equation (4). For the lines with increased power, the sharing ratio of power flow transfer on each line is:

\[ \delta^m_i = \frac{Z^m_i}{\sum_{m \in \Psi} Z^m_i} \]  

where \( \delta^m_i \) is the sharing ratio of line \( m \) when line \( i \) is disconnected; \( \Psi \) is the set of lines with increased power. The entropy of the fault flow distribution is:

\[ B_i = - \sum_{m \in \Psi} \delta^m_i \ln \delta^m_i \]  

The entropy \( B_i \) is used to describe the distribution of \( Z^m_i, n = 1, 2, 3, \ldots, m, \ldots \) When the power is uniformly distributed, each component will shoulder the burden equally. The sharing ratio of each path is \( \delta = 1/N \), where \( N \) is the number of transferable paths. \( B_i \) is the maximum, as is \( \ln N \). Similarly, the smaller the value of \( B_i \) is, the more concentrated the transferred power flow is [32]. It means that the disconnection of line \( i \) will have significant influence on a few lines, which may overload these...
lines and even trigger a cascading failure. Thus, the index $H_i$ can be used to show the distribution of fault flow, i.e.,

$$H_i = \frac{\sum_{m \in \Psi} Z_i^m}{B_i}$$

(13)

### 2.3. The Index of Influence on System Security

Although the disconnections of some lines can cause a significant transfer of power flow, detrimental consequences may not be caused if other lines have large capacities and can successfully share the transferred power. Sometimes, the influence is not great, but the operating condition is close to the safety limits, and the disconnection of a line may force the system into a dangerous operating condition. Hence it is important to consider the redistribution of power flow and the tolerance of the remaining network when evaluating the importance of a line in the whole system. In this paper, the global influence and the influence on each line caused by the removal of line $i$ are analyzed. The index of the influence on line $m$ is:

$$C_i^m = \frac{Z_i^m}{S_m}$$

(14)

where $S_m$ is the remaining available channel of line $m$ before the removal of line $i$. The global influence can be calculated by:

$$C_i^\Psi = \frac{1}{r_0 - 1} \sum_{m \in \Psi} C_i^m$$

(15)

where $r_0$ is the number of lines in the power system, and $C_i^\Psi$ is the average value of $C_i^m$. The influence on system security is:

$$C_i = \frac{1}{2}(C_i^\Psi + C_i^{max})$$

(16)

where $C_i^{max}$ is the maximum value of $C_i^m$, and $C_i$ is the average value of the global influence and the maximal influence.

### 2.4. The Threshold of the Integrated Betweenness

If the removal of line $i$ causes the overload of any other line in the power grid, line $i$ can be considered a critical component [33]. In order to better assess whether the line is critical, three thresholds $D_{th}$, $H_{th}$ and $C_{th}$ are defined next:

1. $D_{th}$:

$$D_{th} = \min_{a \in T_0} |x_a| p_cpa - \min_{b \in T_0} |x_b| p_b$$

(17)

where $x_a$ is the electrical length of line $a$; $p_cpa$ is the remaining available channel of line $a$; $x_b$ is the electrical length of line $b$; $p_b$ is the total active power on line $b$; and, $T_0$ is the set of lines in the power system. For line $i$, $D_i = D_{pi} - D_{p0}$ and $D_{pi}$ has already subtracted $|x_i| p_i$, so min $|x_b| p_b$ is also subtracted from $D_{th}$.

2. $H_{th}$: $H_i$ can reflect the impact caused by a fault on line $i$. To avoid missing critical lines, $H_{th}$ should be the minimum $H_i$ when one remaining line is overloaded after the fault. To get the minimum $H_i$, the entropy $B_i$ of the fault flow distribution should be as large as possible and $\sum_{m \in \Psi} Z_i^m$ in Equation (13) should be as small as possible. Let us assume the smallest remaining available channel of a transmission line is $p_{minp}$, and $p_{minp} = \min_{a \in T_0} p_ms, a \in T_0$. $B_i$ takes the maximum value, i.e., $\ln(r_0 - 1)$, and $\sum_{m \in \Psi} Z_i^m$ takes $p_{minp}$ ignoring the complex conditions. The threshold of $H$ is:
\[ H_{th} = \frac{p_{\text{min,cp}}}{\ln(r_0 - 1)} \]  

(18)

(3) \( C_{th} \): When only one remaining line is overloaded and other lines are not affected, \( C_{i_{\text{max}}} = 1 \) and \( C_{i_{\psi}} = 1/(r_0 - 1) \). The threshold of \( C \) is:

\[ C_{th} = \frac{1}{2}(1 + \frac{1}{r_0 - 1}) \]  

(19)

\( W_{th} \) equals to the sum of normalized \( D_{th}, H_{th} \) and \( C_{th} \), i.e.,

\[ W_{th} = D'_{th} + H'_{th} + C'_{th} \]  

(20)

where \( D'_{th}, H'_{th} \) and \( C'_{th} \) are normalized absolute values of \( D_{th}, H_{th} \) and \( C_{th} \). The transmission line, the integrated betweenness of which is greater than \( W_{th} \), can be regarded as a critical component. The mathematical formulae for \( D_{th}, H_{th} \), and \( C_{th} \) give out small values. Such small threshold values prevent missing the detection of critical lines. After the above process, these lines can be sorted according to \( W \).

3. Identification Process with Integrated Betweenness and CAA

The integrated betweenness of each line is affected by the changes in the grid’s structure or the operating mode. The calculation time is large when the approach is applied to the data of the whole power network. In fact, \( D_i, H_i \), and \( C_i \) are all related to the changes of power flow before and after line \( i \) is removed, where the changes mainly occur on particularly sensitive lines. Therefore, a cyclic addition algorithm (CAA) is proposed to locate the sensitive regions which consist of the transmission lines with large variations of power. The impact on the sensitive regions is analyzed rather than the impact on the whole power network. Thus, the calculation time of the betweenness can be significantly reduced and the proposed identification method is more suited to the real-time analysis.

The process to identify the critical transmission lines is shown in Figure 1. First, the basic data of the grid, such as the structural parameters and the information of power flow, are obtained. The data can be collected by phasor measurement units (PMUs). When the network structure or operating mode changes, the identification process of critical lines in this state will start. Possible changes of the network structure or operating mode include the disconnection/failure of primary equipment (generators, transformers, or lines), the connection/reconnection of primary equipment, and significant redistribution of power flow within the grid. Such changes can be caused by (1) faults, e.g., line trip, generator trip, and line failure; (2) planned adjustments, e.g., the operation of new generators/lines and planned load shedding. Then \( W_{th} \) and the integrated betweenness of each line are calculated: sensitive regions can be located by CAA, and the remaining regions are regarded as non-sensitive regions in this study. Since the influence of the non-sensitive regions on the calculation results is quite small, those regions can be simplified as sources and loads. The number of lines and substations in the system is greatly reduced after the simplification, so that the integrated betweenness \( W \) can be calculated more quickly. With the calculation method introduced in Section 2, \( W \) of each line can be obtained. Finally, the critical lines can be identified.
Select line $i$
Search for the sensitive regions of line $i$ with CAA
Remove the lines and substations outside the sensitive regions and simplify the non-sensitive regions into sources or loads
Obtain the basic data of power network
Calculate $D_i$, $H_i$ and $C_i$ for line $i$

Does the structure or operating mode of power system change?

Yes
Let $i = 1$
Select line $i$

No

Have all lines been analyzed?

Yes
Calculate $D_0$, $H_0$, $C_0$
Calculate $W_0$ and $W$ of each line
Get critical lines whose integrated betweenness is greater than $W_0$ and rank the lines according to $W$

end

Figure 1. The flow chart of the proposed identification method.

**Location of Sensitive Regions with CAA**

The power flow is transferred on all feasible paths, but the transferred power on each path is quite different. As can be seen from previous studies [26–31], when a transmission line is disconnected or removed from the power network, the power is usually transferred to other paths connected with the two-end nodes of the removed line, and mainly along the paths with short electrical distances, as is shown in Figure 2.

![Figure 2](image-url)

**Figure 2.** (a) Power flow of the 6-bus system in normal operation; (b) The redistribution of power flow caused by the removal of line 1–3.
The topological analysis-based method can provide a quick and effective way to identify the network structure and find propagation paths of power flow transfer [34]. Some new approaches, such as the compressive sensing-based approach [35] and graphical learning-based approach [36,37] were proposed in recent years. Based on Dijkstra’s algorithm [38], CAA is proposed in this study. The sensitive regions can be obtained through the cyclic addition of different lines. The following is a detailed description of the process.

Supposing there are \( a \) substations and \( b \) lines, all substations in the power grid can be simplified as nodes and form \( V \) according to graph theory, as is \( V = \{v_1, v_2, \ldots, v_a\} \). All lines are represented by the edge set \( E, E = \{e_1, e_2, e_3, \ldots, e_b\} \). \( V \) and \( E \) constitute \( G(V,E) \), the topology graph of the power grid. All different paths connecting \( v_i \) and \( v_j \) in graph \( G \) form set \( D(G, v_i, v_j) \), and the length of path \( d \) is \( l(d) \). The line reactance is used as the weight of an edge, and \( l(d) \) is the sum of the weights of all edges in path \( d \).

Theoretical principle 1: According to graph theory, if \( G \) cannot be fully connected when node \( v \) is removed, \( v \) is called the cut point in \( G \), such as point \( v_4 \) in Figure 3. Any edge and the paths connected with its two ends can only exist in the same part divided by cut points.

![Figure 3. The cut point in a graph.](image)

Theoretical principle 2: According to the conclusion in Reference [39], any path between two nodes in graph \( G(V, E) \) must be the shortest path between the same nodes in \( G_x(V_x, E_x) \), where \( G_x \) is a sub-graph of \( G \) with a few (including \( b \)) edges. It can be described as:

\[
d \in D(G, v_p, v_q) \iff d = d_{\text{shortest}}(G_x, v_p, v_q) \tag{21}
\]

where \( d_{\text{shortest}}(G_x, v_p, v_q) \) is the shortest path between \( v_p \) and \( v_q \) in graph \( G_x \). Therefore, the path between two nodes in \( G(V, E) \) can be transformed into the shortest path between the same nodes in a sub-graph.

In this study, line \( e_0 \) is defined as the removed line, while \( v_p \) and \( v_q \) are its two-end nodes. Vector \( L \) is built to store the shortest distance between any two nodes which has been found currently. For the graph with \( a \) nodes, there are \( C_a^2 \) elements in \( L \) and the elements can be described as \( L(1–2), L(1–3), L(1–4), \ldots, L(1–a), L(2–3), L(2–4), \ldots, L((a − 1)–a) \), where \( L(i–j) \) is the shortest distance between \( v_i \) and \( v_j \), and \( i \) and \( j \) can be interchanged. Vector \( HJ \) is built to store the successor nodes along the shortest path. Similarly, there are \( C_a^2 \) elements in \( HJ \) and the elements can be described as \( HJ(1–2), HJ(1–3), HJ(1–4), \ldots, HJ(1–n), HJ(2–3), HJ(2–4), \ldots, HJ((a − 1)–a) \). Each sub-graph has corresponding values for \( L \) and \( HJ \).

Step 1: Find cut points and divide the graph into different parts. Find the parts \( G' \) where \( e_0 \) is located. Suppose that there are \( m \) lines and \( n \) nodes in \( G' \). Delete \( e_0 \) and get all the other edges in graph \( G' \). Let \( k = 1 \).

Step 2: Select \( k \) edges, and there are \( C_{n–1}^k \) different selection results. Let \( c = 1 \).

Step 3: Deal with the selection result \( c \), where \( k \) edges and their connected nodes constitute a sub-graph \( G_{kc}' \). If \( v_p \) or \( v_q \) is an isolated node without any connected lines, or \( v_p \) and \( v_q \) are located in different islands in \( G'_{kc}' \), go to step 8, i.e., the sub-graph can be ignored directly; otherwise go to step 4.

Step 4: Take away and rank \( k \) edges. Let \( f = 1 \) and initialize \( L_{kc} \) and \( HJ_{kc} \). When there is no edge in graph \( G'_{kc}' \), the values of the elements in \( L_{kc} \) are equal to an infinite value, as is \( L_{kc} = [\infty, \infty, \infty, \ldots, \infty] \). All elements in \( HJ_{kc} \) are \( \emptyset \), showing there is no path currently.
Step 5: Put edge $f$ back. The two-end nodes of $f$ are $v_u$ and $v_v$, $v_u$ and $v_v \in V'_{kc}$. Next apply the following judgment: If $L_{kc}(u - v) < l(f)$, go directly to step 7. Otherwise, let $L_{kc}(u - v) = l(f)$, $H_{kc}(u - v) = v$, and go to step 6.

Step 6: Deal with $L_{kc}$ and $H_{kc}$:

For $w$ ($1 \leq w \leq n$, $w \neq u$, $w \neq v$):

If $L_{kc}(u - w) > L_{kc}(v - w)$, change the value of $L_{kc}(u - w)$ into $L_{kc}(u - v) + L_{kc}(v - w)$, and change $H_{kc}(u - w)$ into $v$; if $L_{kc}(v - w) > L_{kc}(v - u) + L_{kc}(u - w)$, change the value of $L_{kc}(v - w)$ into $L_{kc}(u - v) + L_{kc}(u - w)$, and change $H_{kc}(v - w)$ into $u$;

If $L_{kc}(u - w) = L_{kc}(u - v) + L_{kc}(v - w) \neq \infty$, insert $v$ into $H_{kc}(u - w)$; if $L_{kc}(v - w) = L_{kc}(u - v) + L_{kc}(u - w) \neq \infty$, insert $u$ into $H_{kc}(v - w)$; then go to step 7.

Step 7: If $f = k$, go to step 8; otherwise $f = f + 1$, then go back to step 5.

Step 8: If $c = C_{m-1}^{k}$, go to step 9; otherwise $c = c + 1$, then go back to step 3.

Step 9: if $k = m-1$, go to step 10; otherwise $k = k + 1$, then go back to step 2.

Step 10: All the paths between $v_q$ and $v_q$ and their lengths can be got. Remove duplicate paths.

Supposing that there are $y$ paths between $v_p$ and $v_q$, the lengths of all paths ($l(1), l(2), ..., l(y)$) can be derived by the above process. According to the principle of electric circuits, the transferred power flow on path $d$ changes inversely with the length $l(d)$. Thus, the power sharing coefficient of edge $i$ is:

$$\xi_i = \sum_{w \in \mu} \frac{1}{l(o)} \sum_{s=1}^{y} \frac{1}{l(s)}$$

where $\mu$ is the set of paths which contain line $i$. Judge whether the power sharing coefficient can satisfy:

$$\xi_{ih} \geq \xi_{th}$$

where $\xi_{ih}$ is the threshold value, $\xi_{th} \geq 0$. If (23) is satisfied, line $i$ will be regarded as a sensitive line, as is closely related to line $e_0$. The sensitive lines should be selected effectively and the sensitive regions which consist of sensitive lines should be fully connected. According to the definition of sensitive lines or transmission sections in Reference [40], the threshold value usually varies between 0.2 and 0.3. Based on the above analysis, the threshold $\xi_{th}$ can be obtained by the following iterative process: $\xi_{th}$ is initialized with 0.3. If the sensitive regions are not fully connected, $\xi_{th}$ will be decreased with a fixed step size, until the requirement of full connection can be satisfied or the value of $\xi_{th}$ is 0.

To improve the accuracy of integrated betweenness and to avoid missing sensitive lines, $\xi_{th}$ can be taken as 0 directly in an extreme situation.

Sensitive regions can be obtained by the above process. For $G'$ with $m$ lines and $n$ nodes, the calculation amount of the cyclic process in CAA in the worst condition is:

$$P = \sum_{k=1}^{m-1} C_{m-1}^{k} \cdot k \cdot (4n - 6)$$

where $P = 1$ means that one element in $L$ or $HF$ is updated once. For $G'$ with $m$ lines and $n$ nodes, $C_{m-1}^{k}$ is the number of sub-graphs with $k$ lines. In the worst case, all sub-graphs can satisfy the requirement in step 3, and none will be ignored. $2(n - 2) + 1$ elements in $L$ and $2(n - 2) + 1$ elements in $HF$ are updated when one transmission line is put back through steps 5 and 6. The process of CAA only contains some simple comparisons based on a pure topology model. Therefore, the calculation time of the identification method can be significantly reduced. At the same time, it should be noticed that CAA is effective for most lines in power systems. For the lines which do not have other paths connected with their two ends, the removal of the lines can result in the islanding phenomenon. In these cases, the injected power of sources and absorbed power of loads in each island should be balanced first.
After that, the calculation of the integrated betweenness can be carried out using the analysis of the whole power network.

4. Test Cases

In this section, IEEE 39-bus system is used as the test system to demonstrate the effectiveness of the proposed method.

4.1. Location of Sensitive Regions

4.1.1. Location of Sensitive Regions with CAA

Line $e_{33}$ is taken as an example and used to show the detailed process to locate the sensitive regions. Firstly, all substations and lines are simplified as nodes and edges. Line $e_{33}$ is the removed line, while $v_{26}$ and $v_{29}$ are its two-end nodes. Divide the graph into different parts with cut points $v_{26}$ and $v_{16}$ in Figure 4. $G'$ which contains $v_{26}, v_{28}, v_{29}, e_{32}, e_{33}$ and $e_{34}$ is found, and then $e_{33}$ is deleted. The edges $e_{32}$ and $e_{33}$ in graph $G'$ are taken away. Let $k = 1$. Select 1 edge, and it can be seen that there is no path between $v_{26}$ and $v_{29}$ in all the sub-graphs with one edge and two nodes. Let $k = 2$. Taking sub-graph $G'_{21}$ as an example, the sub-graph is shown in Figure 5. Since the topology of $G'_{21}$ is simple, the power sharing coefficient of each edge can be obtained directly. To better illustrate the application of CAA, the calculation process is introduced in detail.

![Figure 4. IEEE 39-bus system.](image)

![Figure 5. Topology of sub-graph $G'_{21}$.](image)

With the analysis of step 3, the sub-graph is kept. Take away and rank the 2 edges. Let $f = 1$, and initialize $L_{21}$ and $H_{21}$. The elements in $L_{21}$ are $[L_{21}(26–28), L_{21}(26–29), L_{21}(28–29)]$. The elements in $H_{21}$ are $[H_{21}(26–28), H_{21}(26–29), H_{21}(28–29)]$. $L_{21} = [\infty, \infty, \infty]$ and $H_{21} = [\emptyset, \emptyset, \emptyset]$.

Put edge $e_{34}$ back. The weight of the edge $l(e_{34})$ is 0.0151, which is smaller than $L_{21}(28–29)$. Therefore, let $L_{21}(28–29) = 0.0151$. After the step 5, $L_{21} = [\infty, 0.0151, \infty]$ and $H_{21} = [\emptyset, \emptyset, 29]$. Because

Similarly, put edge $e_2$ back. $L_21 = [0.0474, 0.0625, 0.0151]$ and $H_{21} = [28, 28, 29]$.

The length of the shortest path between nodes $v_{26}$ and $v_{29}$ in graph $G'_{21}$ is 0.0625 according to $L_21(26–29)$. The successor node of $v_{26}$ is $v_{28}$, and the successor node of $v_{28}$ is $v_{29}$, which can be got by $H_{21}(26–29)$ and $H_{21}(28–29)$. Then the shortest path $v_{26}–v_{28}–v_{29}$ can be obtained. The power sharing coefficients of $e_{32}$ and $e_{34}$ are 1. $\xi_{lh}$ is set to 0.3. The sensitive region, which includes line $e_{32}$ and $e_{34}$ is now derived.

By two simple comparisons, the analysis of all lines in the whole network can be simplified as the analysis of two lines. The order of the matrixes, such as $A$ and $B$ in Equation (7), can be reduced from 39 to 3 during the calculation of the integrated betweenness.

4.1.2. Comparison of CAA and Depth-First-Search Method

For the purpose of comparing the performance of CAA and Depth-First-Search (DFS) [41] algorithm, line $e_{33}$ is used as an example. Both algorithms are used separately to find the paths from $v_{26}$ to $v_{29}$ after line $e_{33}$ is removed.

The CAA proposed in the study uses the topological method to search for all possible paths connecting the two-end nodes of the removed line and obtain the power sharing coefficients of different edges. The DFS algorithm explores a graph by starting at a node and going as deep as possible. Such DFS traversal is a type of backtracking method, and therefore exhibits poor performance.

With CAA, the relevant sub-graph can be located at first, which effectively cleans up all useless edges and reduces the search scope. Then, the path and its length can be obtained directly after two comparisons. With DFS, $v_{26}$ is set as the starting node. $v_{25}$, $v_{27}$ and $v_{28}$ can be reached in the next step. The paths through $v_{25}$ or $v_{27}$ are not feasible by several explorations and backtracking. The only feasible path is $v_{26}–v_{28}–v_{29}$.

As can be seen from Section 4.1.1 and the above analysis, the proposed CAA can delete useless lines and sub-graphs to avoid invalid search directions. In this case, the search results with CAA and DFS are identical, but the computation time with CAA is 78.6% shorter than that with DFS.

4.1.3. Accuracy Analysis of the Location Method

(1) To show the validity of the location method, branch $e_{33}$ is removed at 0.5 s. The simulation result based on PSASP is shown in Table 1. As can be seen from the simulation results, there is a large increase in the power transferred on the transmission lines in the sensitive regions, while the increases on un-sensitive lines are much smaller.

**Table 1. Simulation result when line $e_{33}$ is removed.**

<table>
<thead>
<tr>
<th>Line</th>
<th>Power Change (MW)</th>
<th>Line</th>
<th>Power Change (MW)</th>
<th>Line</th>
<th>Power Change (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$e_{34}$</td>
<td>12</td>
<td>$e_{13}$</td>
<td>23</td>
<td>$e_{2}$</td>
</tr>
<tr>
<td>2</td>
<td>189.7265</td>
<td>13</td>
<td>0.584</td>
<td>24</td>
<td>0.6828</td>
</tr>
<tr>
<td>3</td>
<td>186.0633</td>
<td>14</td>
<td>0.103</td>
<td>25</td>
<td>0.6843</td>
</tr>
<tr>
<td>4</td>
<td>2.514</td>
<td>28</td>
<td>0.0999</td>
<td>26</td>
<td>0.6841</td>
</tr>
<tr>
<td>5</td>
<td>4.037</td>
<td>29</td>
<td>0.0051</td>
<td>27</td>
<td>0.6841</td>
</tr>
<tr>
<td>6</td>
<td>2.1787</td>
<td>25</td>
<td>0.0082</td>
<td>28</td>
<td>0.6841</td>
</tr>
<tr>
<td>7</td>
<td>1.964</td>
<td>27</td>
<td>0.0305</td>
<td>29</td>
<td>0.6841</td>
</tr>
<tr>
<td>8</td>
<td>1.3781</td>
<td>16</td>
<td>0.13791</td>
<td>30</td>
<td>0.6843</td>
</tr>
<tr>
<td>9</td>
<td>1.3198</td>
<td>17</td>
<td>0.0565</td>
<td>31</td>
<td>0.6843</td>
</tr>
<tr>
<td>10</td>
<td>1.2928</td>
<td>18</td>
<td>0.0709</td>
<td>32</td>
<td>0.6843</td>
</tr>
<tr>
<td>11</td>
<td>1.2161</td>
<td>19</td>
<td>0.0709</td>
<td>33</td>
<td>0.6843</td>
</tr>
</tbody>
</table>

(2) Line $e_{20}$ is removed at 0.2 s. The simulation result is shown in Table 2. When $\xi_{lh}$ is 0.3, there are $e_4$, $e_5$, $e_6$, $e_{12}$, $e_{16}$, $e_{17}$, $e_{18}$, $e_{19}$, $e_{21}$, $e_{25}$, $e_{26}$, $e_{30}$, and $e_{31}$ in sensitive regions. The first 14 branches with large power variations are included.
4.2. Analysis of Identification Results

4.2.1. Identification Result with Integrated Betweenness and CAA

ξth is set as 0 to insure the accuracy of the identification. The capacity of each line is 5 times the initial active power. The sensitive regions can be located by CAA, and the non-sensitive regions are simplified as sources and loads. Based on that, the integrated betweenness of each line can be calculated. For the lines whose removal can result in islanding phenomenon, the injected power of sources and the absorbed power of loads in each island should be balanced. In this study, the absorbed power of each load and the injected power of each source are reduced in proportion to their initial power and according to the minimum load shedding method as described in Reference [42].

During the identification process, the shortest computation time of W for a single transmission line is 0.515 ms, and the longest time for a single line is 8.416 ms. The computation time of the whole process is 0.061 s. Without CAA, the computation time is 0.286 s when the approach is applied to the data of the whole power network. The results prove that the CAA is effective in reducing the computation time.

The final results of the indices and the betweenness are shown in Table 3. Wth = 0.2546. e6, e14, e15, e19, e24, e26, and e28 cannot be regarded as critical lines. Compared with the results calculated by the global network, the average error rate of index H with the data of sensitive regions is 2.8%, and the average error rate of index C is 1.02%. The average error rate of integrated betweenness W is 2.24%. The accurate identification result can still be obtained through the process of simplification.

Table 2. Simulation result when line e20 is removed.

<table>
<thead>
<tr>
<th>Line</th>
<th>Power Change (MW)</th>
<th>Line</th>
<th>Power Change (MW)</th>
<th>Line</th>
<th>Power Change (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>e19</td>
<td>289.8958</td>
<td>e31</td>
<td>-63.5244</td>
<td>e10</td>
<td>-20.1339</td>
</tr>
<tr>
<td>e21</td>
<td>288.6156</td>
<td>e26</td>
<td>63.2027</td>
<td>e3</td>
<td>15.7069</td>
</tr>
<tr>
<td>e5</td>
<td>237.9716</td>
<td>e4</td>
<td>61.5607</td>
<td>e22</td>
<td>0.1593</td>
</tr>
<tr>
<td>e25</td>
<td>224.116</td>
<td>e9</td>
<td>-48.6027</td>
<td>e23</td>
<td>0.0991</td>
</tr>
<tr>
<td>e6</td>
<td>222.5013</td>
<td>e1</td>
<td>45.4289</td>
<td>e27</td>
<td>0.0972</td>
</tr>
<tr>
<td>e8</td>
<td>-208.162</td>
<td>e2</td>
<td>45.4289</td>
<td>e28</td>
<td>-0.033</td>
</tr>
<tr>
<td>e18</td>
<td>81.8674</td>
<td>e15</td>
<td>45.2833</td>
<td>e33</td>
<td>0.0229</td>
</tr>
<tr>
<td>e12</td>
<td>-81.6569</td>
<td>e14</td>
<td>45.1093</td>
<td>e34</td>
<td>-0.0216</td>
</tr>
<tr>
<td>e17</td>
<td>-75.1401</td>
<td>e7</td>
<td>-28.4259</td>
<td>e35</td>
<td>-0.0111</td>
</tr>
<tr>
<td>e16</td>
<td>-75.1401</td>
<td>e11</td>
<td>-25.1939</td>
<td>e24</td>
<td>0.0187</td>
</tr>
<tr>
<td>e30</td>
<td>-63.6899</td>
<td>e13</td>
<td>-25.0956</td>
<td>e29</td>
<td>-0.0182</td>
</tr>
</tbody>
</table>

Table 3. Calculation result of transmission lines.

<table>
<thead>
<tr>
<th>Line</th>
<th>Index</th>
<th>Line</th>
<th>Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>H</td>
<td>D</td>
<td>H</td>
</tr>
<tr>
<td>e27</td>
<td>0.7439</td>
<td>e18</td>
<td>0.0749</td>
</tr>
<tr>
<td>e3</td>
<td>1.0868</td>
<td>e17</td>
<td>0.0845</td>
</tr>
<tr>
<td>e20</td>
<td>0.6249</td>
<td>e1</td>
<td>0.0454</td>
</tr>
<tr>
<td>e31</td>
<td>0.5744</td>
<td>e2</td>
<td>0.0454</td>
</tr>
<tr>
<td>e4</td>
<td>0.4265</td>
<td>e14</td>
<td>0.0473</td>
</tr>
<tr>
<td>e3</td>
<td>0.2676</td>
<td>e15</td>
<td>0.0042</td>
</tr>
<tr>
<td>e34</td>
<td>0.6789</td>
<td>e9</td>
<td>0.0681</td>
</tr>
<tr>
<td>e12</td>
<td>0.3574</td>
<td>e33</td>
<td>0.2001</td>
</tr>
<tr>
<td>e21</td>
<td>0.1928</td>
<td>e7</td>
<td>0.0397</td>
</tr>
<tr>
<td>e29</td>
<td>0.0523</td>
<td>e27</td>
<td>0.1421</td>
</tr>
<tr>
<td>e11</td>
<td>0.1849</td>
<td>e6</td>
<td>0.0517</td>
</tr>
<tr>
<td>e16</td>
<td>0.1981</td>
<td>e28</td>
<td>0.0334</td>
</tr>
<tr>
<td>e22</td>
<td>0.1972</td>
<td>e29</td>
<td>0.0329</td>
</tr>
<tr>
<td>e8</td>
<td>0.1445</td>
<td>e34</td>
<td>0.0698</td>
</tr>
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<td>e15</td>
<td>0.0232</td>
<td>e19</td>
<td>0.0698</td>
</tr>
<tr>
<td>e10</td>
<td>0.0957</td>
<td>e35</td>
<td>0.0698</td>
</tr>
<tr>
<td>e25</td>
<td>0.0434</td>
<td>e26</td>
<td>0.0014</td>
</tr>
</tbody>
</table>

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4.2.2. Comparison of Lines with Different Integrated Betweenness

Taking lines e_{27}, e_{20} and e_{29} as examples, the increased power of transmission lines at 1 s in IEEE 39-bus system is shown in Figure 6, when e_{27}, e_{20} and e_{29} are removed individually at 0.5 s.

![Figure 6](image)

**Figure 6.** (a) Increased power of transmission lines at 1.0 s when line e_{27} is removed at 0.5 s, $D_{e27} = 0.743978$ $H_{e27} = 1$ $C_{e27} = 0.797568$ $W_{e27} = 2.541546$; (b) Increased power of transmission lines at 1.0 s when line e_{20} is removed at 0.5 s, $D_{e20} = 0.624963$ $H_{e20} = 0.38749$ $C_{e20} = 0.930231$ $W_{e20} = 1.942685$; (c) Increased power of transmission lines at 1.0 s when line e_{29} is removed at 0.5 s, $D_{e29} = 0.052351$ $H_{e29} = 0.540608$ $C_{e29} = 0.356604$ $W_{e29} = 0.949563$.

It can be seen clearly from Figure 6a that the total amount of power flow transfer is quite large and it is mainly concentrated on lines e_{24}, e_{28}, and e_{29} when line e_{27} is removed. It means that the disconnection of line e_{27} has significant influence on the power system. The comparison of Figure 6b,c shows that the transferred power is more concentrated when line e_{29} is removed. But the total transferred power and the global impact is less compared with the removal of line e_{20}. The above phenomena are consistent with the values of $D$, $H$, and $C$, which proves the validity and rationality of the indices.

4.3. Comparison of Identification Results

To verify the effectiveness of the identification method with integrated betweenness, the top 10 lines are attacked. When compared with the attacks performed according to the hybrid flow betweenness (HFB) as introduced in Reference [23], the number of islands, the total value of the
loads, and the average vulnerability of the remaining network are shown in Figure 7. The loss of loads can be determined according to the minimum load shedding method [42]. The average vulnerability of the transmission lines is evaluated using the probability of incorrect tripping caused by the current increment. The linear function of the probability can be obtained according to Reference [43]. In this study, the probability of incorrect tripping is set to 0 when there is no current increment on the transmission line, and the probability is set to 1 when the current is greater than 1.4 times the rated value.

 selon les résultats d'identification de la méthode proposée, la ligne e27 avec des D, H et C élevés sera la première à être déconnectée. Les augmentations de courant sur les lignes e24, e28 et e29 sont 5.268 p.u., 5.8586 p.u. and 5.4645 p.u. Ces lignes présentent des augmentations de courant élevées et auront donc un risque de tripping Incorrect élevé.

Figure 7. (a) The number of the islands under the attacks on the first ten critical lines; (b) the total value of the loads under the attacks on the first ten critical lines; and, (c) the average probability of the incorrect tripping of remaining transmission lines under the attacks on the first ten critical lines.

On the basis of the direction and distribution of power flow, HFB considers the electrical coupling between the lines and the influence caused by a single line fault. Compared with HFB, the integrated betweenness proposed in this study improves the impact analysis of fault flow distribution and considers the tolerance of the remaining network. The top 10 lines are e27, e3, e20, e32, e4, e23, e34, e12, e21, and e29 according to the proposed integrated betweenness and e11, e20, e3, e16, e29, e12, e1, e19, e7, and e5 as according to the HFB. The lines are attacked and disconnected in turn, and the impact of each disconnection on the remaining transmission lines is analyzed.

1. Integrated betweenness attack

According to the identification results of the proposed method, line e27 with high D, H and C will be disconnected first. The current increments on lines e24, e28 and e29 are 5.268 p.u., 5.8586 p.u. and 5.4645 p.u.
5.4645 p.u. These lines exhibit high current increments and will therefore have a high risk of tripping. Then, line e3 is disconnected. It will have impact on several transmission lines, namely e1, e2, e6, e7, e14, e15, e25, e30, and e31. At the same time, the current on e24, e28, and e29 continues to rise and the system risk will be further increased. Following the disconnections of lines e20 and e31, the power flow is redistributed to lines e4, e5, e17, e18, and e19.

Islanding phenomenon occurs when e4 is disconnected. The network is split into a 6-node isolated island and a 33-node system. Although the network is forced to be divided into two parts, the total amount of loads is reduced during the balance of the injected power of sources, and the absorbed power of loads in each island. Therefore, the pressure on the transmission system and the overloads of e1, e2, e7, e14, e15, e17, e18, e19, and e30 are relieved. Subsequently, e23 is disconnected. The node v21 is separated from the network, and the current on e24, e28, and e29 is slightly decreased. After the attacks on e34, e12, e21, and e29, some nodes fall off, but the average vulnerability of the remaining lines keeps increasing.

(2) HFB attack

When line e11 is disconnected, e9, e10, e14, and e15 will have a high risk of tripping. Then e20, e3, e16, e29, e12, and e1 are attacked one by one. Some transmission lines, such as e2, e6, e17, e18, e19, e21, e25, e26, and e30, are affected by the power flow transfer. Most of the current increments on these lines are between 0.2 and 0.3 p.u. and the maximum current increment is 4.4196 p.u. Subsequently, line e19 is disconnected and the node v15 is isolated from the network. Finally, e7 and e5 are attacked and the average probability of incorrect tripping is approximately 0.7.

It can be seen from Figure 7 that the islanding phenomenon is more obvious and the loss of loads is greater under the integrated betweenness attack performed on the IEEE 39-bus system. The lines with high betweenness play critical roles in the power grid. When nearly 30% of the lines are removed under integrated betweenness attack, the loss of load remains at 24%, and the vulnerability of the remaining network is also greatly improved.

4.4. Extreme Cases and Real-Time Behavior

4.4.1. Extreme Cases

IEEE 39-bus system is used as the test system in this section. The injected and absorbed power is changed to present some extreme cases.

(1) All the injected power of generators and the absorbed power of loads are reduced to 0.05 times the initial value. The calculation result is shown in Table 4. Based on the identification method, \( W_{ih} = 2.3149 \), and no line is selected to be a critical transmission line.

<table>
<thead>
<tr>
<th>Line</th>
<th>W</th>
<th>Line</th>
<th>W</th>
<th>Line</th>
<th>W</th>
<th>Line</th>
<th>W</th>
<th>Line</th>
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<tbody>
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<td>0.5680</td>
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<td>e22</td>
<td>1.2320</td>
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</tr>
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</tr>
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<td>e18</td>
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</tr>
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<tr>
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<td></td>
</tr>
</tbody>
</table>

(2) All of the injected power of generators and the absorbed power of loads are increased 4.99 times in respect to their initial values, to bring the system close to its stability limit. The disconnection of any line will result in a serious consequence. In this case, \( W_{ih} = 0.00822 \). The calculation result is shown in Table 5. All lines are regarded as critical transmission lines.
4.4.2. Real-Time Behavior of Power Grids with Different Scales

The computation times of test systems with various scales are statistically analyzed. The systems include IEEE 39-bus system, IEEE118-bus system, and several large-scale systems, which are randomly synthesized by IEEE standard 30-bus and 118-bus system [44]. The standard systems are connected with high-tension transmission lines. Each system is calculated ten times. The average computation time of each test system is shown in Table 6.

<table>
<thead>
<tr>
<th>Test System</th>
<th>Number of Buses</th>
<th>Computation Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE39</td>
<td>39</td>
<td>0.061</td>
</tr>
<tr>
<td>IEEE118</td>
<td>118</td>
<td>0.319</td>
</tr>
<tr>
<td>SYN472</td>
<td>472</td>
<td>1.285</td>
</tr>
<tr>
<td>SYN1062</td>
<td>1062</td>
<td>2.891</td>
</tr>
<tr>
<td>SYN3000</td>
<td>3000</td>
<td>7.639</td>
</tr>
<tr>
<td>SYN7680</td>
<td>7680</td>
<td>32.268</td>
</tr>
<tr>
<td>SYN12000</td>
<td>12,000</td>
<td>73.512</td>
</tr>
</tbody>
</table>

Based on the above data, the computation time can be qualitatively analyzed: the computation time of the system with 39–5000 buses can be of the order of seconds, and the computation time of the system with 5000–10,000+ buses can be of the order of minutes. 

The implementation of “real-time identification of critical lines” depends on two important parts: (1) measurement data acquisition; (2) data processing and analysis. The measurement data can be obtained by wide area measurement system (WAMS). According to Reference [45], WAMS is required to complete the measurement and transmission of PMU data in 30–50 ms for dynamic monitoring and real-time decision-making purposes. Based on the data in Table 6, the time scale of the identification process can be of the order of minutes.

5. Conclusions

In order to implement accurate and fast identification of critical lines, an integrated betweenness is proposed which considers the topological and electrical characteristics of the power network. The indices of power transmission, fault flow distribution, and the influence on system security are used to assess the importance of different lines with respect to the functionality of themselves, relevance with other transmission lines, and system security. Although the three indices are different, they are all related to the changes in the power system before and after the removal of a line. Based on this, the sensitive regions containing lines with large variations of power are located by CAA. The analysis of the whole network is replaced by the analysis of sensitive regions, which reduces the calculation time effectively and provides the possibility for real-time analysis in large-scale power systems.

This study focuses on the identification of critical lines which play important roles in power transmission and whose disconnections have a great influence on system vulnerability. Together with the transmission lines which are heavily overloaded or have high risks of failures, the identification...
results can help formulate plans to avoid blackouts. The simulation results show that the proposed method is effective and practical.

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