A Full Frequency-Dependent Cable Model for the Calculation of Fast Transients

Abdullah Hoshmeh 1,* and Uwe Schmidt 2

1 Department of Electrical Engineering and Information Technology, Chemnitz University of Technology, 09126 Chemnitz, Germany
2 Department of Electrical Engineering and Informatics, University of Applied Sciences Zittau/Goerlitz, 02763 Zittau, Germany; uwe.schmidt@hszg.de
* Correspondence: abdullah.hoshmeh@etit.tu-chemnitz.de; Tel.: +49-371-531-38717

Received: 12 June 2017; Accepted: 27 July 2017; Published: 7 August 2017

Abstract: The calculation of frequency-dependent cable parameters is essential for simulations of transient phenomena in electrical power systems. The simulation of transients is more complicated than the calculation of currents and voltages in the nominal frequency range. The model has to represent the frequency dependency and the wave propagation behavior of cable lines. The introduced model combines an improved subconductor method for the determination of the frequency-dependent parameters and a PI section wave propagation model. The subconductor method considers the skin and proximity effect in all conductors for frequency ranges up to few megahertz. The subconductor method method yields accurate results. The wave propagation part of the cable model is based on a cascaded PI section model. A modal transformation technique has been used for the calculation in the time domain. The frequency-dependent elements of the related modal transformation matrices have been fitted with rational functions. The frequency dependence of cable parameters has been reproduced using a vector fitting algorithm and has been implemented into an resistor-inductor-capacitor network (RLC network) for each PI section. The proposed full model has been validated with measured data.

Keywords: subconductor method; frequency-dependent cable parameters and impedances; PI sections; cable model; time domain

1. Introduction

The safe and reliable operation of cable systems requires the analysis of transient behavior of such systems. Single core cables with a core and a shield are utilized prospectively. The first models for the calculations of wave propagation and their transient behavior were developed in the 1970s. The structure of the line models is divided into two main components, a module for calculation of frequency-dependent parameters and the wave-propagation model.

For calculation of the line parameters, Ametani’s algorithm [1] (cable constants) uses the principal assumptions of Wedepohl [2]. Ametani’s algorithm uses a bundle of equations to determine the coupled cable impedances. The frequency-dependent earth return impedance is considered with a fictive earth conductor given by Carson [3] and Pollaczek [4]. The skin effect within the cable core is implemented using Schelkunoff’s equations [5]. The proximity effect between the conductors and the skin effect in the shield are neglected. Morched et al. [6] also use principal assumptions similar to the ideas of Wedepohl [2] and Ametani [1].

The wave propagation models can be mainly classified into two categories: the models with distributed parameters and the models with lumped parameters. The models with distributed parameters are based on partial differential equations to describe the dependency of current and voltage on the time and distance. These differential equations are also known as telegraph equations [7].
d’Alembert solved the telegraph equations [8]. Using the Bergeron method [9], d’Alembert’s solution
is represented with forward and backward waves, where the line parameters are assumed to be
frequency independent. In addition, the losses are neglected.

Dommel [10] developed the method of Bergeron by considering the line losses using lumped
resistances. Snelson [11] was able to further develop Dommel’s model by taking the frequency
dependency of the line parameters into account. Based on Snelson’s model, Marti J. [12] presented
a model for overhead lines. In this model, a modal transformation has been used to simplify
the modelling. The modal transformations are performed using modal transformation matrices,
whose parameters are generally frequency dependent. However, Marti J. used constant (frequency
independent) modal transformation matrices. The frequency dependency of the modal transformation
matrices are considered later in the model of Marti L. [13]. In order to avoid the difficulties by using
frequency-dependent modal transformation matrices, Noda et al. [14] have developed the line model
directly in the phase domain. Morched et al. [6] have further developed this model and introduced a
universal line model. The model can be used for overhead lines and cables. In order to simplify the
calculations, the universal line model uses a vector fitting algorithm [15,16], which approximates some
of the line parameters (surge admittance and propagation constant) by rational functions.

Another approach (alternatively to the vector fitting algorithm) for the approximation of the line
parameters is based on the algorithm developed in Noda [17]. Noda has shown that this algorithm
can be also applied to both overhead lines [18] and cables [19]. The universal line model gives good
results. However, it sometimes produces numerical instabilities. This happens by approximation of the
propagation constant and especially in the case of short cables, since the associated time delays become
closer, leading to poor fitting. This problem was overcome in [20] by introducing a new method to
calculate the time delays.

A second type for modeling of the line is used by lumping of the line parameters. The lumped
parameters can be combined to build a PI section [10]. The behavior of the models with distributed
parameters is approximated by connecting several PI sections in series, which results in a simplified
representation of a line model with lumped parameters. The number of PI sections depends on the
simulated cable length, the expected highest frequency in the investigated transient and the accepted
calculation error. The principle of the model with lumped parameters or with PI sections is relatively
simple compared to the models with distributed parameters. However, in the traditional PI section
model the line parameters are calculated only at a specified frequency point. In other words, the
frequency dependency of the line parameters is neglected. Therefore, the results of this model will only
be valid for steady state simulations. For simulations of transients, the traditional PI section model has
to be further developed [21].

In this paper, the main steps for the development of a full frequency-dependent three-phase PI
section cable model (3PPI model) is introduced. The frequency-dependent impedances are determined
with an improved sub-conductor method considering the skin and the proximity effect in the core,
the shield and the earth. The frequency dependence of the cable parameters is implemented through
an RLC (resistor-inductor-capacitor) network with a defined number of PI sections. To decouple the
cable system, a modal transformation technique has been used. The calculation time necessary to work
with a large number of PI sections is acceptable. For the simulation in the time domain, a recursive
convolution technique has been used. The differential equations to describe the voltages and currents
along the cable are of the first order, which can be easily solved as state space equations.

2. Algorithm for Calculation of Frequency-Dependent Cable Parameters

2.1. Fundamentals

For any inductive loop, the mutual inductance $M$ of a conductor configuration characterizes the
magnetic influencing of adjacent circuits on each other. The mutual inductance between two conductor
loops $M$ and $N$ as shown in Figure 1 can be determined by Equation (1) (Kuepfmueller [22]).
Figure 1. Equivalent configuration for the determination of the mutual inductance $M$ between linear conductor loops.

$$M_{MN} = \frac{\mu}{2 \cdot \pi \cdot \ell} \left( \frac{r_{14}}{r_{34}} \cdot \frac{r_{23}}{r_{12}} \right)$$  \hfill (1)

Equation (1) is sufficiently accurate if the radii $r_i$ are negligibly small. Otherwise, the conductor cross-sectional area $A$ must be taken into account. Maxwell [23] incorporated the cross-sectional area $A$ through a mean geometric distance $g$. According to Bruederlink [24], for any two areas $A_1$ and $A_2$, shown in Figure 2, their mean geometric distances $g_{MN}$ are given by Equation (2).

$$\ln(g_{MN}) = \frac{1}{A_M \cdot A_N} \int_{A_M} \int_{A_N} \ln r_{mn} \, da_m \, da_n$$ \hfill (2)

Figure 2. Principal configuration for the determination of mean geometric distance $g$.

The mean geometric distance of an area on itself $g_{MM}$ can be obtained by Equation (3).

$$\ln(g_{MM}) = \frac{1}{A_M^2} \int_{A_M} \int_{A_M} \ln r_{mn} \, da_m \, da_n$$ \hfill (3)

For the characterization of conductor loops with common return conductor through earth, the analytical approaches of Carson [3] can be applied on cable systems over narrow frequency ranges only. To incorporate the return conductor into the model, it is reasonable to define a fictive hull cylinder with a radius $R_H$. Figure 3 shows a simplified equivalent configuration introduced by Rees [25], illustrating the conductors $m$ and $n$ as well as the fictive hull cylinder which is the common return.

Figure 3. Equivalent circuit for determination of mutual inductance $M'_{mn}$ with common return (hull cylinder).
The subconductor self impedance $Z'_{mn}$ and the mutual impedances $Z'_{mn}$ are given by Expression (4).

$$
Z'_{mn} = j \cdot \omega \cdot \frac{\mu}{2 \cdot \pi} \cdot \ln \left( \frac{R_H}{S_{mn}} \right)
$$

$$
Z'_{mm} = R'_m + j \cdot \omega \cdot \frac{\mu}{2 \cdot \pi} \cdot \ln \left( \frac{R_H}{S_{mm}} \right)
$$

A resistive component $R'_m$ is thereby included only in the self impedance.

### 2.2. Partial Subconductor Method

The principle of the partial subconductor method as published by Comellini [26] is based on the segmentation of the conductor cross-sectional area into sufficiently small subconductors so that the current density will be nearly homogeneous within each subconductor. The subconductors of the system are inductively coupled through the mutual inductance per unit length $Z'_{mn}$ of all conductor loops. Therefore, it is possible to model skin and proximity effects in the conductors of the cable and in the earth. Figure 4 shows the principle of segmentation.

![Principle of conductor segmentation into subconductors.](image)

The inductive coupling of current and voltage of all subconductors is characterized by the equation system of the subconductor-submatrices as specified in Equation (5) for three conductors A, B, and C.

$$
\begin{bmatrix}
V'_A \\
V'_B \\
V'_C
\end{bmatrix}
= \begin{bmatrix}
Z'_{AA} & Z'_{AB} & Z'_{AC} \\
Z'_{BA} & Z'_{BB} & Z'_{BC} \\
Z'_{CA} & Z'_{CB} & Z'_{CC}
\end{bmatrix}
\begin{bmatrix}
i_A \\
i_B \\
i_C
\end{bmatrix}
$$

(5)

It is possible to incorporate all conductors of the system into the algorithm, for instance any of the cable components such as the core conductor, shield, armor, earth, semiconducting layers and possible return conductors of the system. The conventional subconductor method realizes a “fixed” segmentation with arc segments. Dommel [27] discussed different shapes of segments, for instance circles, squares and so forth. The limitation to frequencies of $f < 10$ kHz is a significant disadvantage of the fixed segmentation. The reason for the limitation to frequencies is named by Lucas [28] described in detail in Schidt et al. [29,30]. The limitation of the conventional sub-conductor method can be avoided by using frequency-dependent segmentation.

### 2.3. Frequency-Dependent Segmentation of Conductors

The segmentation of a cylindrical conductor is realized considering the skin and proximity effect. To determine the segmentation algorithm, the current density $J(f, r)$ of the cylindrical conductor is analytically derived from Maxwell’s equations as a function of the frequency $f$ and the radius $r$. 

---

**Energies 2017, 10, 1158**
Figure 5 illustrates the current density $J_C/J_{\text{max}(C)}$ at different frequencies for a solid, cylindrical copper conductor with a cross-section area $A_C$ of 1000 mm$^2$.

![Figure 5](image)

**Figure 5.** Current density $J_C/J_{\text{max}(C)}$ in a cylindrical conductor at different frequencies $f$.

With increasing frequency $f$ the skin effect grows stronger. At high frequencies, a significant current flow remains directly under the surface only. For instance, at a frequency $f = 1000$ Hz, 90% of the current $I$ is concentrated in the outer 4.5 mm. Therefore, the cylindrical conductor is segmented across its radius $r_A$ as a function of the relative current density $J/J_{\text{max}}$. It is assumed that a constant error of the current density distribution can be achieved by calculating the current density $J(f, r)$ with the outer radius of the $i$-th skin $r_i$ directly dependent on the relative current density $J/J_{\text{max}}$. The number of skins $n$ can be set in accordance to the required accuracy. Nevertheless, it must be considered that the number of skins impacts the algorithm’s computation time. The consideration of the proximity effect is covered by the skin subdivision into arc segments.

As an example, Figure 6 displays the allocation of the skins over the conductor’s radius exemplary for five skins. The determination of the skin radii $r_1$ to $r_5$ follows a $1/n$-increment of the relative current density $J/J_{\text{max}}$. However, the subdivision of the conductors into skins as demonstrated in Figure 6 does not cover the proximity effect. This must be incorporated by another subdivision of the skins into arc segments (see Figure 7). Arnold [31] yields an analytical approach to calculate the proximity effect.

![Figure 6](image)

**Figure 6.** Segmentation of a conductor into five skins.
Those calculations have shown that for the geometric factors $k_s = \ell_{rs}/d_s$ smaller than 2, the relative deviation $s_{mn}/g_{mn}$ is minimized. Figure 8 gives an example for the segmentation into three skins for a frequency of $f = 50$ Hz.

2.4. Earth Segmentation

The segmentation of the earth is based on the analytically calculated current distribution in the earth. The approach of Ruedenberg [33] can directly be applied to underground cable systems. The calculated current density $I_E(f, r)$ is shown in Figure 9 and displays the relative current density $I_E/I_{\text{max}(E)}$ as a function of the radius $r$ at different frequencies between 50 Hz and 1 MHz. The specific earth resistance $\varrho_E$ is assumed to a typical value (in Central Europe) of 150 $\Omega$ m.

The segmentation is again realized by analytical calculation of the conductor current density $I_C/I_{\text{max}(C)}$. For frequencies of $f = 50$ and 1000 Hz, Figure 10 shows how ten skins would be allocated.
To provide a better overview, only every second skin is plotted in Figure 10. At the fictive hull cylinder with the radius $R_H$ the current density $J$ must have declined to zero. As Dommel [34] was able to demonstrate, this condition can be fulfilled for a distance of ten times the penetration depth $\varrho_E$.

The segmentation of the skins is done according to the results of the errors computation discussed in Schmidt [30].

2.5. Shield Segmentation

The shields of single core cables are usually stranded rigid copper conductors illustrated in Figure 11.

![Figure 11. Structure of stranded rigid copper conductor.](image)

In this case, the frequency-dependent segmentation is not necessary since the skin effect is only effective in single wires. The self-impedance per unit length of the single wire with the cross section $A_D = \pi \cdot r_D^2$ can be calculated by Equation (6).

$$Z'_{mm} = R'_{ac(D)} + j \cdot \omega \cdot \frac{\mu}{2\pi} \cdot \ln \left( \frac{R_H}{s_{mm}} \right)$$ (6)

2.6. Calculation of Frequency-Dependent Impedances

Equation (4) describes the impedances $Z'$ of all subconductors in a certain area of all considered subconductors in a certain area and thus the inductive coupling between all subconductors. The reference conductor (or common return conductor) of all loops is the radius of the fictive hull cylinder $R_H$. The complete impedance matrix of a system appears as represented in Equation (5) and contains the subconductor matrices of each individual phase. The rank of the impedance matrix $Z'$ is equal to the number of all subconductors in the enclosed area $n_{all}$. A resistive component $R'$, which characterizes the direct current resistance per unit length $R'_m$ of the electrical subconductor $m$ as a function of the specific resistance and the subconductor area $A_m$, is merely included in the self impedances $Z'_{mm'}$. The direct current resistances $R'_m$ of all subconductors are comprised in a column vector $R'$. The impedance matrix $Z'$ can be expressed according to Equation (7).

$$Z' = R' + j \cdot \omega \cdot \frac{\mu}{2\pi} \cdot (\ln R_H - \ln s)$$ (7)

The indices of the centroid distance elements in Matrix (8) designate the respective subconductor. The subscripted index names the $m$-th subconductor of the current-carrying loop and the superscripted index names the $n$-th subconductor influenced by the current-carrying loop.

$$s = \begin{bmatrix}
  s_{A1} & \ldots & s_{A_{A_H}} & s_{A1} & \ldots & s_{A_{N_{n_{all}}}} \\
  s_{A_{A_H}} & \ldots & s_{A_{A_{A_H}}} & s_{A_{A_{A_H}}} & \ldots & s_{A_{N_{n_{all}}}} \\
  \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
  s_{A_{A_{A_{A_H}}}} & \ldots & s_{A_{A_{A_{A_{A_H}}}}} & s_{A_{A_{A_{A_{A_{A_H}}}}}} & \ldots & s_{A_{N_{n_{all}}}} \\
  \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
  s_{A_{N_{1}}} & \ldots & s_{A_{N_{A_{A_H}}}} & s_{A_{N_{N_{1}}}} & \ldots & s_{A_{N_{N_{n_{all}}}}} \\
  s_{N_{N_{1}}} & \ldots & s_{N_{A_{A_{A_H}}}} & s_{N_{N_{N_{1}}}} & \ldots & s_{N_{N_{N_{n_{all}}}}} \\
  \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
  s_{N_{N_{A_{A_{A_{A_{A_H}}}}}}} & \ldots & s_{N_{A_{A_{A_{A_{A_{A_{A_H}}}}}}}} & s_{N_{N_{N_{N_{1}}}}} & \ldots & s_{N_{N_{N_{N_{n_{all}}}}}} \\
  \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
  s_{N_{N_{n_{all}}}} & \ldots & s_{N_{A_{A_{A_{A_{A_{A_{A_{A_H}}}}}}}}} & s_{N_{N_{N_{n_{all}}}}} & \ldots & s_{N_{N_{N_{N_{n_{all}}}}}}
\end{bmatrix}$$ (8)
The subconductor matrices of the centroid distances \(s_{mn}\) and the admittance matrix \(Y\) are symmetrical. Therefore, to calculate the centroid distances \(s\), it is sufficient to determine the elements above the main diagonal and to reflect them at the main diagonal. For unknown currents \(i\) of the cable system and known voltages \(v\), a system of equations (with reference to Equation (5)) can be obtained as given by Equation (9).

\[
i = Y' \cdot v' \tag{9}
\]

The admittance matrix \(Y'\) is the inverse of the impedance matrix \(Z'\). The voltage across all subconductors in a conductor \(K\) is the same. It is:

\[
V_k' = V_{k1}' = V_{k2}' = \ldots = V_{kn}' \tag{10}
\]

The sum of the subconductor admittance matrices \(Y'_{mn}\) and \(Y'_{mm}\) can be realized using basic matrix operations demonstrated in Relation (11).

\[
Y'_{\text{red}} = T \cdot Y' \cdot T^{-1} \tag{11}
\]

The number of rows in the transformation matrix \(T\) according to Equation (12) is equal to the number of conductors \(N\). The number of elements per row represents the number of subconductors \(n\) in conductor \(K\). The reduced impedance matrix \(Z'_{\text{red}}\) can be obtained from the reduced admittance matrix \(Y'_{\text{red}}\).

\[
Z'_{\text{red}} = \left( Y'_{\text{red}} \right)^{-1} \tag{13}
\]

2.7. System Impedances

The reduced impedance matrix \(Z'_{\text{red}}\) (13) contains the submatrices of all conductors. It is useful to modify the matrix containing all submatrices to a \((6 \times 6)\) matrix. Expression (14) shows the structure of the reduced system with all submatrices for the core (C), shield (S) and earth (E).

\[
\begin{bmatrix}
Z'_{cc} & Z'_{cs} & Z'_{ce} \\
Z'_{sc} & Z'_{ss} & Z'_{se} \\
Z'_{ec} & Z'_{es} & Z'_{ee}
\end{bmatrix}
\begin{bmatrix}
i_c \\
i_s \\
i_e
\end{bmatrix}
\]

\[
\begin{bmatrix}
Z'_{cc} & Z'_{cs} & Z'_{ce} \\
Z'_{sc} & Z'_{ss} & Z'_{se} \\
Z'_{ec} & Z'_{es} & Z'_{ee}
\end{bmatrix}
\begin{bmatrix}
i_c^p \\
i_s^p \\
i_e^p
\end{bmatrix}
\]

The voltage vector \(v'\) describes longitudinal voltages measured against the hull cylinder. The equation system (14) can be partitioned into impedances of the cable system and impedances of return conductors (15).

\[
\begin{bmatrix}
Z'_{cc} & Z'_{cs} & Z'_{ce} \\
Z'_{sc} & Z'_{ss} & Z'_{se} \\
Z'_{ec} & Z'_{es} & Z'_{ee}
\end{bmatrix}
\begin{bmatrix}
i_c^p \\
i_s^p \\
i_e^p
\end{bmatrix}
\]
The reference potential is the potential of the fictive hull cylinder \( U_H = 0 \). It can be expressed in Equation (16).

\[
\begin{align*}
\psi_p' &= Z_{cc}^p \cdot i_p^p + Z_{ce}^p \cdot i_e^p \\
0 &= Z_{el}^p \cdot i_p^p + Z_{ee}^p \cdot i_e^p
\end{align*}
\]  

(16)

Equation (16) can be rearranged to obtain (17) for the current \( i_e^p \):

\[
i_e^p = -\left( Z_{ve}^p \right)^{-1} \cdot \left( \psi_p^p - Z_{cc}^p \cdot i_p^p \right)
\]  

(17)

Inserting Equation (17) into (16) results in Equation (18) for the voltage \( u_p^p \).

\[
\begin{align*}
u_p^p &= (Z_{cc}^p - Z_{ce}^p \cdot (Z_{ee}^p)^{-1} \cdot Z_{ve}^p) \cdot i_p^p \\
u_p^p &= Z_{mod} \cdot u_p^p
\end{align*}
\]  

(18)

The modified matrix \( Z_{\text{mod}} \) has the rank of the sum of all cable conductors. The matrix is a \((6 \times 6)\) matrix single core cable system \((3 \times \text{core}, 3 \times \text{shield})\). It is also possible to reduce the coefficient matrix of equation system (14) to a \((3 \times 3)\) matrix. In case of earthed shields on both cable ends, the return conductor submatrices of the partitioned matrix (15) contains the shield submatrices. The earth and shield influence is considered in all modified impedance matrices \( Z_{\text{mod}} \). In principle, this approach can also be realized for one-sided shield earthing (Schmidt [30]).

### 2.8. Impact of Semiconducting Layers

The influence of semiconducting layers is formulated in only very few publications. Ametani [35] determined the impact of semiconducting layers by modifying Schelkunoff’s method [5]. Semiconducting layers can also be considered with the subconductor method. A one-skin segmentation for the inner semiconducting layer is shown in Figure 12.

![Figure 12. One-skin segmentation of the inner semiconducting layer.](image)

Segmentation can be realized also with more than one skin. The specific resistances of semiconducting layers can be \( \varrho_{sl} = 0.01 \ldots 10 \, \Omega \cdot \text{m} \) according to Ametani [35]. Nevertheless, cable manufacturers specify particular values between \( \varrho_{sl} = 10 \) and \( 100 \, \Omega \cdot \text{m} \). The calculated resistance per unit length \( R_{ac(C)}' \) for relevant specific semiconducting layer resistances \( \varrho_{sl} \) is shown in Figure 13.

![Figure 13. Resistance per unit length \( R_{ac(C)}' \) considering the semiconducting layer.](image)
2.9. Main Insulation Capacitance

The single core cable capacitance shown in Figure 14 can be calculated by Equation (19) for the cross-linked polyethylene (XLPE) main insulation $C'_{CS}$:

$$
C'_{CS} = 2 \cdot \pi \cdot \varepsilon_0 \cdot \varepsilon_{r(h)} \cdot \frac{1}{\ln \left( \frac{r_2}{r_1} \right)}
$$

(19)

and for the sheath capacitance $C'_{sh}$ (20).

$$
C'_{sh} = 2 \cdot \pi \cdot \varepsilon_0 \cdot \varepsilon_{r(m)} \cdot \frac{1}{\ln \left( \frac{r_4}{r_3} \right)}
$$

(20)

![Diagram of a single core cable configuration](image)

Figure 14. Configuration of a single core cable.

The capacitances depend on the cylinder geometry, the relative permittivity $\varepsilon_r$ of the insulation and the polarization. The frequency-dependent effects of polarization are investigated in several publications (Liu [36], Hadid and Schmidt [37]). The frequency range of polarization types are given in Table 1 (see also Liu [36]).

<table>
<thead>
<tr>
<th>Polarization</th>
<th>Frequency Range $f_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electronic</td>
<td>$\approx 10^{15}$ Hz</td>
</tr>
<tr>
<td>Ionic</td>
<td>$\approx 10^{12}$ Hz</td>
</tr>
<tr>
<td>Orientational</td>
<td>$\approx 10^9$ Hz</td>
</tr>
<tr>
<td>Interfacial</td>
<td>mHz . . . kHz</td>
</tr>
</tbody>
</table>

Table 1. Frequency ranges of polarization.

The table shows that electronic, ionic, and orientational polarization will not contribute significantly to the capacitances of single core cables. A typical interfacial polarization occurs between the XLPE insulation and the semiconducting layers. This effect is of relevance in a frequency range $f \leq 1$ Hz. The interfacial polarization can be ignored in frequency ranges $f \geq 10$ Hz. Therefore, the frequency dependency of the capacitances can be generally neglected in an algorithm for frequency-dependent parameters. A typical interfacial polarization occurs between the XLPE insulation and the semiconducting layers. This effect is of relevance in a frequency range $f \leq 1$ Hz. The interfacial polarization can be ignored in frequency ranges $f \geq 10$ Hz. Therefore, the frequency dependency of the capacitances can be generally neglected in an algorithm for frequency-dependent parameters. The semiconducting layers have an influence on the capacitance $C'_{CS}$, but for transient calculations it can be neglected. Hadid and Schmidt [37] and Wagenaars [38] confirm this approach.
2.10. Sheath Capacitances

2.10.1. Earth Installation

Ametani [1], Wedepohl [2], Gustavsen [39], and Marti L. [13] neglected the earth resistance \( R_E \) and the earth capacitance \( C_E \). In Schmidt [30], this approach is confirmed for specific earth resistances \( \varrho_E < 500 \Omega \cdot m \) and frequencies \( f < 100 \text{ kHz} \). The sheath capacitance per unit length \( C'_{sh} \) can be calculated with Equation (20).

2.10.2. Air Installation

For correct calculation of shield-shield capacitance \( C_{SS} \), the sheath capacitance \( C_{sh} \) and the air capacitance \( C_E \) have to be considered. The sheath capacitance \( C_{sh} \) is significantly larger than the air capacitance \( C_E \). Therefore, the sheath capacitance \( C_{sh} \) can be neglected (Schmidt [30]). This is also valid in case of small phase distances. The sheath capacitance \( C_{sh} \) can be calculated with the charge simulation methods of Steinbigler [40], and Probst [41].

3. Wave Propagation Model

3.1. Approach

The wave propagation in a multi-conductor cable or transmission line can be represented in the frequency domain with second-order differential equations from Marti J. [42]:

\[
\frac{d^2 V}{dx^2} = Z' Y_L \frac{dV}{dx} = \lambda V
\]

\[
\frac{d^2 I}{dx^2} = Y_L Z' \frac{dI}{dx} = \lambda I
\]

The longitudinal impedances in the phase domain \( Z' \) are calculated using the partial subconductor method as in Equation (7). The shunt admittance in the phase domain \( Y_L \) represents the capacitance equations given in Equations (19) and (20). The index “L” stands for lateral. In order to solve the differential Equations (21) and (22), a modal transformation technique has been proposed in Marti J. [42]. As a result, the coupling between the cable conductors disappears and the cable system can be considered as decoupled single conductors. In the modal domain, the Equations (21) and (22) can be rewritten as:

\[
\frac{d^2 V_m}{dx^2} = T_v^{-1} Z' Y_L T_v \frac{dV_m}{dx} = \lambda V_m
\]

\[
\frac{d^2 I_m}{dx^2} = T_i^{-1} Y_L Z' T_i \frac{dI_m}{dx} = \lambda I_m
\]

where \( \lambda \) is a diagonal matrix which represents the eigenvalues matrix of \( Z' Y_L \) and \( Y_L Z' \). For the transformation into modal domain the modal transformation matrices \( T_v \) and \( T_i \) are used. The relation between the matrices is given by Marti J. [13]:

\[
T_v = \left[ T_i^{-1} \right]^{tr}
\]

Therefore, it is sufficient to calculate only one of them. In this paper, the results for \( T_v \) will be calculated as in Chrysochos et al. [43].

In order to make the calculation in the modal domain, the quantities \( Z', Y_L, V \) and \( I \) have to be transformed into this domain. From Equations (23) and (24), it can be derived:

\[
T_v^{-1} Z' Y_L T_v = \lambda
\]

\[
T_i^{-1} Y_L Z' T_i = \lambda
\]
With simple matrix perturbation, the above equations are then rewritten as:

\[
T^{-1}vZ'(T_iT_i^{-1})Y'T_i = \lambda \tag{28}
\]

\[
T^{-1}Y_i'(T_iT_i^{-1})Z'T_i = \lambda \tag{29}
\]

By rearranging the terms and by using the Equation (25) in (28) and (29), the longitudinal impedance and the shunt admittance in the modal domain can be defined as:

\[
Z'_m = T_i't'Z_i'T_i \tag{30}
\]

\[
Y_{l,m}' = T_i^{-1}Y_i'[T_i^{-1}]^{tr} \tag{31}
\]

Both matrices in Equations (30) and (31) are diagonal. The frequency dependence of the elements of \(Y_{l,m}'\) cannot be neglected (as assumed for the elements of \(Y_i'\)). This is related to the frequency dependent elements of \(T_i\). The elements of \(Z'_m\) are also strongly frequency-dependent. In order to take the frequency dependency of \(Z'_m\) and \(Y_{l,m}'\) into account, their diagonal elements need to be approximated by mathematical functions using the vector fitting algorithm (VF) of Gustavsen and Semlyen [15], and Gustavsen [16]. For example, the first diagonal element in \(Z'_m\) can be approximated using VF as:

\[
Z_{m,fit} = d + j\omega h + \sum_{q=1}^{N} \frac{p_q}{j\omega - d_q} + \sum_{b=N+1}^{N+M} \left( \frac{s_{b,\text{re}} + j s_{b,\text{im}}}{j\omega + s_{b,\text{re}} - j s_{b,\text{im}}} + \frac{s_{b,\text{re}} - j s_{b,\text{im}}}{j\omega + s_{b,\text{re}} + j s_{b,\text{im}}} \right) \tag{32}
\]

The accuracy of \(Z_{m,fit}'\) depends on the number of the real terms \(N\) and the number of the complex terms \(M\). \(Z_{m,fit}'\) can be reproduced with an electrical network. In Hoshmeh et al. [21], an RL network has been used. Using such a network allows us to employ only the real poles and residues in the approximation (the complex terms are discarded). Although the fitting is achieved with a low order of approximation, the accuracy is affected. In this paper, an RLC network shown in Figure 15 is introduced with regards to all the poles and residues in the approximation including the complex terms.

![Figure 15](image)

**Figure 15.** Representation of the frequency dependence of the cable longitudinal impedance.

As can be seen, the network has been divided into two sections. The elements of section I can be evaluated in a similar way as in Hoshmeh et al. [21]. The elements of section II are obtained as follows:
The first diagonal element of $Y'_{L,m}$ is similarly approximated. Furthermore, $R'_0$ and $L'_0$ are assumed to be zeros in $Y'_{L,m \text{ fit}}$. After calculating $Z'_{\text{fit}}$ and $Y'_{L,m \text{ fit}}$, they will be combined to form a developed PI section. Figure 16 shows a representation for the developed PI section after regarding the cable length, where $v_{m,s}$ and $v_{m,r}$ are the voltages in the modal domain at the sending and receiving ends, respectively. One single conductor in the developed cable model can then be represented by cascading a number of PI sections.

\[
R'_b = \begin{cases} 
-2 s_{b,\text{re}}^3 s_{b,\text{im}}^2 - 2 s_{b,\text{im}}^3 s_{b,\text{re}} s_{b,\text{im}} \\
\delta_{b,\text{im}}^2 (s_{b,\text{re}}^2 + s_{b,\text{im}}^2)
\end{cases} \quad b = N+1 \ldots N+M
\]

\[
L'_b = \begin{cases} 
2 s_{b,\text{re}}^3 \\
\delta_{b,\text{im}}^2 (s_{b,\text{re}}^2 + s_{b,\text{im}}^2)
\end{cases}
\]

\[
C'_b = \begin{cases} 
1 \\
2 s_{b,\text{re}}
\end{cases}
\]

\[
R''_b = \begin{cases} 
2 s_{b,\text{re}}^2 \\
\delta_{b,\text{im}} s_{b,\text{im}} - \delta_{b,\text{re}} s_{b,\text{re}}
\end{cases}
\]

For the remaining diagonal elements of $Z'_{m}$ and $Y'_{L,m}$, the same procedure is used to form the other conductors in the developed cable model. From the Equations (23)–(25), the voltages and the currents are transformed into the modal domain as follows:

\[
V_m = T^r T^i V
\]

\[
I_m = T^r T^i I
\]

Equations (34) and (35) are still in the frequency domain. However, since the simulations in the power system are necessary in the time domain, these equations have to be transformed into this domain. For the voltages in the Equation (34), the convolution integral is applied as:

\[
v_m(t) = \int_{-\infty}^{\infty} t_1^r (t - u) v(u) \, du
\]

Evaluation this integral point by point is time consuming. By approximating the elements of $T^r$ using rational functions, an efficient recursive convolution method is used from Marti J. [12] and the above integral can be rewritten as a sum of exponentials:

\[
v_m(t) = \int_{-\tau}^{\infty} b e^{-l(u-\tau)} v(u) \, du
\]

where $b$ and $l$ are the approximations parameters of the rational functions. The wave travel time $\tau$ for the impulse from one end of the cable to the other one is related to the imaginary term of the propagation function $\gamma$:

\[
\tau = \sqrt{\frac{\gamma}{\gamma}}
\]
The voltage vector $v_m(t)$ is calculated recursively from $v_m(t - \Delta t)$ as:

$$v_m(t) = a_1 v_m(t - \Delta t) + a_2 v(t - \tau) + a_3 v(t - \tau - \Delta t)$$ \hspace{1cm} (39)

In this relation, $a_1$, $a_2$ and $a_3$ are constants that mainly depend on $b$, $l$ and $\Delta t$ Marti J. [42]. The same procedure is used to transform the currents in the Equation (35) into the time domain. In order to execute the time domain simulations, the differential equations for single conductors in the developed cable model have to be formulated. As shown in Hoshmeh et al. [21], the differential equations can be represented using state space techniques. A numerical backward Euler rule of integration can then be applied to find the solution for the related state space equations of Meyer and Dommel [44]. To solve the cable equations with the rest of the network, which is always defined in phase quantities, the calculated voltages and currents in the modal domain must be transformed back to phase quantities.

Figure 17 shows an overview for the realization of the 3PPI model. Starting from defining $Z'$ and $Y_L$, the cable model part is executed only once to provide the PI sections for the calculation part. In this part, the computation is evaluated every $\Delta t$. For the first one, the vector of the voltages $v$ is calculated using the network model and the vector of the currents $i$. By applying the recursive convolution between $v$ and the approximation of $t_i^*$ ($\simeq t^*_i$), the vector of the voltages $v_m$ is calculated. Using the PI sections and $v_m$, the state space equations are formulated and solved by means of numerical backward Euler rule of integration to yield the current vector $i_m$. The recursive convolution between $i_m$ and the approximation of $t_i$ ($\simeq t_i$) provides the vector of currents $i$ for the next $\Delta t$.

![Figure 17. Principle of the three-phase cable model. VF: vector fitting algorithm.](image)

4. Validation of the Full Frequency-Dependent Cable Model

For the validation of the 3PPI model, a DC-Test on a three-phase cable with a length of $\ell \approx 2.5 \text{ km}$ in trefoil formation has been used. The test is performed by charging the cable conductors up to 1 kV. After that, the charged conductors are connected to earth through the circuit breaker (s2) and the currents $i_{\text{core}(A)}$, $i_{\text{core}(B)}$, $i_{\text{shield}(A)}$ and $i_{\text{shield}(B)}$ at the sending end are measured. The test configuration is shown in Figure 18. The cable parameters are given in Table 2.
The capacitances are calculated with Equations (19) and (20). The calculated frequency-dependent self resistance $R'_{\text{self}}$ and self inductance $L'_{\text{self}}$ are shown in Figure 19. The simulations with the 3PPI model have been executed with 75 PI sections. The order of approximation for $Z'_m$ fit and for $Y'_L$ fit is 8. The simulation time step is set to $\Delta t = 10^{-7}$ s. In this simulation, two facts should be replicated correctly: the wave travel time $\tau$, which is related to the imaginary term of $\gamma$ and the attenuation, which is related to the real term of $\gamma$. For the sake of clearness, Figure 20 shows only a comparison between the outcomes of 3PPI Model with the measured currents for the core and shield A. The comparison shows that the model results match very well with the measured data. This means that the wave travel time and attenuation are replicated correctly in the developed cable model which proves its validity. For example, in our case the simulated wave travel time is 14.8 $\mu$s and the measured value is 14.5 $\mu$s. The small deviation between the travel times (0.3 $\mu$s) can also be minimized by increasing the number of PI sections. However, this will increase the calculation time.

In addition, it should be noted that a certain variation between simulation and measurement is always to be expected in general. This is mostly related to the lack of accurate information about some cable data, for example the thickness of inner and outer insulation screens and also their conductivity and permittivity. The inaccuracy in the cable data will affect the accuracy of the calculated longitudinal impedance $Z'$ and shunt admittance $Y'_L$. This will be reflected in the simulation results of the cable model.

The required number of PI sections in the model depends on the expected frequencies in the transients, the cable length and the accepted calculation error. With the proposed model, calculation times of few seconds have been achieved.
5. Conclusions

In this paper, a full frequency-dependent cable model for calculations of transients was introduced. The frequency-dependent cable parameters are determined with an improved partial subconductor algorithm. The algorithm is able to handle a large number of conductors. Complete cable systems can be implemented including components like shields, armor, earth and other grounding conductors in parallel through the earth. If necessary, semiconducting layers can be realized in form of additional conductors. The skin and the proximity effect is considered in all conductors.

The improved partial subconductor method merely allows the calculation of the inductive coupling between conductor loops. The capacitive coupling is realized by other analytical methods. The frequency dependency of a XLPE insulation capacitance can be neglected for calculations in frequency ranges $f < 5 \text{ MHz}$. 

The wave propagation cable model based on lumped parameters. A modal transformation technique was employed in the cable model to transform the cable system into decoupled single conductors. Every single conductor in the modal domain is represented through cascaded developed...
PI sections, where the frequency dependence of cable parameters is implemented through an RLC network in every PI section.

For an efficient computation time, the simulations in the time domain are executed using a recursive convolution technique. The number of PI sections in the full frequency-dependent cable model depends on the simulated cable length, the expected highest frequency in the investigated transient, and the accepted calculation error.

A comparison of the outcomes of proposed 3PPI model with measured data has shown a good match, which validates the approach used in developing this model.

A next important step is to compare the results of the developed full Frequency-dependent PI section cable model with other existing cable models, which is the goal of our next publication.

**Author Contributions:** Abdullah Hoshmeh developed the wave propagation model; Uwe Schmidt developed the subconductor method. The publication costs of this article were funded by the German Research Foundation/DFG and the Technische Universität Chemnitz in the funding programme Open Access Publishing.

**Conflicts of Interest:** The authors declare no conflict of interest.

**References**


© 2017 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).