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Estimation of Transformers Health Index Based on the Markov Chain

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Abstract: This paper presents a study on the application of the Markov Model (MM) to determine the transformer population states based on Health Index (HI). In total, 3195 oil samples from 373 transformers ranging in age from 1 to 25 years were analyzed. First, the HI of transformers was computed based on yearly individual oil condition monitoring data that consisted of oil quality, dissolved gases, and furanic compounds. Next, the average HI for each age was computed and the transition probabilities were obtained based on a nonlinear optimization technique. Finally, the future deterioration performance curve of the transformers was determined based on the MM chain algorithm. It was found that the MM can be used to predict the future transformers condition states. The chi-squared goodness-of-fit analysis revealed that the predicted HI for the transformer population obtained based on MM agrees with the average computed HI along the years, and the average error is 3.59%.

Keywords: transformers; Health Index (HI); Markov Model (MM); nonlinear optimization; transition probabilities; deterioration performance curve; chi-squared goodness-of-fit; asset management

1. Introduction

Transformers are counted among the important assets in a power system network, failures of which could lead to costly consequences. Failures of transformers can be initiated by several factors such as design issues, unusual loadings, electrical faults, and advanced degradation of insulations. According to References [1,2], the degradation of transformers is a complex phenomenon that can be affected by several factors. Nowadays, the majority of utilities have implemented Condition-Based Management (CBM) to closely monitor the condition states of transformers. Through this approach, the management strategies of the assets can be improved and the cost can be reduced as compared to previous Time-Based Management (TBM). CBM utilizes overall condition monitoring data from transformers and provides possible actions that can be carried out by utilities [2–4]. Under CBM, a single quantitative assessment known as Health Index (HI) is normally formulated to provide the overall condition of transformers. HI normally consists of multiple input parameters such as oil condition monitoring data, loadings, design, location, and electrical/mechanical integrities [5–11].

HI provides a comprehensive condition assessment of transformers as compared to Dissolved Gases Analysis (DGA), which mainly focuses on the identification of faults [5]. Conventionally, HI is used to determine the current state of transformers and there is a potential to utilize HI for future states predictions. Common mathematical approaches such as regression, fitting, and extrapolation techniques are not suitable due to the overreliance on the data, which may affect the reliability of predictions [12–14]. Currently, there are still less studies that have been carried out to model the future condition states of transformers based on HI. Other studies, such as those in References [6,9,15–17], mainly focused on the utilization of the HI to determine the future reliability of transformers and its impact on the power system network. The Markov Model (MM) is identified as one of the prediction methods that can be used to determine the future states of transformers based on HI. It is based on a probability decision process where future decisions on maintenance schemes depend on actual assets performances [14,18,19]. MM had been widely implemented in References [14,18–28] to model the deterioration of different types of equipment. In civil engineering, MM had been applied in References [14,18–24] to model the degradation of bridge deck and elements, pavement, water piping components, and steel hydraulic structures. MM had also been utilized in References [25–28] for electrical equipment such as modeling the condition of switchgear oils, the identification of faults, and transformers spare units. In this study, an innovative approach is proposed that utilizes MM to determine the deterioration performance curve based on computed HI from the transformer population. The approach can be used to estimate the future condition of the transformer population with less complexity and the prediction data can be updated dynamically. In total, the oil condition monitoring data from 373 distribution transformers with ratings of 33 kV and 30 MVA are used for the case study. Next, the HI is computed based on a scoring method and the future condition states of transformers are predicted based on MM.

2. Condition Assessment and Health Index (HI)

The overall condition of transformers is normally monitored through the Health Index (HI). HI is defined by Reference [29] as an approach to quantify transformers condition monitoring information for asset management purposes. Nowadays, HI is adopted by most of the utilities in the world [5,7,8,10,11,30]. The conventional concept of HI formulation is based on a scoring method that is based on weighting and ranking techniques [29–31]. There were also a number of advanced methods that had been proposed to determine the HI [32,33]. These techniques are quite complex and require extensive information to compute the HI. In this study, the scoring method was chosen for the computation of HI due its simplicity, adaptability with the readiness of data, and the fact that it is most commonly used by utilities nowadays. Figure 1 shows the HI computation principle for a single condition monitoring data based on the scoring method [30].

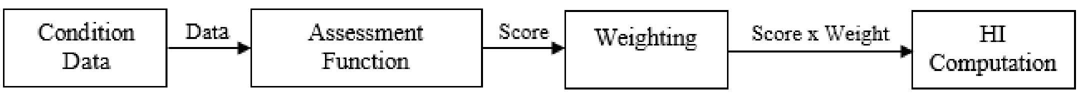

Figure 1. Health Index (HI) scoring method computational principles for single condition monitoring data (adapted from [30]).

The condition data are extracted from the condition monitoring information and physical observations. The assessment function is defined based on standards, guidelines, historical information, and theoretical knowledge. Expert judgement and statistical record are usually utilized to determine the weighting factors. In this study, the oil quality parameters considered were AC breakdown voltage, moisture in oil, acidity, color, and interfacial tension. In total, seven gases were considered, including hydrogen, methane, ethane, ethylene, acetylene, carbon monoxide, and carbon dioxide. First, the score and weighting factors for individual parameters were obtained according to the corresponding ranges in References [5,31]. Next, the factors for oil quality and dissolved gases were computed according to References [5,31]. The next step was to determine the factors for oil quality and dissolved gases in oil according to Equation (1).

$$DGF \text{ or } OQF = \frac{\sum_{i=1}^{n} S_j \times W_j}{\sum_{i=1}^{n} W_j} \quad (1)$$

where W_j is the weighting factor for each parameter, n is the number of parameters in each factor, and S_j is the score for each parameter. Finally, the rating codes for both parameters were determined from the rating code table in References [5,31]. For furanic compounds, the rating codes were determined directly from the rating code table in References [5,34]. Based on the rating codes for all parameters, the final HI was computed according to Equation (2). Equation (2) is based on References [5,31,34], where the modification was carried out by the removal of percentage ratios for transformers and tap changers, as only transformers data could be obtained in this study.

$$HI = \frac{K_{DGA} HIF_{DGA}}{4K_{DGA}} + \frac{K_{OQA} HIF_{OQA}}{4K_{OQA}} + \frac{K_{FA} HIF_{FA}}{4K_{FA}} \quad (2)$$

where K is the rating given to each factor, and HIF is the score of each factor.

3. Development of the Markov Model (MM)

3.1. Markov Chain Modeling Concept

In this study, MM is implemented to determine the future condition of the transformers population based on HI. The overall process of the approach in this study can be seen in Figure 2.

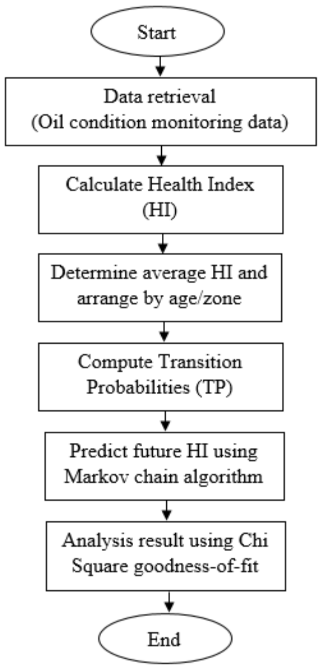

Figure 2. Modeling process of deterioration performance curve based on the Markov Model (MM).

According to References [18,35,36], the Markov decision process is normally characterized as a memoryless process where it predicts the future condition of equipment as a probabilistic estimate. The Markov chain depends on the transition probabilities given as P_{ij} [18,36]. P_{ij} is the probability of equipment decaying from state condition i to j in a specific interval time. A set of transition probabilities can be represented in a form known as the transition matrix, $\boldsymbol{P}$. $P_{ij}(t)$ has the same value in a specific year and each state probability must be equal to 1, for example, $P_{11} + P_{12} = 1$, $P_{22} + P_{23} = 1$, $P_{33} + P_{34} = 1$, and $P_{44} + P_{45} = 1$. Equation (3) shows the formulation of the transition matrix for five state conditions used in this study.

$$\boldsymbol{P} = \begin{bmatrix} P_{11} & 1-P_{11} & 0 & 0 & 0 \\ 0 & P_{22} & 1-P_{22} & 0 & 0 \\ 0 & 0 & P_{33} & 1-P_{33} & 0 \\ 0 & 0 & 0 & P_{44} & 1-P_{44} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

Once the yearly HI of all transformers were computed based on Equation (2), the average HI based on age was determined and plotted. The plot trend was defined as the computed transformers life deterioration performance curve and used for the modeling purposes of MM. The HI indicator scales and states used for the MM were obtained from References [5,31], and can be seen in Table 1.

Table 1. HI indicator scales and states (adapted from [5,31]).

State	Health Index	Condition	Description
1	85–100%	Very Good	Some aging or minor deterioration of a limited number of components.
2	70–84%	Good	Significant deterioration of some components.
3	50–69%	Fair	Widespread significant or serious deterioration of specific components.
4	30–49%	Poor	Widespread serious deterioration.
5	0–29%	Very Poor	Extensive serious deterioration.

Several assumptions were made to analyze the HI of transformers based on the MM. First, the deterioration process of transformers was considered as a monotonic and irreversible process. Thus, the condition of transformers either remained in its existing condition group or moved to the next state condition group. In order to develop the homogeneity of the deterioration performance curve, the transition probabilities were determined based on the transformers age groups. This zoning technique was applied to avoid over- and under-estimations of the transformers conditions [14,18,19]. In total, five zones of transformers age were identified, of which the transition matrix, $\boldsymbol{P}$, was assumed to be homogenous. The last zone of the transition matrix was used for future prediction. For further simplification of the Markov chain process, the final state condition, P_{55}, was set as 1 based on the assumption that all transformers would end up in the very poor condition. Next, the future HI of transformers were computed based on the initial HI condition, as shown in Equation (4).

$$\boldsymbol{H}_{n+1} = \boldsymbol{H}_n \times \boldsymbol{P} \times \boldsymbol{R}^T \quad (4)$$

where $\boldsymbol{H}_{n+1}$ is the next condition at the specific interval, $\boldsymbol{H}_n$ is the current condition and $\boldsymbol{R}^T$ is the matrix transform of the HI state condition. The input data for the matrix transform was obtained from Table 1, where $\boldsymbol{R}$ = [100 84 69 46 29]. MM is also able to predict the future condition state for a number of intervals, t, from the initial state, $\boldsymbol{H}_0$, and transition matrix, $\boldsymbol{P}$, which can be seen in Equation (5). In this study, all transformers at age 0 were considered to be at the initial state where $\boldsymbol{H}_0$ = [1 0 0 0 0] for zone 1. Since the transformers condition measurements were performed every year, t was set as 1.

$$\boldsymbol{H}_t = \boldsymbol{H}_0 \times \boldsymbol{P}^t \quad (5)$$

3.2. Derivation of Transition Probabilities

Estimation of the transition probabilities, P_{ij}, is crucial since it is the core element of the MM process. The transition probabilities matrix can be determined by heuristic or statistic techniques. In this study, a statistical technique known as the nonlinear optimization technique was implemented.

The objective of this technique is to identify the values of four parameters, P_{11}, P_{22}, P_{33}, and P_{44}, that would minimize the absolute differences between the computed and predicted HI data for each transformers group [14,18,37–40]. The function can be seen in Equation (6).

$$\min \sum_{t=1}^{N} | A(t) - B(t,P) | \quad (6)$$

where N is the number of year in each zone, P is the transition probabilities (P_{11}, P_{22}, P_{33}, P_{44}), $A(t)$ is the average or computed HI at time t, and $B(t,P)$ is the predicted values of condition HI by MM at time t. Once the transition matrix in the first zone was determined, the transition probabilities of the second zone were computed based on Equations (5) and (6) through the assumption that the last state condition in the previous zone became the initial state condition for the next. The process was repeated through to the last group of transformer conditions. Finally, the deterioration performance curve was obtained.

4. Application of Markov Modeling

In this study, the condition monitoring data from 3195 oil samples measured from 373 transformers with voltage and power ratings of 33/11 kV and 30 MVA were tested. The range of the transformers' age was between 1 and 25 years and the distribution of oil samples data can be seen in Figure 3. The computed HI of transformers in its age and zone are shown in Table 2.

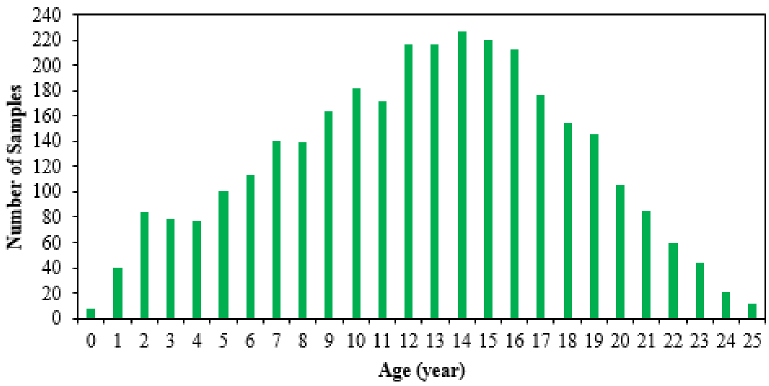

Figure 3. Distribution of oil sample data.

Table 2. Computed HI by age and zones.

Zone	Number of Oil Sample	Transformer Age (Year)	Computed HI (%)
1	40	1	89.38
	84	2	82.80
	79	3	82.57
	78	4	69.46
	101	5	74.33
2	113	6	68.82
	140	7	67.60
	139	8	65.53
	164	9	60.85
	182	10	61.09
3	171	11	56.26
	216	12	58.35
	216	13	55.59
	227	14	54.36
	220	15	54.41

Table 2. *Cont.*

Zone	Number of Oil Sample	Transformer Age (Year)	Computed HI (%)
4	212	16	53.55
	177	17	49.78
	154	18	50.24
	146	19	48.99
	106	20	49.05
5	85	21	49.18
	60	22	50.48
	44	23	57.82
	21	24	56.80
	12	25	51.41

Next, the nonlinear optimization was performed based on Equation (6). This technique was carried out through the combination of the transition probabilities (P_{11}, P_{22}, P_{33}, P_{44}), the solution of which was based on the least error count. This technique was adopted in several MM studies for other applications [14,18,37–40]. The first two zones of computed HI were used to compute the transition matrix for training and application purposes. The computed HI for zones 3, 4 and 5 were used to validate the predicted HI obtained by the Markov chain algorithm. An example of the first set of the computation for zone 1 is described in the following section. First, the transition matrix in Equation (7) was computed using Equation (6) and Table 2.

$$\boldsymbol{P} = \begin{bmatrix} 0.2889 & 0.7111 & 0 & 0 & 0 \\ 0 & 0.7717 & 0.2283 & 0 & 0 \\ 0 & 0 & 0.9626 & 0.0374 & 0 \\ 0 & 0 & 0 & 0.9900 & 0.0100 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

Next, the computed transition matrix was used to determine the condition state from years 1 to 5. The computational process for each year can be seen in Equations (8)–(12).

$$\boldsymbol{H}_1 = \boldsymbol{H}_0 \times \boldsymbol{P}^1 = (0.2889,\ 0.7111,\ 0.0000,\ 0.0000,\ 0.0000) \tag{8}$$

$$\boldsymbol{H}_2 = \boldsymbol{H}_0 \times \boldsymbol{P}^2 = (0.0835,\ 0.7542,\ 0.1623,\ 0.0000,\ 0.0000) \tag{9}$$

$$\boldsymbol{H}_3 = \boldsymbol{H}_0 \times \boldsymbol{P}^3 = (0.0241,\ 0.6414,\ 0.3284,\ 0.0060,\ 0.0000) \tag{10}$$

$$\boldsymbol{H}_4 = \boldsymbol{H}_0 \times \boldsymbol{P}^4 = (0.0070,\ 0.5121,\ 0.4626,\ 0.0183,\ 0.0001) \tag{11}$$

$$\boldsymbol{H}_5 = \boldsymbol{H}_0 \times \boldsymbol{P}^5 = (0.0020,\ 0.4002,\ 0.5622,\ 0.0354,\ 0.0002) \tag{12}$$

The process was repeated for zone 2 where the last condition state computed in year t = 5 in Equation (12) was used as an initial condition state for zone 2. Equation (13) shows the transition matrix computed for zone 2.

$$\boldsymbol{P} = \begin{bmatrix} 0.5051 & 0.4949 & 0 & 0 & 0 \\ 0 & 0.8923 & 0.1077 & 0 & 0 \\ 0 & 0 & 0.7315 & 0.2685 & 0 \\ 0 & 0 & 0 & 0.9900 & 0.0100 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

The transition matrix in Equation (13) was used to predict the future HI for zones 3 to 5. The initial state in all zones calculated using Equation (5) can be seen in Table 3.

Table 3. Initial state of each zone.

Zone	Initial State				
Zone 1	1.0000	0.0000	0.0000	0.0000	0.0000
Zone 2	0.0020	0.4002	0.5622	0.0354	0.0002
Zone 3	0.0001	0.2278	0.2136	0.5442	0.0144
Zone 4	0.0000	0.1289	0.0991	0.7261	0.0459
Zone 5	0.0000	0.0729	0.0515	0.7918	0.0838
Zone 6	0.0000	0.0413	0.0282	0.8067	0.1238
Zone 7	0.0000	0.0233	0.0157	0.7969	0.1640

The predicted HI obtained by MM for a period of 25 years are shown in Table 4 and Figure 4. It was found that majority of the predicted HI values are quite close to the computed HI values. There are slight deviations for several of the predicted HI values at the ages of 3, 4, 24, and 25 years. Further analysis was carried out based on chi-squared goodness-of-fit test, as shown in Equation (14), in order to determine the goodness-of-fit between the computed and predicted HI [18,19].

$$X^2 = \sum_{i=0}^{k} \frac{(R_i - E_i)^2}{E_i} \tag{14}$$

where k is number of observations, E_i is the computed value of the ith observation, R_i is the predicted value of the ith observation, and X^2 is a chi-squared distribution coefficient with $k-1$ degrees of freedom. At probability α of 0.05, X^2 is 4.19, which is lower than the chi-squared critical value, which is 36.42.

Table 4. Computed and predicted HI.

Age	Computed HI (%)	Predicted HI (%)
1	89.38	88.62
2	82.80	82.90
3	82.57	79.25
4	69.46	76.53
5	74.33	74.35
6	68.82	70.66
7	67.60	67.60
8	65.53	65.02
9	60.85	62.84
10	61.09	60.96
11	56.26	59.34
12	58.35	57.92
13	55.59	56.66
14	54.36	55.56
15	54.41	54.57
16	53.55	53.69
17	49.78	52.89
18	50.24	52.17
19	48.99	51.51
20	49.05	50.91
21	49.18	50.35
22	50.48	49.84
23	57.82	49.37
24	56.80	48.94
25	51.41	48.53

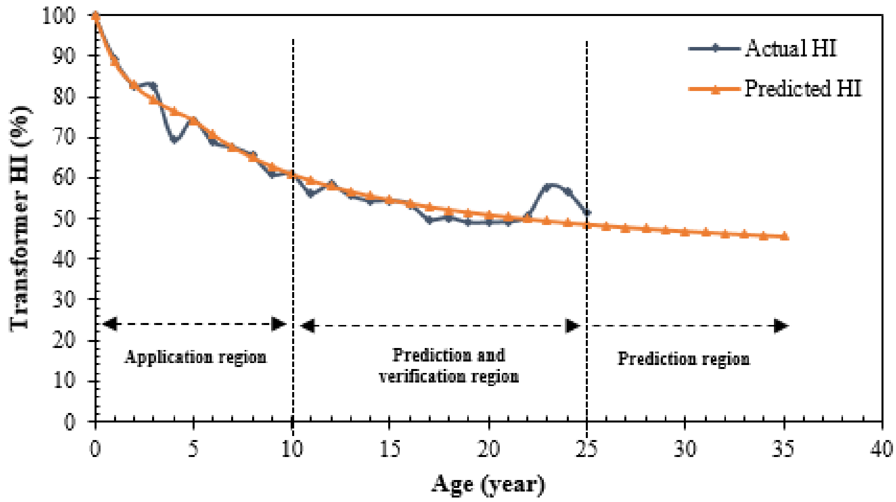

Figure 4. Comparison between computed and predicted HI.

Based on the case study, it was found that the transformer population is in very good and good conditions during the first six years of service, as shown in Table 1 and Figure 4. The transformer population is in fair condition between seven and 21 years of service. After 22 years of service, the transformer population starts to enter poor condition. The prediction reveals that the transformer population remains in poor condition even after 35 years of service. The same analysis as [12] was carried out to determine the average percentage error between the computed and predicted HI condition curves based on Equation (15), as shown in Figure 5.

$$\text{Average Percentage Error } (\%) = \frac{\sum_{t=n}^{25} \left(\frac{|Y_n - X_n|}{|Y_n|} \times 100 \right)}{n} \tag{15}$$

where Y_n is the computed HI, X_n is the predicted HI, and n is the age of the transformer. The overall average percentage error from zones 1 to 5 is 3.59%, while from zones 3 to 5 it is 4.51%. By subtracting 100% with predicted error determined for zones 3 to 5, the accuracy of the HI prediction based on MM for the transformer population is 95.49%. The application of MM to estimate the future states of transformers based on HI is a promising approach for the assets management of utilities. It is shown that with limited data, MM is able to predict the future states of transformers based on HI. This study can be further validated in the future if the HI from utilities can be obtained. Based on this information, the prediction of HI can be accurately determined based updated transition probabilities.

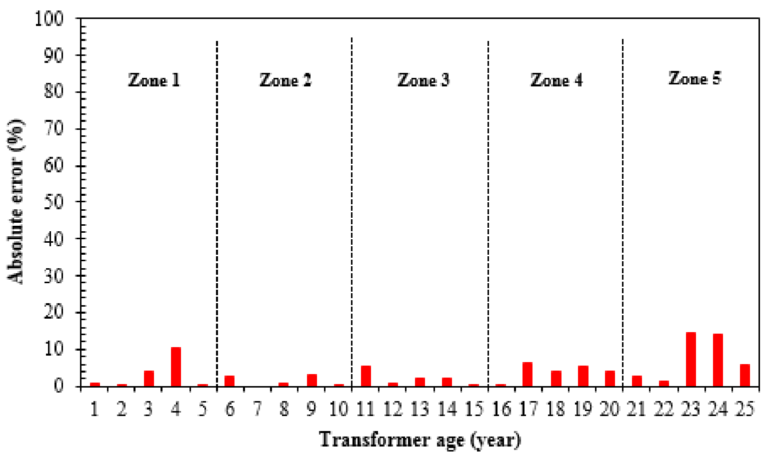

Figure 5. Absolute error between computed and predicted HI.

5. Conclusions

The application of MM to predict the transformers deterioration performance curve based on HI was carried out in this study. It was found that MM can be used to estimate the future states of transformers based on HI. The transition probabilities obtained by the nonlinear optimization technique show that the predicted HI is quite good and the prediction accuracy could reach up to 95.49%. Based on the predicted HI condition curve, state scale, and recommendations in References [5,6,11,31], the planning for the maintenance, repair, and replacement processes could be considered after 22 years, when the transformer population starts to enter the poor condition. Overall, MM provides a less complex approach for the prediction of transformers HI and can be easily implemented by utilities that utilize HI for their optimal asset management strategy. In addition, MM is a dynamic approach where the projected HI can be updated with the updated transition probabilities.

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Conflicts of Interest: The authors declare no conflict of interest.

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