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A Novel Methodology for Estimating State-Of-Charge of Li-Ion Batteries Using Advanced Parameters Estimation

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Received: 4 October 2017; Accepted: 26 October 2017; Published: 1 November 2017

Abstract: State-of-charge (SOC) estimations of Li-ion batteries have been the focus of many research studies in previous years. Many articles discussed the dynamic model's parameters estimation of the Li-ion battery, where the fixed forgetting factor recursive least square estimation methodology is employed. However, the change rate of each parameter to reach the true value is not taken into consideration, which may tend to poor estimation. This article discusses this issue, and proposes two solutions to solve it. The first solution is the usage of a variable forgetting factor instead of a fixed one, while the second solution is defining a vector of forgetting factors, which means one factor for each parameter. After parameters estimation, a new idea is proposed to estimate state-of-charge (SOC) of the Li-ion battery based on Newton's method. Also, the error percentage and computational cost are discussed and compared with that of nonlinear Kalman filters. This methodology is applied on a 36 V 30 A Li-ion pack to validate this idea.

Keywords: dynamic model of Li-ion battery; variable forgetting factor recursive least square (RLS) estimator; multiple forgetting factors RLS estimator; state-of-charge (SOC) estimation using Newton's method

1. Introduction

Batteries have become a common power source in many applications. This is especially true of Li-ion batteries, which have many merits, such as low weight and high current density. The dynamic model of batteries has been the topic of many recent publications. Some of these publications [1–4] studied the electrochemical modeling technique (ECM), which depends on the internal structure of the battery. Since the structure of the battery is difficult to be obtained, it is hardly used in the modeling. Others studied an empirical modeling technique (EPM) as in [5], which does not clarify the behavior of the internal states of the Li-ion battery (black box). The intuitive way to model the Li-ion battery is the electrical modeling technique (EM), which can be quickly built and depict the electrical characteristic of the Li-ion battery [6].

In [7], two resistor-capacitor (RC) networks are used to establish an equivalent circuit of the Li-ion battery model, and this model is parameterized using a numeric optimization method. In [8], the same model is used, but it recommended using more RC circuits in the model to increase the accuracy. In [9], a leakage resistance is added to the RC network in the battery model, and an experimental procedure is used to determine the internal resistance, leakage resistance, and the values of the RC elements. All of the previous articles considered that the model parameters are variable, but in [10], the model parameters of the Li-ion battery are considered constant from 20% state-of-charge (SOC) to 100% SOC, while the models have nonlinear behavior in the interval from 0% SOC to 20% SOC. The recent articles [11–17] discussed the uncertainty in the modeling, the nonlinear relation between open circuit voltage (OCV) and SOC, and the sensor noise. In [11,13], the fixed forgetting factor recursive least square estimator of Li-ion battery parameters was used to estimate the parameters with minimum error, but the change rate of each parameter to reach the true value was not taken into consideration, which leads to a poor estimation of the parameters.

The nonlinear relation between OCV and SOC was linearized as in [16,17] whereas the others dealt with this nonlinear relationship as in [11,13,18]. All of these articles used different types of Kalman filters (KF) to estimate the states changes of the Li-ion battery, especially SOC, except [4], which used a proportional–integral (PI) observer. Due to this nonlinearity between OCV and SOC, nonlinear KF types were employed, such as an extended Kalman filter (EKF) [11], unscented Kalman filter (UKF) [19], and particle filter (PF) [20]. All of these KFs are affected by significant noise in the measurements, which can tend toward divergence in the estimation (blow up).

In this paper, the change rate of parameters estimation is taken into consideration, and two solutions are proposed to solve this problem. The first solution is a variable forgetting factor used with the recursive least square estimator instead of a fixed forgetting factor, while the second solution defines a vector of forgetting factors for a recursive least squares estimator. After the estimation, an algorithm based on Newton's iteration method is established as a direct way to estimate SOC of the Li-ion battery and independent of measurements noise.

This paper is organized as follows. The dynamic model of the Li-ion pack is presented in Section 2, and the two solutions for parameters estimation are proposed. Section 3 describes the new idea for estimating SOC using Newton's method, while Section 4 describes the test bench layout and clarifies its components. Finally, the results are drawn and discussed.

2. Dynamic Model of Li-Ion Battery

2.1. Li-Ion Battery Equivalent Circuit

The accurate equivalent circuit of the Li-ion battery in [10] is used in this paper, where the equivalent circuit of the Li-ion battery is represented as in [11,13,17,21] and depicted in Figure 1, where R_0 represents the internal resistance of the Li-ion battery, R_{p1} and C_{p1} indicate the activation polarization resistance and capacitance, respectively, and R_{p2} and C_{p2} are the concentration polarization resistance and capacitance, respectively.

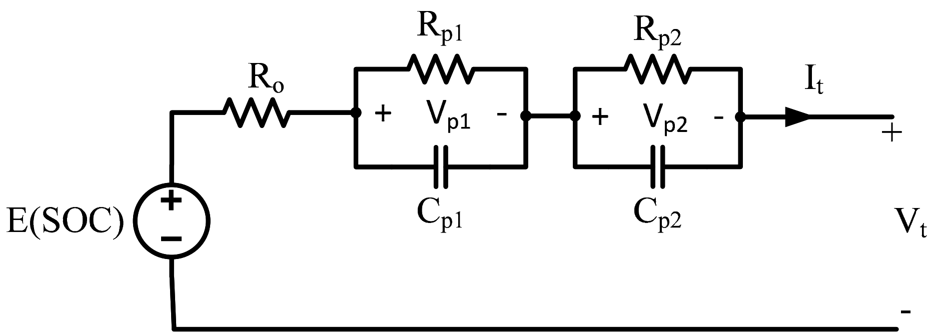


Figure 1. Equivalent circuit of Li-ion battery.

From Figure 1, the differential equations that represent this circuit can be deduced as:

$$\dot{SOC} = -\frac{1}{Q_n} I_t \tag{1}$$

$$\dot{V}_{P_1} = -\frac{1}{R_{p_1} C_{p_1}} V_{p_1} + \frac{1}{C_{p_1}} I_t \tag{2}$$

$$\dot{V}_{P_2} = -\frac{1}{R_{p_2}C_{p_2}}V_{p_2} + \frac{1}{C_{p_2}}I_t \quad (3)$$

$$V_t = E(SOC) - V_{p_1} - V_{p_2} - R_o I_t \quad (4)$$

where V_t and I_t are the battery terminal voltage and the battery current, respectively, V_{p1} is the applied voltage on activation polarization branch, V_{p2} is the applied voltage on concentration polarization branch, and $E(SOC)$ is the OCV, and a function of SOC. Finally, Q_n represents the nominal capacity of the battery.

Because of the output equation in (4), the system becomes nonlinear. To overcome this dilemma and estimate the parameters, the output equation can be rearranged and rewritten as:

$$v_d = V_{p_1} + V_{p_2} + R_o I_t$$

where $v_d = E(SOC) - V_t$.

In this case, the system becomes linear, and the transfer function can be obtained quickly as:

$$G(s) = \frac{v_d}{I_t} = \frac{R_0 s^2 + \left(\frac{R_0(\tau_1+\tau_2)+R_{p1}\tau_2+R_{p2}\tau_1}{\tau_1\tau_2}\right)s + \frac{R_0+R_{p1}+R_{p2}}{\tau_1\tau_2}}{s^2 + \left(\frac{\tau_1+\tau_2}{\tau_1\tau_2}\right)s + \frac{1}{\tau_1\tau_2}} \quad (5)$$

where $\tau_1 = R_{p1}C_{p1}$ and $\tau_2 = R_{p2}C_{p2}$.

The recursive least square estimator can be easily used, if the previous transfer function is discretized using bilinear transformation $\left(z^{-1} = \frac{2/T_s - s}{2/T_s + s}\right)$ as:

$$G(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \quad (6)$$

As in [13], the discretized transfer function after the transformation is compared with Equation (5), and the dynamic model parameters are derived as:

$$R_0 = \frac{b_0 - b_1 + b_2}{1 - a_1 + a_2}$$

$$\tau_1\tau_2 = \frac{T_s^2}{4}\left(\frac{1 - a_1 + a_2}{1 + a_1 + a_2}\right)$$

$$\tau_1 + \tau_2 = T_s\left(\frac{1 - a_2}{1 + a_1 + a_2}\right)$$

$$R_0 + R_{p1} + R_{p2} = \frac{b_0 + b_1 + b_2}{1 + a_1 + a_2}$$

$$R_0(\tau_1 + \tau_2) + R_{p1}\tau_2 + R_{p2}\tau_1 = \frac{T_s(b_0 - b_2)}{1 + a_1 + a_2}$$

Because of lack of accurate measurements, the battery model can be described as an output error model as in [22], where the error appears in the measurements due to low precision equipment, and it is also known as an autoregressive with exogenous (ARX) model

$$y(t) = w(t) + e(t) \quad (7)$$

$$w(t) + f_1 w(t-1) + \ldots + f_{n_f} w(t - n_f) = b_1 u(t-1) + \ldots + b_{n_b} u(t - n_b) \quad (8)$$

From (7) and (8), the output becomes:

$$y(t) = \frac{B(z)}{F(z)}u(t) + e(t)$$

where $F(z) = 1 + f_1 z^{-1} + \ldots + f_{n_f} z^{-n_f}$ and $B(z) = b_1 z^{-1} + \ldots + b_{n_b} z^{-n_b}$.

Consequently, the natural predictor is described as:

$$y(t|\theta) = \frac{B(z)}{F(z)}u(t)$$

Which is the same as (6).

For calculating v_d, the OCV should be determined as a function of SOC. Most of the articles used polynomial interpolation techniques as [11,13,21] to determine the relationship between OCV and SOC even though the rational interpolation is stronger than the polynomial interpolation method [23–25]. The rational function is a mixer between two polynomials, and its form is represented as:

$$f(x) = \frac{a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n}{b_0 + b_1 x + b_2 x^2 + \ldots + b_m x^m}$$

The Levenberg–Marquardt algorithm is used to estimate these coefficients. This algorithm is also known as damped least square method, and is mainly used to solve nonlinear least square problems. For estimating the parameters β of the model curve $f(x,\beta)$, the sum of the squares of the deviations is minimized:

$$\hat{\beta} = \mathrm{argmin} S(\beta) = \mathrm{argmin} \sum_{i=1}^{m} (y_i - f(x_i, \beta))^2$$

In each iteration, β is replaced by a new iteration $\beta + \delta$, so the function update will be $f(x_i, \beta + \delta)$. This function can be linearized using Taylor expansion as:

$$f(x_i, \beta + \delta) \approx f(x_i, \beta) + J_i \delta$$

where $J_i = \frac{\partial f(x_i,\beta)}{\partial \beta}$.

Finally, the solution of this iteration technique becomes:

$$\left(J^T J\right)\delta = J^T [y - f(\beta)] \tag{9}$$

2.2. Parameters Estimation of Li-Ion Battery

Some articles, such as [11,13], used recursive least square estimator with a fixed forgetting factor, which may tend to poor estimation because of different change rates for each parameter. Therefore, this paper introduces two solutions for this problem. The first solution is using recursive least square estimator with variable forgetting factor, and the second one is recursive least square estimator with a vector of forgetting factors.

2.2.1. Recursive Least Square Estimator with Variable Forgetting Factor

In [26], the solution is based on a signal-to-noise ratio (SNR), where SNR is defined as:

$$SNR = \frac{\sigma_y^2}{\sigma_v^2}$$

where σ_y^2 is the variance of $y(n)$ and σ_v^2 is the variance of the white output noise.

The estimated SNR within the adaptive filtering context can be defined as:

$$SNR_f = \frac{\sigma_{\hat{y}}^2}{\sigma_e^2}$$

where $\sigma_{\hat{y}}^2$ is the variance of the estimated output $\hat{y}(n)$, and σ_e^2 is the variance of the error signals.

At steady state conditions:

$$SNR \approx SNR_f$$

Since $y(n)$ and $v(n)$ are uncorrelated, the output variance has been written as:

$$\sigma_d^2 = \sigma_y^2 + \sigma_v^2 \tag{10}$$

From the equality of *SNR* and SNR_f at steady state conditions and Equation (10), the output noise variance can be derived as:

$$\sigma_v^2 = \frac{\sigma_e^2 \sigma_d^2}{\sigma_{\hat{y}}^2 + \sigma_e^2}$$

For evaluating the variance of output noise, the other variances should be estimated as:

$$\hat{\sigma}_d^2(k) = \alpha \hat{\sigma}_d^2(k-1) + (1-\alpha) d^2(k)$$

$$\hat{\sigma}_{\hat{y}}^2(k) = \alpha \hat{\sigma}_{\hat{y}}^2(k-1) + (1-\alpha) \hat{y}^2(k)$$

$$\hat{\sigma}_e^2(k) = \alpha \hat{\sigma}_e^2(k-1) + (1-\alpha) e^2(k)$$

where α is a weighting factor with $\alpha = 1 - 1/(KM)$, K is a positive number >1, and M is the sample width.

Also, the variance of the parameters $\hat{\sigma}_\theta^2$ is discussed, which has a role in evaluating the forgetting factor λ as well as the other variances. The variance of the parameters is defined as:

$$\hat{\sigma}_\theta^2(k) = \alpha \hat{\sigma}_\theta^2(k-1) + (1-\alpha) \theta^2(k)$$

Finally, the practical variable forgetting factor is derived as

$$\lambda(k) = \begin{cases} \min\left(\frac{\hat{\sigma}_\theta^2(k)\hat{\sigma}_v^2(k)}{\left| \varepsilon + \hat{\sigma}_e^2(k) - \hat{\sigma}_v^2(k) \right|}, \lambda_{max} \right) & \zeta(k) > \varepsilon \\ \lambda_{max} & \zeta(k) \le \varepsilon \end{cases} \tag{11}$$

where $\zeta(k)$ is a convergence parameter with $\zeta(k) = \left| \hat{\sigma}_d^2(k) - \hat{\sigma}_{\hat{y}}^2(k) - \hat{\sigma}_e^2(k) \right|$ and ε is a small positive number.

2.2.2. Multiple Forgetting Factors Recursive Least Square Estimator

The recursive least square estimator with a single forgetting factor, the gain L is:

$$L_k = \frac{P_{k-1}\varphi(k)}{\lambda + \varphi(k)^T P_{k-1} \varphi(k)}$$

And the covariance matrix P is:

$$P_k = \frac{1}{\lambda}\left(I - L_k \varphi(k)^T \right) P_{k-1}$$

However, in recursive least square estimator with multiple forgetting factors, the gain L becomes as [27]:

$$L_k = \frac{F_\lambda\left(P_{k-1}{}^{-1}\right)^{-1}\varphi(k)}{\lambda + \varphi(k)^T F_\lambda\left(P_{k-1}{}^{-1}\right)^{-1}\varphi(k)}$$

And the covariance matrix also becomes:

$$P_k = \left(I - L_k\varphi(k)^T\right)F_\lambda\left(P_{k-1}{}^{-1}\right)^{-1}$$

where F_λ is the forgetting map, which is defined as follows:

$$\begin{aligned} F_\lambda &: S_p^+ \to S_p^+ \\ R_{k-1} &\to F_\lambda(R_{k-1}) \end{aligned}$$

Here, S_p^+ denotes the cone of positive definite matrices of dimension p and $\lambda = \begin{bmatrix} \lambda_1 & \ldots & \lambda_p \end{bmatrix}^T \in \mathbb{R}^p$ is the forgetting vector while $P_k = R_k^{-1}$.

Let $R_{k-1} = \begin{bmatrix} R_{k-1,1} & \cdots & R_{k-1,15} \\ \vdots & \ddots & \vdots \\ R_{k-1,51} & \cdots & R_{k-1,5} \end{bmatrix}$, and since the parameters are uncorrelated with each other therefore, the mapping will be as in [12]:

$$F_\lambda(R_{k-1}) = \begin{bmatrix} \lambda_1 R_{k-1,1} & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 R_{k-1,2} & 0 & 0 & 0 \\ 0 & 0 & \lambda_3 R_{k-1,3} & 0 & 0 \\ 0 & 0 & 0 & \lambda_4 R_{k-1,4} & 0 \\ 0 & 0 & 0 & 0 & \lambda_5 R_{k-1,5} \end{bmatrix}$$

In general, the diagonal updating is:

$$[F_\lambda(R_{k-1})]_{i,j} = \begin{cases} 0 & if\ \lambda_i \neq \lambda_j \\ [R_{k-1}]_{i,j}\lambda_i & \text{otherwise} \end{cases}$$

However, in [27], the author proposed that the parameters are correlated, and the mapping is changed to be:

$$F_\lambda(R_{k-1}) = \begin{bmatrix} \lambda_1 R_{k-1,1} & \cdots & \lambda_5 R_{k-1,15} \\ \vdots & \ddots & \vdots \\ \lambda_5 R_{k-1,51} & \cdots & \lambda_5 R_{k-1,5} \end{bmatrix}$$

where $\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4 > \lambda_5$.

So generally, the mapping becomes:

$$[F_\lambda(R_{k-1})]_{i,j} = \min\left(\lambda_i, \lambda_j\right)[R_{k-1}]_{i,j}$$

3. Estimation of SOC

This paper proposes a new idea for estimating SOC of the Li-ion battery, which is based on Newton's iteration method and independent on the measurement noise variance. Firstly, Equation (5) is transformed to state space representation in continuous form as in Equation (12); then, the state space representation is transformed from continuous form to discrete form, as in Equation (13).

$$\begin{bmatrix} \dot{V}_{p1} \\ \dot{V}_{p2} \end{bmatrix} = \begin{bmatrix} \frac{-1}{R_{p1}C_{p1}} & 0 \\ 0 & \frac{-1}{R_{p2}C_{p2}} \end{bmatrix} \begin{bmatrix} V_{p1} \\ V_{p2} \end{bmatrix} + \begin{bmatrix} \frac{1}{C_{p1}} \\ \frac{1}{C_{p2}} \end{bmatrix} I_t \qquad (12)$$
$$V_d = V_{p1} + V_{p2} + R_0 I_t$$

$$\begin{bmatrix} V_{p1} \\ V_{p2} \end{bmatrix}_{k+1} = \begin{bmatrix} e^{\frac{-T_s}{R_{p1}C_{p1}}} & 0 \\ 0 & e^{\frac{-T_s}{R_{p2}C_{p2}}} \end{bmatrix} \begin{bmatrix} V_{p1} \\ V_{p2} \end{bmatrix}_k + \begin{bmatrix} R_{p1}\left(1 - e^{\frac{-T_s}{R_{p1}C_{p1}}}\right) \\ R_{p2}\left(1 - e^{\frac{-T_s}{R_{p2}C_{p2}}}\right) \end{bmatrix} I_{t,k} \qquad (13)$$
$$V_{d,k} = V_{p1,k} + V_{p2,k} + R_0 I_{t,k}$$

According to Newton's iteration method, if $f(x)$ is supposed to have a simple zero x^* in $[a,b]$, and f is continuously differentiable on $[a,b]$, then there is an interval about x^* such that Newton's method converges to x^* for any x_0 in that interval. Therefore, this method is appropriate to estimate SOC where $SOC \in [0,1]$ in the OCV relationship.

This method is clarified and implemented in an algorithm, which is depicted in Figure 2.

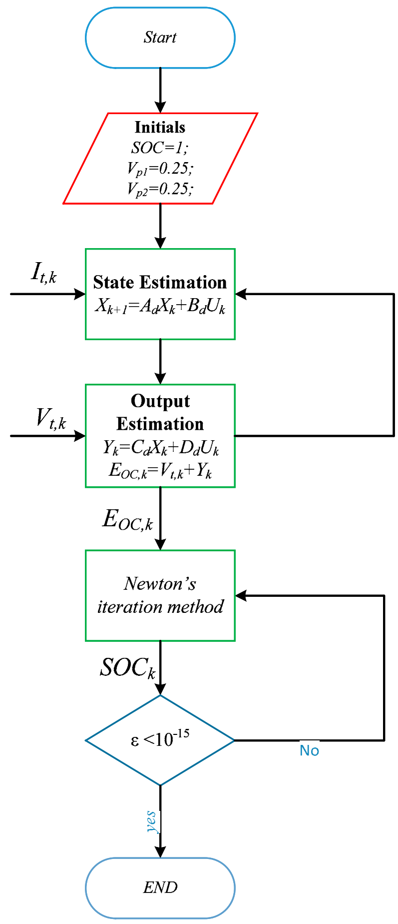
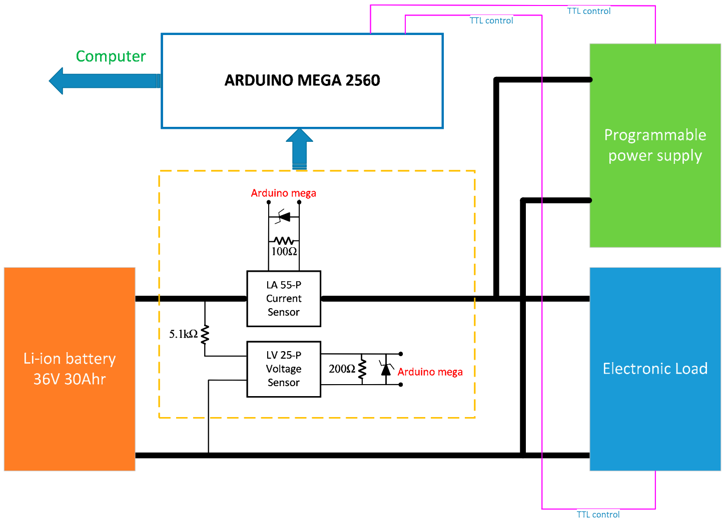


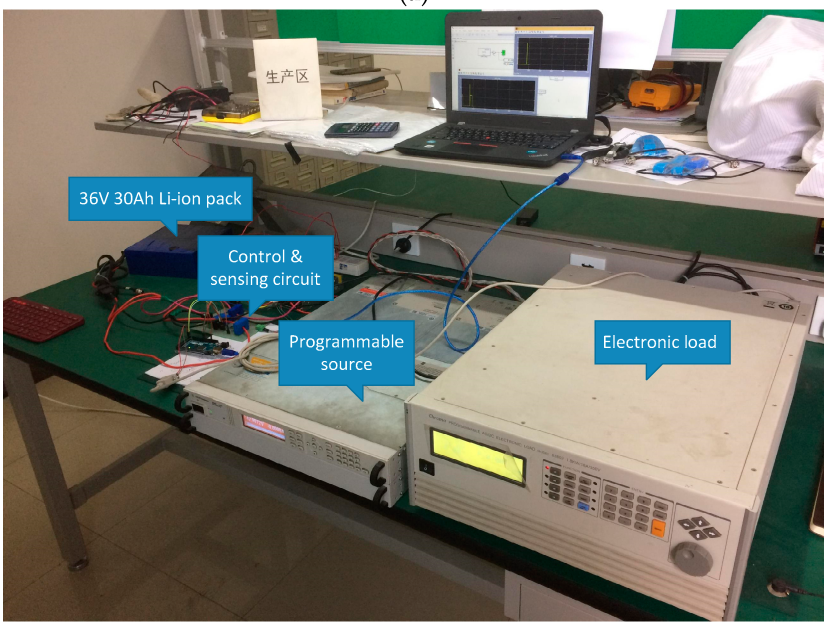
Figure 2. Algorithm for estimating state-of-charge (SOC) of the Li-ion battery using Newton's iteration method.

4. Case Study System and Experiment Setup

The experimental setup is a 36 V, 30 A, Li-ion pack, and the layout of the test bench is described as in Figure 3. This test bench consists of a programmable power supply and electronic load. The programmable power supply is a solar array simulator, which acts as a constant current source. This power supply is a 510 W module with maximum output voltage 65 V and maximum output current 8.5 A. The electronic load is working as a constant current load with the utmost current 18 A.



(**a**)



(**b**)

Figure 3. (**a**) Experiment layout for testing the battery; (**b**) Experiment test bench of Li-ion battery.

In this test bench, the microcontroller (Arduino mega 2560) is used to interface with the MATLAB environment in a computer, as well as control the switching between the programmable power supply and the electronic load through transistor-transistor logic (TTL) control. This bench is built up to plot the terminal voltage, with 8 A pulsating current for extracting the OCV values to the corresponding state of charge. Also, it is used to validate the dynamic model of the Li-ion pack by a hybrid pulse power characterization (HPPC) test, where charging and discharging are applied consequently on the Li-ion pack.

5. Results and Discussion

From the test bench, the terminal voltage is obtained by adjusting the electronic load to withdraw 8 A as a constant current load, and generating a pulsating current via TTL control of the electronic load, which is clarified in Figures 4 and 5.

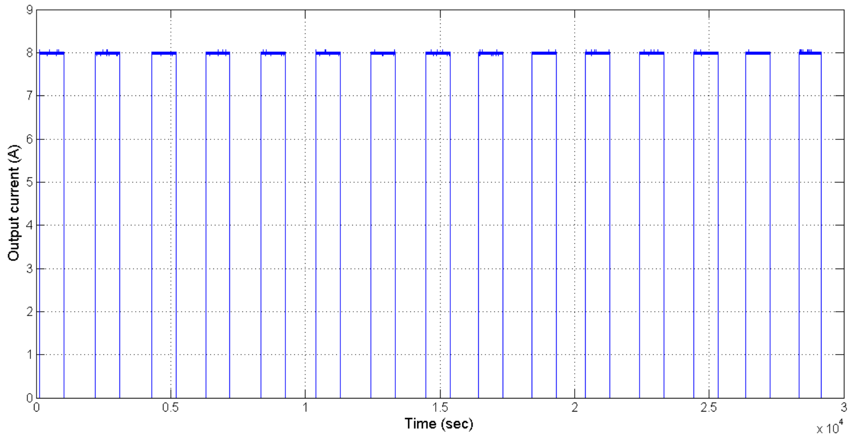


Figure 4. The pulsating current with constant amplitude (8 A) drawn by the electronic load.

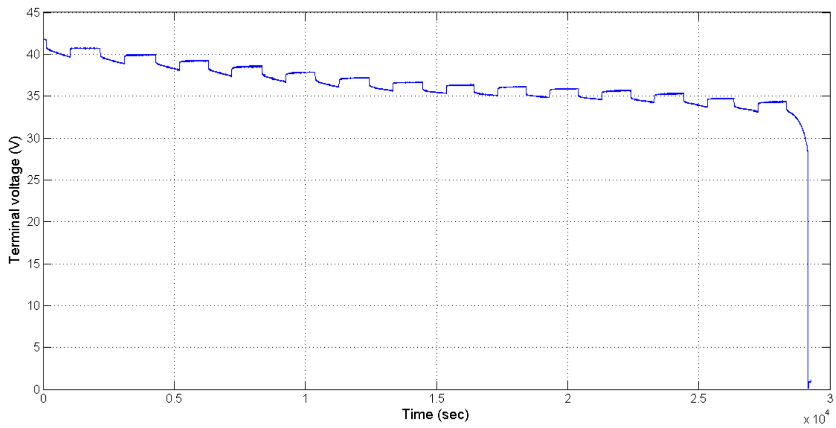


Figure 5. The measured voltage via Arduino mega 2560.

After the terminal voltage values are extracted under current pulsating test, the relationship between OCV and SOC can be estimated by applying Equation (9), and it is derived as:

$$E(SOC) = \frac{-226.8SOC^4 + 488SOC^3 - 334.1SOC^2 + 77.47SOC + 0.6258}{SOC^5 - 8.696SOC^4 + 15.54SOC^3 - 9.959SOC^2 + 2.222SOC + 0.02141} \tag{14}$$

Fractional forms with various orders had been tried, but this particular form of E(SOC) was chosen to fit the experimental points with minimum error, and it is drawn as a fitted curve as in Figure 6 during the whole discharge period. From subtracting Figure 5 from Figure 6, v_d can be easily calculated and drawn, as shown in Figure 7.

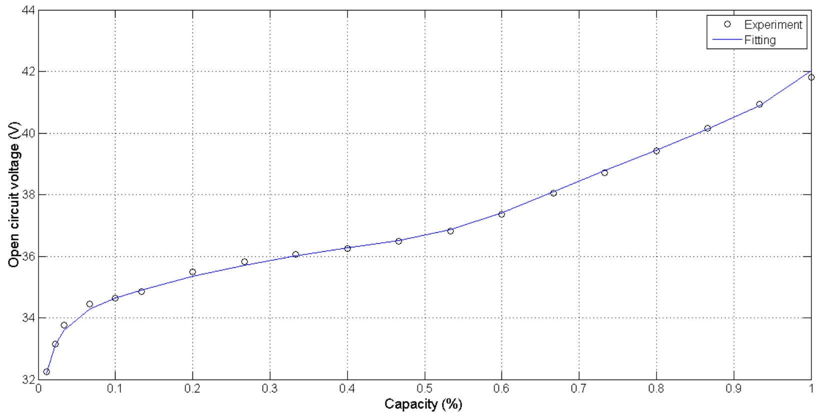


Figure 6. A comparison between experimental measurements and the fitted curve.

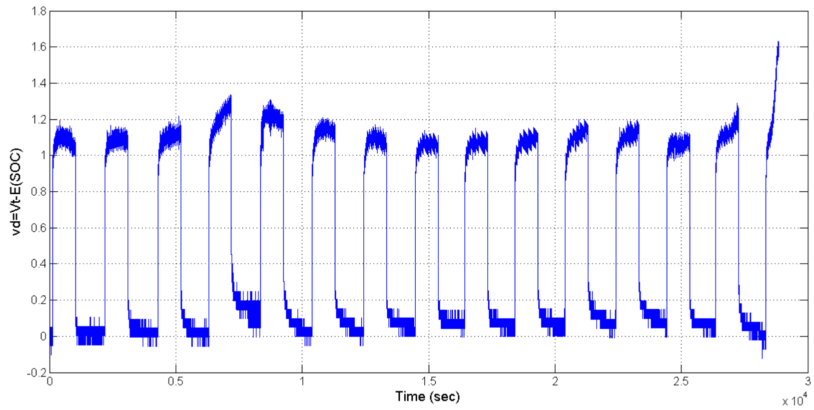


Figure 7. The subtraction between the open circuit voltage (OCV) and the terminal voltage (v_d).

Due to the different change rates of each parameter to reach its true value, two solutions are proposed in Section 2.2 to estimate these parameters. The first solution, which is described as a variable forgetting factor recursive least square estimator, is applied on our system, and the estimation is converged as in Figure 8. The error in voltage between the measured voltage and the estimated one is less than 0.04 V most of the time, as depicted in Figure 9. In Figure 10, it is evident that all of the parameters converged to a steady state value.

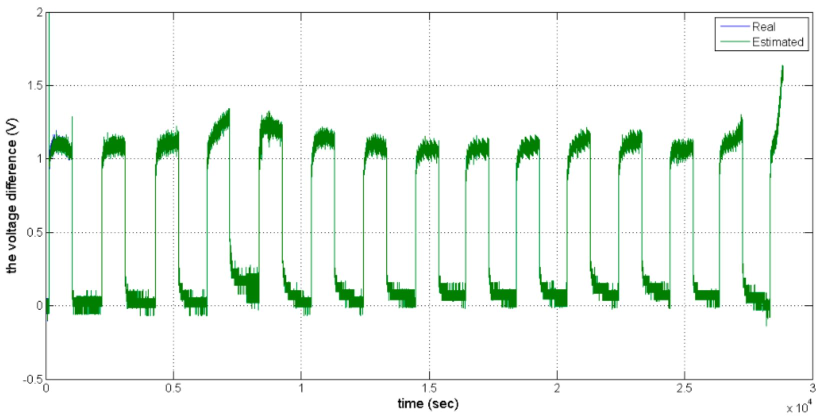


Figure 8. The parameter estimation using variable forgetting factor.

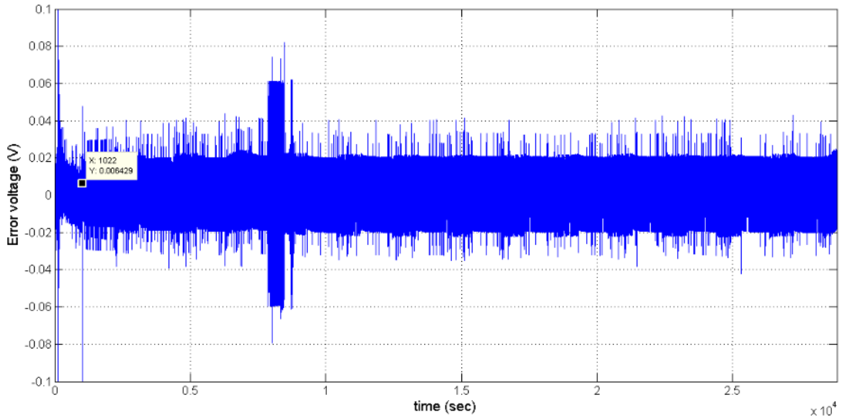


Figure 9. The error in voltage between the measured output and the estimated output.

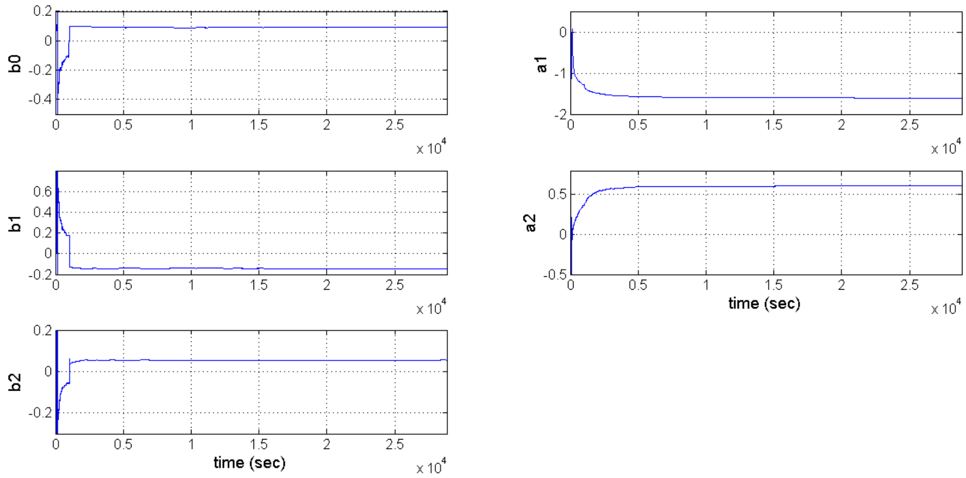


Figure 10. The estimated parameters of a discrete transfer function $G(z)$.

The second solution, which is based on establishing a vector of forgetting factors one for each parameter, to overcome the different change rate is also applied on the system. The vector mapping based on the correlation among the parameters is employed, and the estimated results are demonstrated in Figure 11. The error between the measured voltage and the estimated one reached at the end was less than 0.04 V, as in the previous technique, which is depicted in Figure 12. Consequently, the parameters converge to the true value, whenever the change rate of each parameter is different.

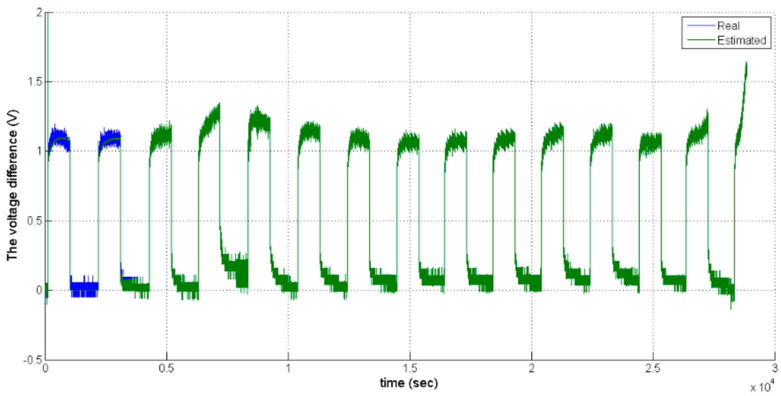


Figure 11. The parameter estimation using multiple forgetting factors.

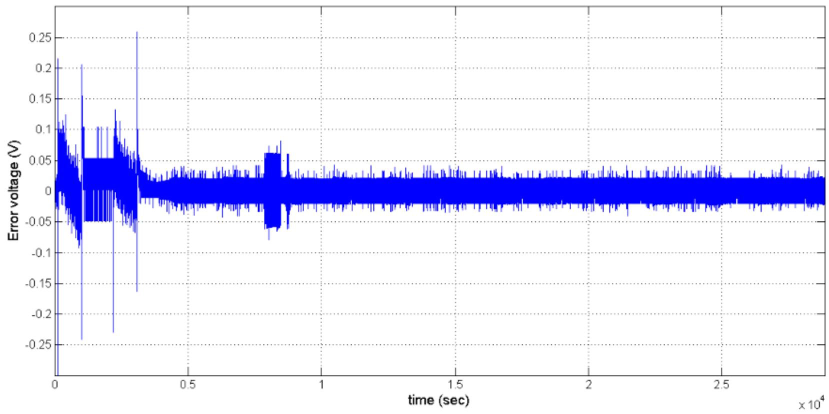


Figure 12. The error in voltage between the measured output and the estimated output.

From Figures 10 and 13, it is observed that the parameters of the Li-ion battery model are constant, and it can be said that the transfer function $G(z)$ represented the dynamic model of the Li-ion battery as:

$$G(z) = \frac{0.08664 - 0.1402z^{-1} + 0.0573z^{-2}}{1 - 1.63z^{-1} + 0.6312z^{-2}} \tag{15}$$

For validating the dynamic model of the Li-ion battery, the hybrid pulse power characterization (HPPC) test is employed, as in [15]. This test is used to determine power capability over the battery useable voltage range using a test profile that incorporates both discharge and charge pulses. In Figure 14, it is shown that the simulated model performance of the Li-ion battery matched the performance of the experimental setup, with a max error about ±0.2 V, as in Figure 15. These results show more accurate results than [15].

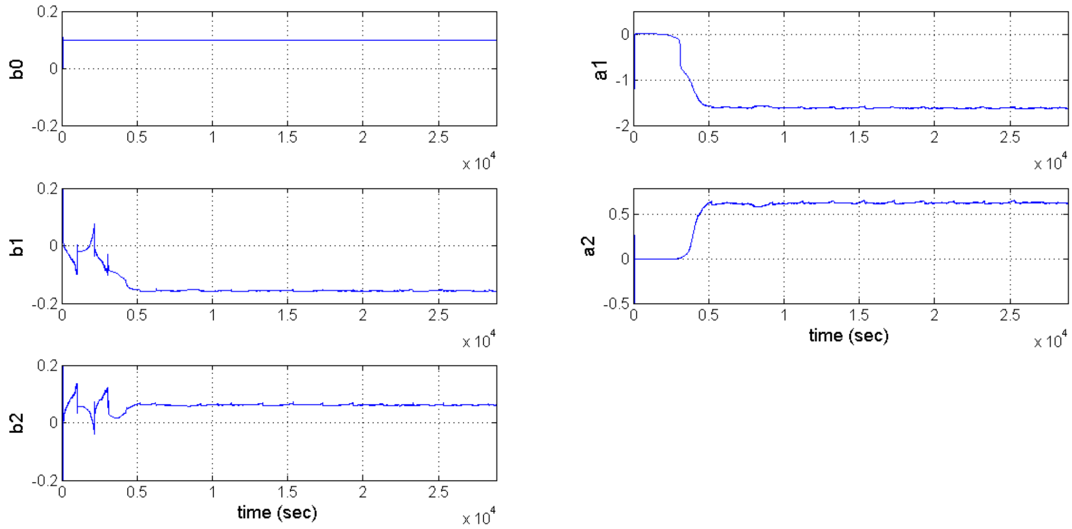


Figure 13. The estimated parameters of a discrete transfer function $G(z)$.

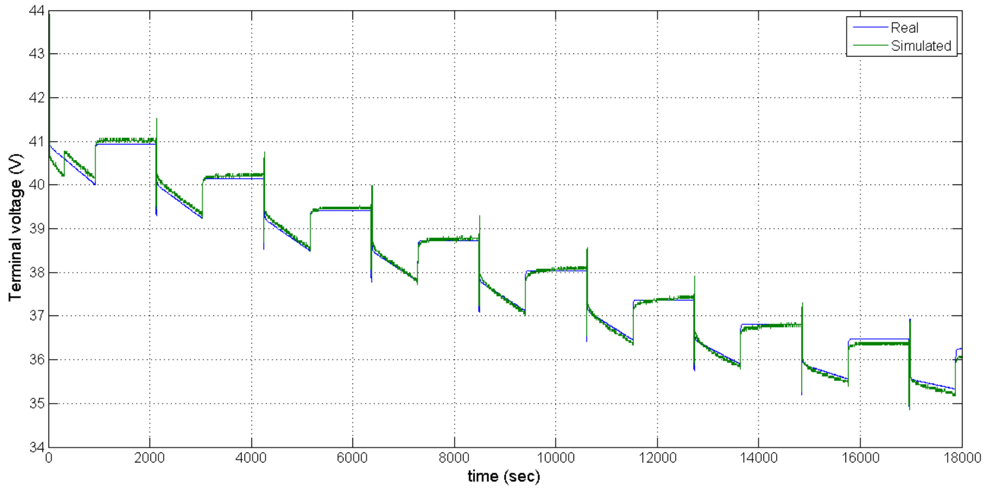


Figure 14. The comparison between actual response of the battery and the simulation under the hybrid pulse power characterization (HPPC) test.

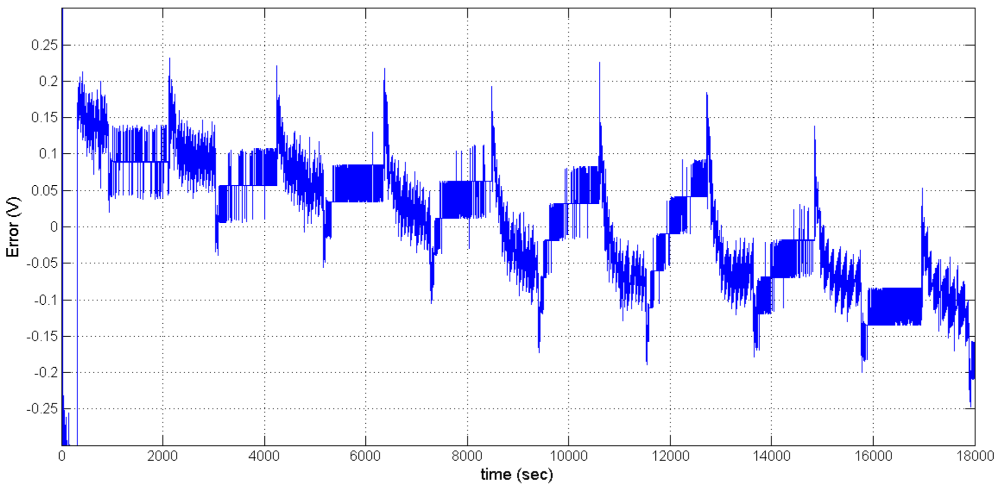


Figure 15. The error between the real model and the simulated one under HPPC.

After validating the dynamic model with an acceptable error, a new methodology is proposed to estimate SOC independent on the noise. Its algorithm is shown in Figure 2, where it runs via the MATLAB environment. This method is applied on a 36 V 30 A Li-ion pack during the whole discharge period (T = 28,850 s), and pulsating current 8 A, as in Figure 16, which gives an error around 3% via the effective working period (20% SOC to 100% SOC), as shown in Figure 17. In [14], a comparative study among nonlinear Kalman filters (EKF, UKF, cubature KF (CKF) and PF) was established, which is applied on a UR18650A Li-ion cell. This comparative study discussed the computational cost and the root mean square error (RMSE) percentage in SOC estimation. The percentages of RMSE of SOC for different types of KFs are clarified in Table 1, where the error percentage is from 4.48 in EKF to 3.05 in PF.

SOC is a dimensionless value from 0% to 100%, and it is independent of the capacity of the battery. Also, the comparative study is applied on the Li-ion battery with the same electrical model and the same current profile as in [14]. Consequently, the proposed technique has an acceptable RMSE percentage in this comparative study, as in Table 1. For the computational cost calculation, it depends on how long one loop of the algorithm takes from start to end [28]. In [14], EKF as an example takes 0.41 s for one full discharge scenario (4000 points); that means, it takes 1.025×10^{-4} s/pt. However, the proposed method takes 6.3945 s for one full discharge period (146,434 points); that means it takes 4.3668×10^{-5} s/pt. From this result, it is deduced that the computational cost of this method is low.

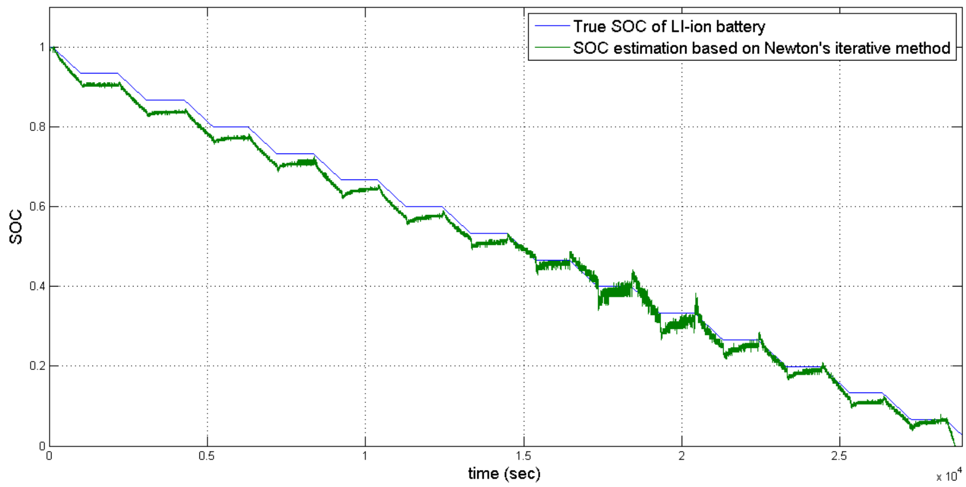


Figure 16. SOC estimation based on Newton's iteration method.

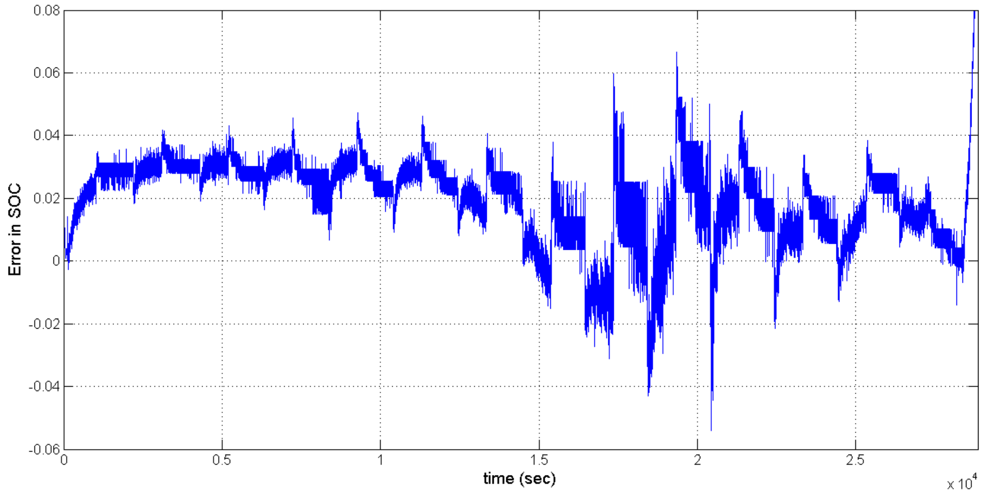


Figure 17. The error in the estimation of SOC.

Table 1. Efficiency of the applied state estimation methods [14].

Method	%RMSE (SOC)	Computational Time (s)
EKF	4.48	0.41
UKF	4.02	1.27
CKF	3.31	0.88
PF	3.05	10.43 (N = 300)

There is no doubt that the biggest problem in the nonlinear Kalman filters is the effect of the measurements noise, which can cause the filter divergence. This algorithm overcomes this issue because it is independent of the measurement's noise, and its computational cost is low, as in EKF. If this algorithm error compares with that of nonlinear Kalman filters which are mentioned in [14], it is deduced that the algorithm gives good estimation with an acceptable error.

6. Conclusions

In this publication, the estimated parameters of the Li-ion battery reach its true value accurately when the change rate of each parameter is taken into consideration by applying the recursive least square estimator with a variable forgetting factor or a vector of forgetting factors. In the first solution, the forgetting factor is derived depending on the signal to noise ratio and the variance of the parameters σ_θ allowing the parameters to reach its true values. However, in the second solution, the vector of forgetting factors is determined by criteria of selection $(\lambda_1 > \lambda_2 > \lambda_3 > \lambda_4 > \lambda_5)$ to permit parameters reaching its true values.

The estimation of SOC is based on Newton's method, which is an iteration method. The advantage of using this method is avoidance of the effect of the measurement noise in the estimation process. Besides this advantage, the algorithm built based on this method estimates the state-of-charge with an acceptable error percentage around 3%, and a low computational cost, 6.3945 s, during the whole discharge period.

Acknowledgments: This work was supported by the National Natural Science Foundation of China (grant numbers 51420105011, 51677110).

Author Contributions: Ibrahim M. Safwat contributes in this manuscript with the idea, the experimental tests, the simulations, and writing the manuscript. Weilin Li contributes in this manuscript with the English language revision and the manuscript organization. Xiaohua Wu provides the first author with the apparatuses and the technical revision.

Conflicts of Interest: The authors declare no conflicts of interest.

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