

# **Hedging performance and multiscale relationships in the German electricity spot and futures markets**

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**ABSTRACT** We explore optimal hedge ratios and hedging effectiveness for the German electricity market. Given the increasing attention that wavelets received in the financial market, we concentrate on the investigation of the relationship, covariance/coherence evolution and hedge ratio analysis, on a time-frequency-scale approach (discrete and continuous), between electricity spot and futures. Simpler approaches are also used for comparison purposes like the naïve, OLS and the dynamic multivariate GARCH model in order to account for risk reduction through hedging.

Results allow us to conclude that: dynamic hedging strategies provide higher variance reductions in terms of hedging effectiveness; there is poor correlation among spot and futures, not being homogeneous across scales, which condition the effectiveness of the hedging strategy; the long-horizon hedge ratio does not converge to its long run equilibrium of one. Wavelets poor fit in variance reduction is attributed to low coherence and to statistical relationships between spot and futures electricity series.

The instability found in various aspects of market comovements may imply serious limitations to the investor's ability to exploit potential benefits from hedging with futures contracts in electricity markets. Moreover, much variation in the contemporaneous relationship among spot and futures may highlight inadequacy in assuming (short-term) relationships in both markets, which might account for the difficulty in achieving profitable active trading.

**KEYWORDS:** Dynamic and Static Hedging; Electricity Futures and Spot Prices; Discrete and Continuous Wavelets Coherence and Phase; Optimal Hedge Ratio; Multivariate GARCH

**JEL CLASSIFICATION:** C32; G10

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## **1. INTRODUCTION**

Electricity markets are of considerable interest and challenging in terms of modeling and hedging due to non-storability, strong seasonal fluctuations, price spikes and their highly volatile price behavior (Huisman, Huurman and Mahieu, 2007)<sup>1</sup>. Hedgers in these markets use futures to reduce the risk from variations in the spot market (Torró, 2009; Zanotti, Gabbi and Geranio, 2010). Since futures are a derivative security from the spot market, it should be safe to say that both are subject to the same impacts from market fundamentals, and that both series should be highly co-integrated.

We investigate the relationship between spot and futures contracts in the German electricity market, EEX (European Electricity Exchange), in terms of covariance and coherence, while analyzing hedge ratios at various time scales resorting to wavelet analysis. Wavelets are used here in both discrete and continuous versions to examine the data at different time locations and resolution levels, while comparing reconstructed price series based on different levels of information detail. The ability to decompose financial data at several time-scales is the wavelets' main advantage.

Early studies assume a constant hedge ratio through time simply estimating it by using OLS or a naïve strategy. However, given the time varying nature of covariance in many financial markets (Lee, 1999), this constancy assumption appears inappropriate (Lee and Leuthold, 1983). Afterwards, improvements appeared (Wang and Low, 2003) in the adoption of bivariate GARCH frameworks, introducing dynamics to the analysis.

Traders make singular decisions at different time scales and the dynamic relationship structure of spot and futures will itself vary at different time scales associated with those different investment horizons. In fact, the dynamics of commodity markets have always been influenced by the interactions of traders with different time horizons, which react to the arrival of new information in a heterogeneous manner. Results attained here should be of interest to local intraday traders, hedge managers, international investors, as well as monetary and regulatory authorities, all of whom operate on very different time scales. Wavelets enable us to conclude if scales are important contributors to the overall variability of a series, and lead us to explore the hypothesis

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<sup>1</sup> The authors provide an overview on (hourly specific) day-ahead price characteristics.

that decomposing both price series and hedge ratio into various time scales will be helpful<sup>2</sup>.

Our empirical results show a weak feedback relationship between electricity spot and futures markets regardless of time scales. While wavelet variance shows that the futures markets are less volatile than the spot market, wavelet correlation varies over investment horizons and remains very low. The magnitude of the correlation decreases as the wavelet time scale increases, indicating that spot and futures markets are found to be fundamentally different and their relation not homogeneous across scales. The economic interpretation may be that the arrival of information in the market does not resolve price uncertainty in electricity markets. More uncertainty is added, and over a longer amount of time the basis risk increases (Dewally and Marriott, 2008). As such, the noise in the market does not tend to be canceled over time.

We also found higher comovement of spot and futures series at lower frequencies (high scales). As such, heterogeneous trading needs to be considered in the analysis of spot and futures price data in electricity markets and different hedging strategies also need to take into account this different scale behavior. Given our findings, if long-run adjustments were taken out, short-run movements would be little correlated. As such, only with a considerable time span spillovers are transmitted between markets.

Attained results show that dynamic hedging strategies provide higher variance reductions in terms of hedging effectiveness, as compared to those attained by static ones, given the poor correlation between spot and futures prices in electricity markets, and a very different volatile behavior. In sum, low correlation between spot and futures prices conditions the effectiveness of the hedging strategy. As such, even if with poor gains in terms of hedging effectiveness, the MGARCH hedging strategy shows to be the most reliable among the others in terms of hedging effectiveness, at least for the German electricity market considered here, even better than using wavelet analysis. As such, similar to Maharaj et al. (2008), we may say that econometric sophistication does not boost hedging effectiveness for electricity markets. These results offer additional insights and allow us to understand the properties and characteristics that shape effectiveness of electricity futures, valuable for electricity hedgers.

The time variation pattern documented in this study may carry some important implications for hedging. On one hand, the instability found in market comovements may imply

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<sup>2</sup> Given previous empirical findings favoring the use of wavelets (In and Kim, 2006a, 2006b), we are interested to know if decomposing electricity series into different time scales provides additional hedging gains, which is not confirmed by the empirical findings presented here.

serious limitations to the investor's ability to exploit potential benefits from hedging with futures contracts in electricity markets. On the other hand, much variation in the contemporaneous relationships among spot and futures base prices may also highlight inadequacy in assuming relationships in both markets, at least for the short-run, which might account for the difficulty in achieving profitable active trading.

The paper develops as follows. Section 2 presents previous theory and evidence about the studied relationship between spot and futures prices and hedge effectiveness. In section 3 the hedge ratio and its effectiveness are discussed. Section 4 presents the research methods employed in the empirical part of the work (other than the OLS and naïve hedge ratios): discrete and continuous wavelet analysis and the multivariate GARCH BEKK model. In section 5 the relevant data and empirical findings are presented, while section 6 concludes this work.

## **2. THEORY AND EVIDENCE**

Spot electricity markets deal with immediate delivery, while futures trade is based on delivery at some future point in time. Market conditions (continuous demand and supply balance, for example) alongside a series of other factors (seasonality and outages, among others) are often blamed for causing inherent uncertainties in electricity markets. Future (and forward) markets are devised to provide hedging mechanisms to deal with these uncertainties. In a way, futures reflect market's expectations about future market conditions and therefore the gap between spot and future prices is often used as a signal for describing the general market condition.

When we are in the presence of a non-storable commodity such as electricity, futures contract valuation and its use for risk management purposes are even more difficult. Cash-and-carry arbitrage lacking ends up creating a looser relationship between spot and futures prices, especially when futures maturity increases (Torró, 2009). On the other hand, electricity spot price behavior is characterized in a special way in the literature (jumps, high volatility, mean-reversion, seasonalities, positive skewness, as well as heteroskedasticity; see Huisman, Hurman and Mahieu, 2007, for example). These effects combined end up producing a lower than usual correlation between spot and futures prices, and might produce a poor performance when hedging spot price risk using futures contracts (Byström, 2003; Moulton, 2005; Torró, 2009).

Conclusions from existent studies measuring the efficiency of futures markets vary considerably. Reviewed literature shows no uniformity regarding results provided by the

accessible measuring methods, while the selected method can slightly bias the results (Zanotti, Gabbi and Geranio, 2010). More advanced models tend to confirm market efficiency but older ones may be prone to reject it. In general, it seems that commodity, energy and even power markets are not especially efficient (STEM, 2006). Avsar and Goss (2001) study market efficiency for the PJM and the California Power Markets and cannot reject the efficient market hypothesis for the period July 1998 – March 1999, although they cannot accept it for the whole data period. For the same PJM electricity market, Longstaff and Wang (2004) perform an empirical analysis with hourly data on spot and day-ahead forward prices. They find that forward risk premia vary systematically through the day and are related to agent's measures of economic risk like volatility of unexpected changes in demand, spot prices and total revenues.

Although it seems logical to assume that there are several time periods in decision making, economic and financial analysis have been restricted to at most two time scales (the short and the long run), due to the lack of analytical tools to decompose data into more than two time scales (Kim and In, 2005). Bierbrauer et al. (2007) use spot and futures price data from the German EEX power market to test the adequacy of various one and two-factor models for electricity spot prices. For short and medium term periods their results underpin the frequently stated hypothesis that electricity futures quotes are consistently greater than the expected future spot, a situation denoted as Contango. Worthington and Higgs (2004) found significant innovation and volatility spillovers between futures and spot market indices for Australian electricity regions, confirming the presence of strong ARCH and GARCH effects. Also, Shawky, Marathe and Barret (2003) investigate the statistical properties of wholesale electricity spot and futures prices traded on the New York Mercantile Exchange (NYMEX) for delivery at the California-Oregon Border. By using daily data they find that many of the characteristics of the electricity market can be viewed to be broadly consistent with efficient markets using a GARCH specification to estimate minimum variance hedge ratios.

Without resorting to wavelets, several have been the attempts to model the hedge ratio in the literature of commodities. Moschini and Myers (2002) assumed that the investor takes out futures positions and holds the position for a week. At the end of the week, the investor reevaluates the futures position and chooses a new hedge ratio for the following week. Hence, the hedge ratio must be adjusted every week to reflect time varying volatility. They reject the null of a constant hedge ratio and that time variation in optimal hedge ratio can solely be explained by deterministic seasonality and time to maturity effects, using weekly corn cash and futures prices, developing

modified BEKK parameterization for the Bivariate GARCH (q,r) model. Ripple and Moosa (2007) examine the effect of the maturity of the futures contract used as the hedging instrument on the effectiveness of futures hedging, using daily and monthly data on the WTI crude oil futures and spot prices (NYMEX). They use as measures of hedging effectiveness the near-month contract and the six-month contract, to conclude that futures' hedging is more effective when the near month contract is used, and that hedge ratios are lower for near-month hedging. Dewally and Marriott (2008) adopt a sample reduction technique to analyze short and long run hedging in base metal markets finding that the short run hedge ratio and hedging effectiveness increase with the hedging horizon and that the long term horizon limit to the optimal hedging ratio is not converging to one but is slightly higher.

Although the appropriate way to calculate hedge ratios remains a controversial issue in the literature, the major methodologies for hedging with futures contracts have been OLS, VAR, VECM and multivariate GARCH (MGARCH) (Moschini and Myers, 2002; Moulton, 2005; Torr , 2009; Huisman, Mahieu and Schlichter, 2009; among others).

Regarding European Power Markets, the largest number of studies exist for Nord Pool, the most developed power market in Europe since its foundation in 1993 (Bystr m, 2003). Bystr m concludes that traditional simple price hedging models are almost equally efficient as the most advanced ones. Therefore, hedging at Nord Pool (or whatever power futures markets) does not request more advanced models than other financial markets though underlying product features differ noticeably from other financial or commodities products. Torr  (2009) obtained an acceptable performance by increasing hedging duration and closing futures positions as near as possible to their final settlement, using weekly futures contracts and weekly spot prices (the average spot price for the 7 days in the week) for the period from 1998 to 2007 in the Scandinavian (Nord Pool) electricity market. In Moulton (2005), the underlying spot to the Californian futures was the average of peak hour's spot prices in a month. On the other hand, Bystr m (2003) uses weekly spot price risk, hedged with weekly futures using only one-week hedges duration for the NordPool market. The intention was to study the short term hedging performance (one-week holding period), and since short-term future contracts are more liquid as well as more correlated with the underlying spot prices than the longer term contracts, futures with three weeks left to maturity are chosen for the hedging investigations. Zanotti, Gabbi and Geranio (2010) estimate optimal hedge ratios through OLS, na ve and multivariate GARCH models for the German, French and Scandinavian electricity markets to conclude that the choice of the hedge ratio estimation

model is crucial on determining hedging effectiveness, using monthly contracts.

Connor and Rossiter (2005) point out that, in the context of commodity markets, long-horizon traders will essentially focus on price fundamentals that drive overall trends, whereas short-term traders will primarily react to incoming information within a short-term horizon. Hence, market dynamics in the aggregate will be the result of the interaction of agents with heterogeneous time horizons. Most of the empirical studies ignore the dependence of the optimal hedge ratio on the hedging horizon even though individuals and institutions, which use futures contracts for hedging purposes, do not have the same hedging horizon (Lien and Shrestha, 2007).

The present work differs from previous ones about hedging in electricity markets since we examine the relationship between spot and futures electricity markets over various time horizons using a recent empirical technique, wavelet analysis, thus employing a different testing methodology compared with previous studies that investigate spot and futures electricity markets relationships.

Applying continuous wavelet analysis to examine these factors has at least three salient features. First, the main advantage of using wavelet analysis is the ability to decompose the data into several time scales (investment horizons). Owing to the different decision making time scales among traders, the true dynamic structure of the relationship between the spot and futures electricity markets itself will vary over different time scales associated with those different horizons. Although it has been recognized that there are several time periods in decision making, most of the financial analyses have been restricted to at most two time scales: the short and the long run. Second, wavelet covariance decomposes the covariance/correlation between two stochastic processes over different time scales which allows us to examine it. Third, cross-wavelet coherency-phase analysis allows analyzing transient dynamics for the association between two time series. Wavelet analysis provides a way to investigate the relationship between spot and futures returns, as well as the estimation of the hedge ratio on a time scale-by-scale basis. This allows the estimation of the hedge ratios for different hedging horizons, while by using the wavelet coherence we are able to analyze the existent patterns between spot and futures prices at different time scales. Moreover, wavelet analysis does not suffer from the sample reduction problem faced by Chen, Lee and Shrestha (2004) and later recognized by Lien and Shrestha (2007), when matching the data frequency to the hedging horizon.

While becoming popular, Wavelet analysis has been used to study a number of issues like the relation between stock returns and economic activity (Gallegati, 2008), the relation between

financial variables and real economic activity (Kim and In, 2005), the relation between stock and futures returns for several markets (In and Kim, 2006a, 2006b), the study of the CAPM model (Aktan et al., 2009); and the relation between economic variables in the macro economy and the effects of monetary policy (Conraria, Azevedo and Soares, 2008). Studies using wavelet analysis that were applied to stock and future returns tend to favor the use of wavelet analysis, like those of In and Kim (2006a, 2006b), and Aktan et al. (2009). However, Maharaj et al. (2008), using wavelet analysis applied to hedge ratios, show that on the basis of the variance ratio test and variance reduction, econometric sophistication does not boost hedging effectiveness. Their study was performed using data from crude oil, soybeans and the S&P500 index.

A few studies that consider the effect of the length of the hedging horizon on the optimal hedge ratio include those by Ederington (1979), Malliaris and Urrutia (1991), Geppert (1995) and Lien and Tse (2000, 2002). These studies find that within-sample hedging effectiveness tends to increase as the investment horizon increases. However, all these studies, except Geppert (1995), consider only two to three different hedging horizons. By opposition, Chen, Lee and Shrestha (2004), In and Kim (2006, 2006a), Fernandez and Lucey (2007) and Fernandez (2008), all rely on hedging performance at different time scales using financial and commodities futures, but none of these studies has been applied to electricity markets. For example, Lien and Shrestha (2007) estimate optimal hedge ratios for different horizons for 23 different futures contracts using wavelet analysis. They conclude that the performance of the wavelet hedge ratio improves with the increase in the length of the hedging horizon. Chen, Lee and Shrestha (2004), when matching the data frequency to the hedging horizon, found that almost all of the total return variance (more than 90%) can be attributed to shorter time scales, which holds for both spot and futures returns. Moreover, Geppert (1995) finds that correlation between cash and futures increase as we move from high to low frequency. To justify this he argues that given a co-integrated relationship between spot and futures prices, which is made up of both permanent and transitory components, over longer horizons the permanent component ties futures and spot prices together while the effect of the transitory component becomes negligible. Therefore, most of the previous studies of wavelets applied to hedge ratio estimation conclude that the optimal hedge ratio increases in line with the time horizon and tends to approach the Naïve hedge ratio which is 1: *“This means that the investor would sell a larger number of futures contracts to achieve the optimal hedge ratio as the frequency of the hedge increases”* (Geppert, 1995; Cotter and Hanly, 2010). But this would have implications on the cost of the hedging strategy, which increases with scales (in this sense, hedging

a single 20-day period would be more expensive than a single 1-day period). We show here that the same does not apply to electricity markets.

In sum, the present study distinguishes from previous ones in at least four aspects. First, we apply wavelet analysis to study electricity spot and futures returns on a scale-by-scale basis, which has been revealed to be useful given the different time investment horizons and hedging decisions among traders. Second, we wanted to see if previous empirical findings for index and commodities markets were similar or not to electricity given its special characteristics. Third, given the usefulness of wavelet analysis we wanted to examine the lead-lag relationship between spot and futures electricity markets, and extend previous analysis to more than just two time scales (short and long-run). Finally, we want to check if the estimation method has a significant effect on risk reduction and hedging effectiveness.

### **3. THE HEDGE RATIO AND ITS EFFECTIVENESS**

In this section we present the adopted hedge ratio and hedging effectiveness measure. To derive the minimum variance (MV) ratio, suppose an individual has taken a fixed position in some asset and that this person is long one unit of the asset. Let  $h_t$  represent the short position taken in the futures market at time  $t$  under the adopted hedging strategy. Ignoring daily resettlement, the hedger's objective within this framework is to minimize the variance of the change in the value of the portfolio:

$$\text{MinVar}(\Delta h_t) = \text{Var}(\Delta S_t - h_t \Delta F_t) = \text{Var}(\Delta S_t) + h_t^2 \text{Var}(\Delta F_t) + 2h_t \text{Cov}(\Delta S_t, \Delta F_t) \quad (1)$$

where  $\Delta h_t$  is the change in the value of the hedge portfolio during time  $t$ ,  $\Delta S_t = S_{t+1} - S_t$  and  $\Delta F_t = F_{t+1} - F_t$  the changes in the log of the spot and futures prices at time  $t$ , respectively, and  $h_t$  is the optimal hedge ratio.

Suppose the hedger decides to pursue a hedging strategy. The optimal hedge is then determined by differentiating expression (1) with respect to  $h_t$ ,

$$h_t^* = -\frac{\text{Cov}(\Delta S_t, \Delta F_t)}{\text{Var}(\Delta F_t)} \quad (2)$$

This corresponds to the conventional hedge ratio, when changes in both spot and futures prices are homoskedastic. The use of  $h_t$  assumes that the covariance and variance of futures returns remain constant over time. We consider only the Minimum Variance (MV) hedge ratio given that most of the existing studies analyze the MV hedge ratio, which will also allow us for comparison.

Finally, it can also be shown that, under some normality and martingale conditions, most of the hedge ratios based on other criteria (expected utility, extended mean-Gini coefficient, and generalized semi-variance) converge to the MV hedge ratio (Chen, Lee and Shrestha, 2001).

We consider the degree of hedging effectiveness, proposed by Ederington (1979), measured by the percentage reduction in the variance of spot price changes. Therefore, the degree of hedging effectiveness, denoted as EH, can be expressed as

$$EH = \frac{Var(\Delta S_t) - Var(\Delta h_t)}{Var(\Delta S_t)} = \rho_{sf,t}^2 \quad (3)$$

where  $\rho_{sf,t}^2$  is the square of the correlation between the change in spot and futures prices.

## 4. RESEARCH METHODS

Simple and widely used approaches to compute the hedge ratio are the naïve one-to-one and ordinary least squares (OLS). The naïve hedge ratio consists of taking an equal and opposite position in futures relative to the position in the spot, whereas the OLS-based hedge ratio is obtained from a linear regression model. Both routes assume that the hedge ratio will remain constant through time. However, for the past 20 years, the finance literature has reported the existence of time-varying (conditional) volatility. Hence, recent studies have resorted to GARCH-type (Gagnon, Lypny and McCurdy, 1998; McMillan, 2005) and stochastic volatility models (Lien and Wilson, 2001) to characterize the behavior of hedge ratios over time. A novel approach, also aimed at capturing the time-varying nature of a hedge ratio, is wavelet analysis (In and Kim, 2006a, 2006b; Lien and Shrestha, 2007). It may be that price volatility at different time scales will be key to pricing derivative instruments linked to commodity prices (Dewally and Marriott, 2008).

### 4.1. Wavelet analysis

Wavelets are relatively new signal processing techniques (Percival and Mofjeld, 1997; Percival and Walden, 2000; and Gençay, Selçuk and Witcher, 2002) in economics and finance, taking their roots from filtering methods and Fourier analysis. But wavelets combine information from the time-domain and frequency-domain, being very flexible and do not make strong assumptions

concerning the data generating process for the series under investigation. In wavelet analysis, we settle on the Maximal Overlap Discrete Wavelet Transform (MODWT) based on the Daubechies least asymmetric wavelet filter of length 8 (LA(8)), decomposing our data up to level 7<sup>3</sup>. Daubechies family of wavelets has compact support, which is an important characteristic allowing wavelets to more parsimoniously describe functions with cusps and spikes (Lien and Shrestha, 2007).

For a given integer J (which represents the level of resolution), the basis functions  $\psi_{j,k}$  are obtained through scaling and translation of wavelet  $\psi(t)$  (known as the mother wavelet) as follows

$$\psi_{j,k}(t) = 2^{-j/2} \psi\left(\frac{t-2^j k}{2^j}\right); \quad (4)$$

$$\int \psi(t) dt = 0, j = 1, 2, \dots, J$$

where  $\lambda_j = 2^{j-1}$  represents scaling, k represents translation (or shift), and t represents time. Therefore, basis functions are double sequences of functions. This allows us to visualize the process in a way which is not possible using other transform techniques.

There are two types of discrete wavelet transforms: The discrete wavelets transform (DWT), which uses orthonormal transformation of the original series; and the MODWT. Unlike DWT, MODWT involves a highly redundant non-orthogonal transformation and leads to J transform coefficient vectors each of length N, where N is not required to be an integer multiple of 2. The maximum J allowed should be the largest integer less than  $\log_2(N)$ , where N is the sample size.

Given the mentioned method, we can decompose the spot return series ( $\Delta S_t$ ) and futures return series ( $\Delta F_t$ ) into different time scales as follows:

$$\Delta S_t = B_{j,t}^S + D_{j,t}^S + D_{j-1,t}^S + \dots + D_{1,t}^S \quad (5)$$

$$\Delta F_t = B_{j,t}^F + D_{j,t}^F + D_{j-1,t}^F + \dots + D_{1,t}^F$$

These equations represent a decomposition of a series x (t) into different series each associated with a different time scale, while here  $x(t) = \{S_t; F_t\}$ . This process of decomposition is referred to a multi-resolution analysis (MRA). By applying OLS we can estimate J regressions using the J decompositions:  $D_{j,t}^S = \theta_{j,0} + \theta_{j,1} D_{j,t}^F + \varepsilon_j$ . Then, the minimum variance hedge ratio associated with the j<sup>th</sup> time scale is given by the estimate of  $\theta_{j,1}$ . In the present work we use weekly data, therefore, the time scale  $\lambda_1$  represents 2 weeks' interval,  $\lambda_2$  represents 4 weeks' interval, and so on.

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3 To capture the annual seasonality behavior of electricity spot prices, a length of 64 weeks is enough.

With the MODWT wavelet transformation, if a series of length  $T=64$  (more than one year in weeks) is considered, there would be 64 wavelet coefficients at each scale. Retaining all possible times at each scale in MODWT decomposition has the advantage of retaining the time invariant property of the original series.

High frequency components describe the short-term dynamics while low-frequency components represent the long-term behavior of the series. Lowest scales will mimic the short-term fluctuations of the original time series. The upper scales of the data will be associated with the trend components of the spot and futures prices, and, therefore, such scales will be relevant to investors with longer term horizons. By contrast, the lower scales will be the focus of interest of investors with short-term horizons.

The MODWT can also be used to define an analog of the variance of a time series. The MODWT can be used to compute unbiased estimators of the variance, covariance and correlation parameters. For a sample of size  $N$  and a wavelet filter of length  $L$ , the wavelet variance  $\sigma_x^2(\lambda_j)$ ,

$\sigma_x^2(\lambda_j) = \frac{1}{2\lambda_j} \text{Var}(d_{j,t})$  can be estimated by  $\tilde{\sigma}_x^2(\lambda_j)$  where  $\tilde{\sigma}_x^2(\lambda_j) = \frac{1}{N} \sum_{t=L_j-1}^{N-1} \tilde{d}_{j,t}^2$  and  $d_{j,t}$  are the  $j^{\text{th}}$  level

MODWT coefficients. As might be expected, an estimator for the wavelet covariance,

$\gamma_x(\lambda_j) = \frac{1}{2\lambda_j} \text{Cov}(d_{1,j,t}, d_{2,j,t})$ , of  $X_t = (x_{1,t}, x_{2,t}) = (\Delta S_t, \Delta F_t) = (s_t, f_t)$  for scale  $\lambda_j$  is  $\tilde{\gamma}_x(\lambda_j) = \frac{1}{N} \sum_{t=L_j-1}^{N-1} \tilde{d}_{1,j,t} \tilde{d}_{2,j,t}$ ,

where  $d_{1,j,t}$  and  $d_{2,j,t}$  are the  $j^{\text{th}}$  level MODWT coefficients corresponding to  $x_{1,t}$  and  $x_{2,t}$  series, respectively. The above estimates are invariant with respect to circular shifts of the time-series.

The estimates for the wavelet variance and covariance can be used to estimate the wavelet

correlation. In particular,  $\tilde{\rho}_x(\lambda_j) = \frac{\tilde{\gamma}_x(\lambda_j)}{\tilde{\sigma}_1(\lambda_j) \tilde{\sigma}_2(\lambda_j)}$  is an unbiased estimator of the wavelet

correlation. As with the usual correlation coefficient between two random variables,  $|\tilde{\rho}_x(\lambda_j)| < 1$ .

The wavelet correlation is analogous to its Fourier equivalent, the complex coherency (Gençay, Selçuk and Whitcher, 2002).

Finally, given the wavelet variance and covariance between two series, the minimum variance hedge ratio at scale  $\lambda_j$  can be calculated using

$$h_j^d = -\frac{\tilde{\gamma}_x(\lambda_j)}{\tilde{\sigma}_2^2(\lambda_j)} \tag{6}$$

In this specification,  $h_j^d$  indicates the wavelet multiscale hedge ratio, which can be varying

depending on the wavelet scales (or investment horizons).

#### 4.2 .Continuous time wavelets

There are two classes of wavelet transforms; the continuous wavelets transform (CWT) and its discrete counterpart (DWT). The DWT is a compact representation of the data and is particularly useful for noise reduction and data compression whereas the CWT is better for feature extraction purposes. To further analyze the relationship between electricity prices the continuous wavelet transform is also used. In this part of the work we also decompose the data series up to level 8.

The continuous wavelet transform, with respect to the wavelet  $\psi$ , is a function  $W_x(s, \tau)$  defined as:

$$W_x(s, \tau) = \int_{-\infty}^{+\infty} x(t) \frac{1}{\sqrt{s}} \psi^* \left( \frac{t - \tau}{s} \right) dt \quad (7)$$

where  $*$  denotes the complex conjugate form. Wavelet coefficients are given by this transformation. The mother wavelet  $\psi(\cdot)$  serves as a prototype for generating other window functions. The term translation,  $\tau$ , refers to the location of the window (indicates where the wavelet is centered). As the window shifts through the signal, the time information in the transform domain is obtained. The term scaling,  $s$ , refers to dilating (if  $|s| > 1$ ) or compressing (if  $|s| < 1$ ) the wavelet (controls the length of the wavelet).

The Morlet wavelet allows good identification and isolation of periodic signals, as it provides a balance between localization of time and frequency (Grinstead, Moore and Jevrejeva, 2004). This is a complex wavelet, as it yields a complex transform, with information on both amplitude and phase, essential for studying synchronisms between different time series. The Morlet wavelet in its simplified version is defined as:

$$\psi_\eta(t) = \pi^{-1/4} e^{i\eta t} e^{-t^2/2} \quad (8)$$

The wavelet transform performs what is called time-frequency analysis of signals. In other words, it can estimate the spectral characteristics of signals as a function in time. Dealing with discrete time series  $\{x_n, n = 0, \dots, N-1\}$  of  $N$  observations with a uniform time step  $\delta t$ , the integral in (7) has to be discretized, and the CWT of the time series  $\{x_n\}$  becomes

$$W_m^x(s) = \frac{\delta t}{\sqrt{s}} \sum_{n=0}^{N-1} x_n \psi^* \left( (n-m) \frac{\delta t}{s} \right), m = 0, 1, \dots, N-1 \quad (9)$$

As evidenced by Torrence and Compo (1998), Percival and Walden (2000) and Conraria, Azevedo and Soares (2008), when applying the CWT to a finite length time series we inevitably suffer from border distortions. This is due to the fact that the values of the transform at the beginning and at the end of the series are always incorrectly computed, involving missing values of the series which are then artificially prescribed. The region in which the transform suffers from these edge effects is called the cone of influence, and as such results in this area must be interpreted carefully.

The wavelet power spectrum can be interpreted as depicting the local variance of a time series and the cross-wavelet power of two times series depicts the local covariance between these series at each scale or frequency. For more general data generating processes one has to rely on Monte Carlo simulations.

The phase for wavelets shows any lag or lead relationships between components, and is defined as

$$\psi_{x,y} = \tan^{-1} \frac{I\{W_n^{xy}\}}{\Re\{W_n^{xy}\}}, \psi_{x,y} \in [-\pi, \pi] \quad (10)$$

where  $I$  and  $\Re$  are the imaginary and real parts, respectively, of the smooth power spectrum. Phase differences are, therefore, useful to characterize phase relationships between two time series. For a complete interpretation of the difference of phase between the analyzed series we suggest the reading of Barbosa and Blitzkow (2008, pp. 28-29) who interpret the meaning of the phase angles, and Conraria, Azevedo and Soares (2008). The vectors in the cross-wavelet coherency indicate the phase difference between the two series. However, we need to know which of the time series is processed first for the scheme to be valid. In the present work, pictures show the cross-coherency between two series. The name of the series presented first is our first series, the other one being the second we consider.

The concept of coherence is fundamental and quite important in all the fields dealing with fluctuating quantities. Given that correlation is defined as the relation of two or more time series, we could say that those series that are highly correlated are coherent. The degree of coherence is a measure of how closely  $X$  and  $Y$  are related by a linear transformation. Thus,  $X$  and  $Y$  are closely related by a linear transformation if and only if their degree of coherence is close to its maximum value of unity. The two random variables  $X$  and  $Y$  are said to be completely coherent if and only if  $|\rho|=1$  and completely incoherent if and only if  $|\rho|=0$ , where  $\rho$  is the correlation coefficient. The caveat is that this correlation may not be contemporaneous, but may involve a lead or a lag. A

measure of the magnitude of this lead or lag is the phase lead.

In time-series, the degree of coherence of two time series  $x(t)$  and  $y(t)$  with zero time-average values is the magnitude of their temporal correlation coefficient. Coherence provides information about the stability of the true relationship between the two signals with respect to power asymmetry and phase relationship and not direct information about this relationship. Correlation, on the other hand, may be calculated over a single epoch or several epochs and affected by phase, independently of amplitudes.

Following Torrence and Compo (1998) we define the wavelet coherency of two time series as

$$R_n^2(s) = \frac{|S(s^{-1}W_n^{xy}(s))|^2}{S(s^{-1}|W_n^x(s)|^2) \cdot S(s^{-1}|W_n^y(s)|^2)} \quad (11)$$

where  $S$  is a smoothing operator in both time and scale.

The cross-wavelet coherence gives an indication of the correlation between rotary components that are rotating in the same direction as a function of time and periodicity. Coherences near one show a high similarity between the time series, while coherences near zero show no relationship.

### ***4.3. Multivariate GARCH model***

When static hedging strategies are used (like OLS and naïve strategies), each moment the agent faces decision whether to hedge with the current future price estimate or wait for new information. When we refer to static hedging we mean that once the hedge is created it is not changed after that. So, static hedging means that the hedge ratio  $h$  remains constant over time. The static hedging strategy determines the equilibrium point or neutral point of the dynamic hedging strategy, but when the position taken in derivatives changes over time, the hedging strategy is dynamic. MGARCH models are used to capture the time-varying dynamic behavior between both series. It is developed to examine the joint processes relating spot and futures returns.

We will assume that the market is incomplete; therefore not all the risks are hedgeable through trading the underlying spot. If the market were complete, given sufficient initial capital, all claims could be replicated by trading the spot dynamically. Static derivatives hedges do not add anything to dynamic hedges in complete markets, but of course they are very valuable tools in

realistic incomplete market models, where there may be risk factors that cannot be eliminated just by dynamic trading of the underlying spot. By incorporating static hedges, we enlarge the set of feasible hedging strategies that the investor can choose from and allow for a better hedging performance.

In terms of dynamic hedging strategies, Bollerslev, Engle and Wooldridge (1988) generalized the univariate GARCH to a multivariate dimension to simultaneously model the conditional variance and covariance of two interacted series. This multivariate GARCH model is thus applied to the calculation of dynamic hedge ratios that vary over time based on the conditional variance and covariance of the spot and futures prices. Engle and Kroner (1995) present various MGARCH models with variations to the conditional variance-covariance matrix of equations. Generalized from GARCH(1,1), a standard M-GARCH(1,1) model is expressed as:

$$\begin{bmatrix} h_{ss,t} \\ h_{sf,t} \\ h_{ff,t} \end{bmatrix} = \begin{bmatrix} c_{ss,t} \\ c_{sf,t} \\ c_{ff,t} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} \varepsilon_{s,t-1}^2 \\ \varepsilon_{s,t-1}\varepsilon_{f,t-1} \\ \varepsilon_{f,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \times \begin{bmatrix} h_{ss,t-1} \\ h_{sf,t-1} \\ h_{ff,t-1} \end{bmatrix} \quad (12)$$

where  $h_{ss}$ ,  $h_{ff}$  are the conditional variance of the errors ( $\varepsilon_{s,t}$ ,  $\varepsilon_{f,t}$ ) from the mean equations, where:

$$\begin{aligned} \varepsilon_t | \phi_{t-1} &\sim BN(0, H_t) \\ \varepsilon_t &= \begin{bmatrix} \varepsilon_{s,t} \\ \varepsilon_{f,t} \end{bmatrix}; H_t = \begin{bmatrix} h_{ss,t} & h_{sf,t} \\ h_{fs,t} & h_{ff,t} \end{bmatrix} \end{aligned} \quad (13)$$

Karolyi (1995) suggests that the BEKK (Baba, Engle, Kraft and Kroner) model allows the conditional variance and covariance of the spot and futures prices to influence each other, and, at the same time, does not require the estimation of a large number of parameters to be employed. The model also ensures the condition of a positive semi-definite conditional variance-covariance matrix in the optimization process which is a necessary condition for the estimated variance to be zero or positive. The BEKK parameterization for the MGARCH(1,1) model is written as:

$$\begin{bmatrix} h_{ss,t}^2 & h_{sf,t}^2 \\ h_{fs,t}^2 & h_{ff,t}^2 \end{bmatrix} = \begin{bmatrix} b_{ss} & b_{sf} \\ 0 & b_{ff} \end{bmatrix}^T \begin{bmatrix} b_{ss} & b_{sf} \\ 0 & b_{ff} \end{bmatrix} + \begin{bmatrix} c_{ss} & c_{sf} \\ c_{fs} & c_{ff} \end{bmatrix}^T \begin{bmatrix} \varepsilon_{s,t-1}^2 & \varepsilon_{s,t-1}\varepsilon_{f,t-1} \\ \varepsilon_{f,t-1}\varepsilon_{s,t-1} & \varepsilon_{f,t-1}^2 \end{bmatrix} \begin{bmatrix} c_{ss} & c_{sf} \\ c_{fs} & c_{ff} \end{bmatrix} + \begin{bmatrix} g_{ss} & g_{sf} \\ g_{fs} & g_{ff} \end{bmatrix}^T \begin{bmatrix} h_{ss,t-1}^2 & h_{sf,t-1}^2 \\ h_{fs,t-1}^2 & h_{ff,t-1}^2 \end{bmatrix} \begin{bmatrix} g_{ss} & g_{sf} \\ g_{fs} & g_{ff} \end{bmatrix} \quad (14)$$

where  $h_{ss,t}^2$ ,  $h_{ff,t}^2$  and  $h_{sf,t}^2$  are the conditional variance and covariance of the errors ( $\varepsilon_{s,t}$ ,  $\varepsilon_{f,t}$ ) from mean equations. Conditional variance and covariance only depend on their own lagged squared residuals and lagged values. The MGARCH model incorporates a time-varying conditional covariance and variance between the spot and futures prices and hence generates more realistic time-varying hedge ratios.

As an alternative empirical distribution to the normal one we will also use the bivariate t-student distribution in the multivariate-GARCH BEKK model used here  $\varepsilon_t | \phi_{t-1} \sim t(0, H_t, \nu)$  where  $\nu$  is the degrees of freedom parameter of a conditional bivariate t-student distribution.

Bivariate GARCH modeling allows to model not only the conditional second moments, but also the cross moments, with special relevance, in our case, to the contemporaneous covariance between electricity spot and futures. That's why the conditional, on time t-1 available information, error term vector follows a bivariate normal law, and for comparison purposes also a bivariate t distribution, being  $H_t$  the positive definite variance covariance matrix dependent on time,  $\varepsilon_t | \phi_{t-1} \sim t(0, H_t, \nu)$ , where  $\nu$  is the degrees of freedom parameter of a conditional bivariate t-student distribution.

## **5. DATA AND EMPIRICAL RESULTS**

We use German electricity spot and futures prices from the European Electricity Exchange (EEX) in Leipzig. Our data is composed of daily closing prices for each series obtained directly from EEX.

In the spot market, hourly power contracts are traded daily for physical delivery in the next 24 hour period, being this price known as the system price. EEX also trades electricity futures and options. At the moment, the most important are: monthly, quarterly and yearly futures all based on peak and base data, traded by both hedgers and speculators. The settlement of futures can take place either in cash or physical according to their contract specifications<sup>4</sup>. From a liquidity perspective only the cash settled futures can be considered liquid. That is why we only take these into consideration. The present study focuses on monthly futures, taking one price per day. We use base<sup>5</sup> data for futures contracts.

We transform the available data into weekly German returns. We do this for two reasons. First the data is more stable than daily data. Second, various studies suggest that weekly hedges are

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4 Physical settlement occurs in the German Base Load Future, the German Peak Load Future, the French Base Load Future and the French Peak Load Future. Cash settled futures are the Phelix Base Future and the Phelix Peak Future.

5 "Base data" is the average daily price for the 24 hours in the day. As such, a base contract ensures delivery around the clock and a peak contract delivery between 8 am and 8 pm (see EEX website for more details).

more efficient than daily hedges. Butterworth and Holmes (2005) found that increasing hedging period length from daily to weekly and monthly increased the hedging effectiveness. Avci and Çinko (2010) findings for the Turkish index indicate that future weekly hedge periods are more effective than daily hedges in terms of risk reduction criteria. Malliaris and Urrutia (1991) found that hedging horizon and data frequency are important in hedging effectiveness. While Benet (1992) states that short hedging periods were more effective, Ripple and Moosa (2007) found that the use of the most recent contract was more effective than the use of more distant contracts.

The futures market offers derivative products that do not comprise physical settlement of electricity during the delivery period. But these are primarily used as hedging instruments against market uncertainties. The underlying of these futures contracts is the so called Phelix Index (Physical Electricity Index) that is calculated from spot market prices on a daily basis. Depending on the corresponding products on the spot market, the index distinguishes between base load and peak load. The Phelix base day price is an equally weighted average of all 24 hourly spot prices for that particular day. The Phelix base month price is the mean of all Phelix base day prices of that month<sup>6</sup>. These arithmetic averages over a specific period are the reference prices in all cash settlement calculations at expiration of derivative contracts. The delivery period specifies the Phelix Index that serves as underlying. For example, in the case of Phelix base month futures with maturity in December 2005, the reference price at maturity is the value of the Phelix base month index in December 2005.

The EEX was formed by the merger between LPX Leipzig Power Exchange and the Frankfurt-based European Energy Exchange in 2002 and is the most important energy exchange in continental Europe today (see EEX), with more than 200 trading participants from 20 countries. Its generation technology mix in 2006 consisted of 4% hydropower, 27% nuclear, 61% conventional thermal, and 8% of other sources. EEX has a leading position in terms of the intensity of growth and the speed of expansion.

The data period analyzed is from 18 June 2004 until 15 July 2008. For the German electricity markets 6 monthly futures contracts can be traded daily. However, similar to Ripple and Moosa (2005) we use only the near-month and the six-month contract in the EEX market, allowing

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<sup>6</sup> There is a base and peak version for every future. Thus a month future ensures for example the delivery of electricity with a constant around the clock delivery rate of 1 MW on any delivery day of a calendar month (base) or an all delivery days from Monday until Friday from 8 am to 8 pm (peak). The Phelix Base and Phelix Peak Index are the underlying for the cash settled base and peak future, respectively.

us to construct 2 different futures data series. For the spot, only one price per week (the base price for that day) was considered. In the empirical application futures with different maturities (1 and 6 months base data, B1 and B6, respectively) are considered to hedge the spot price variation and a unique hedging length is considered: one week. Similar to us, Zanotti, Gabbi and Geranio (2010) use as hedging instruments futures contract with 1 month to expiration, being the shorter term instruments available. However, short term instruments are generally more liquid than long term and more correlated with underlying spot. To create a time series of futures prices and avoid delivery or thin markets effect we roll over to the next 1 (6) month futures contract 1 week before the expiration of the previous future contract.

Liquidity is important when analyzing data on electricity markets since electricity exchanges are wholesale markets and the number of market participants is limited. Two measures of liquidity can be used for future markets: the open interest and the traded volume. As discussed and analyzed in Pietz (2009), open interest averaged 28 TWh in 2002 and 356 TWh in 2008, an astonishing increase of open interest of almost 1300 percent in six years and speaks for a liquid and well developing market. As for the number of traded contracts a typical pattern may be observed. Trading mainly takes place in futures with short time-to-delivery. As such, the maximum in traded contracts is reached in the days just before the start of the delivery period and decreases thereafter.

Table 1: Summary statistics for spot and futures return

<b>Variable</b>	<b>Mean</b>	<b>St. dev.</b>	<b>Variance</b>	<b>Skewness</b>	<b>Kurtosis</b>
Spot	0.089	21.06	4.435	74.31	2044.36
FB1	0.072	2.94	0.086	444.11	16098.50
FB6	0.088	3.28	0.108	140.66	1975.43

FB1 and FB2 stand for Future Base data for 1 and 6 months, respectively, while St. dev. stands for standard deviation. Values presented are in percentage terms.

Both future returns and spot returns have means very close to zero, and we may say that the unconditional distribution of spot returns and futures are non-normal as evidenced by skewness and high excess kurtosis. Table 1 evidences the existent differences between electricity spot and futures returns in the German market, and is mostly visible by the highly different volatility values, as measured by standard deviation.

Variances are 0,086% and 0,108% for the futures series, base 1 month and base 6 months, respectively, and 4,435% for the spot, showing that spot prices have higher volatility than futures. Volatility is not even constant over time<sup>7</sup> leading to the idea of using time varying models in hedge ratio choices. Skewness signs are all positive. The high excess kurtosis values suggest that we are in the presence of leptokurtic distributions, which means we have heteroskedasticity present in the data.

Notice that if the arbitrage condition holds, the total variances of spot and futures returns should be close to each other. However, Lien and Shrestha (2007) stress that for financial assets, total futures return variances are higher than the total spot return variances. For commodities like Hogs and Cotton, the total futures return variances are significantly less than the total spot return variances. The same happens in electricity markets. They argue that the extent to which an arbitrage can be implemented depends upon, among other factors, the liquidity<sup>8</sup> of the spot and futures markets. *“In the case of commodities, the futures markets are more liquid than the spot markets. Consequently, variances of futures returns are much smaller than those of spot returns for commodities”* (Lien and Shrestha, 2007). However, for electricity the difference is huge suggesting a high lack of liquidity in spot markets. In fact, total volume traded in the spot power market was 203 TWh in 2009 against a total volume traded in the power derivatives market of 1025 TWh, which may explain these findings. Still, there is a lack of explanation for the causes behind the differences in total variances and liquidity for electricity markets. A more detailed analysis between both spot and futures series will demand for a new (and deeper) thorough empirical study.

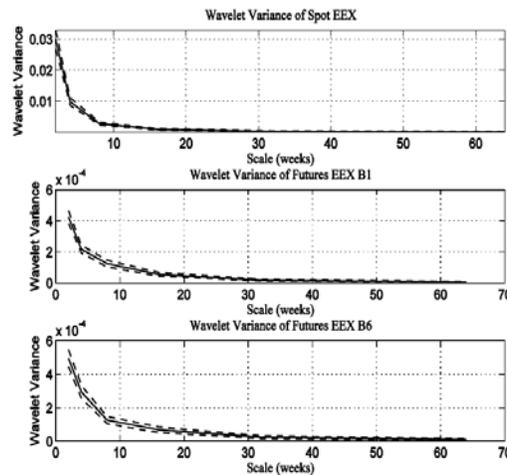
Turning one step further into our empirical investigation, original data has been transformed by the wavelet filter (LA(8)) up to time scale 7 to study correlation in the various time scales.

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<sup>7</sup> Plots for the changing variance over time are not presented here in order to save space but will be provided upon request.

<sup>8</sup> Liquidity is an important feature of a well functioning market. We can define it as the ability to quickly buy or sell a desired commodity or financial instrument without causing a significant change in its price and without incurring significant transaction costs. A key feature of a liquid market is that it has a large number of buyers and sellers willing to transact at all times. This is not at all the case of electricity markets where we have a low number of generators and electricity special characteristics like the continuous balance between demand and supply.

Figure 1: Estimated wavelet variance of spot and futures returns



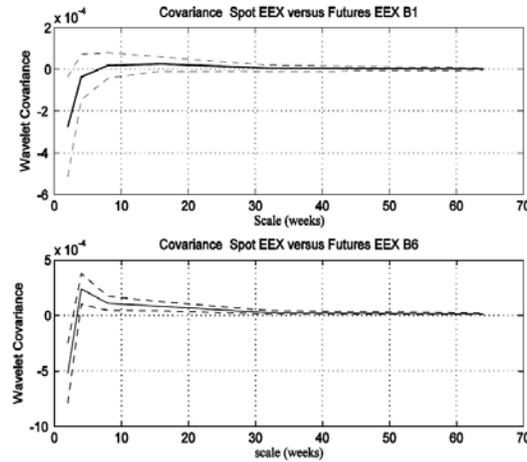
Wavelet variances for the spot and futures returns are plotted on the different time scales on the x-axis. The dotted lines represent the approximate 95% confidence interval. Since the confidence interval does not use any information regarding the distribution of the wavelet variance, these are robust to non-Gaussianity.

Wavelet variance analysis enables us to identify which scales are the most important contributors to the overall variability of the data (Percival and Walden, 2000). Variances of both spot and futures markets decrease as the wavelet scale increases, which is in accordance with previous literature using wavelets applied to financial assets and commodities. The variance versus wavelet scale curves show a broad peak at the lowest scale (d1) in both markets. More specifically, a wavelet variance in a particular time scale indicates the contribution to sample variance. It also shows that the spot market is more volatile than the futures market regardless of the time scale. This is in opposition to the results of In and Kim (2006a) who find that the futures markets is more volatile than the stock market for the S&P500 index.

Overall, the movements of covariance increase as the time scale increases (at low scales), becoming stable around 0 at high scales (lower frequencies) as can be seen in figure 2. The correlation between futures and spot returns shows an erratic behavior, in particular when more observations (at the lower scale) are available. Since at low scales (high frequencies) there are more observations for calculating the wavelet variance, covariance and correlation, there should be more wavelet variation at the lower scales and therefore we should expect less measured correlation between futures returns and stock returns (In and Kim, 2006a). However, the plot shows that the covariance is not very high; being almost always close to 0, which means that the

hedge may turn out to be bad, as it will be showed in the hedging computations (table 2).

Figure 2: Estimated wavelet covariance between the spot and futures returns



Wavelet covariance between the spot and futures returns is plotted on the different time scales on the x-axis with approximate 95% confidence interval (dotted lines).

At scales d1-d3, wavelet covariance increases rapidly, and at scales d4-d6, wavelet covariance between spot and futures returns gradually converges to zero. The covariance plot also allows us to indicate that the spot and futures in electricity markets are found to be fundamentally different.

After the previous empirical analysis, concerning the variance and covariance of spot and futures electricity prices resorting to the MODWT techniques, we start presenting and discussing the results attained by applying the continuous Morlet wavelet. The continuous wavelet power spectrum for the spot base, and futures base for 1 and 6 months was computed.

Looking at the time scale decomposition of these variables some interesting facts are revealed. Most of the action in the series occurred at high scales (low frequencies). There are no clear and general structural changes occurring for all the series at once in the years under analysis, since the red power is spread through all of them. However, some interesting aspects deserve to be mentioned.

For the 6 month futures contracts we see a significant power event at the beginning of 2007 for scales d6 and d7. These power events are associated with periods of high volatility, and given the period in which they occurred we can attribute them to the sub-prime crisis and all the instability caused in energy markets. The wavelet power spectrum for the spot shows that both

series have higher variance at high scales.

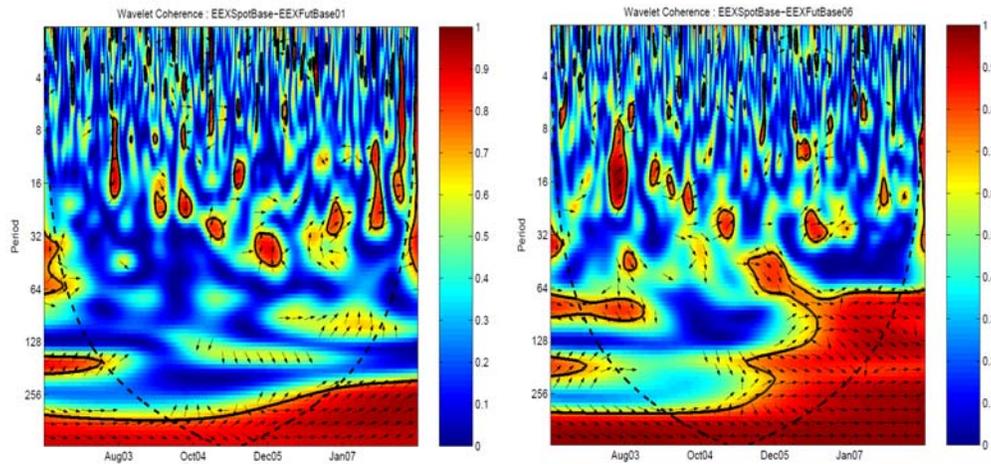
It is clear that the different time series have different characteristics in the time-frequency domain, but volatility for all of them is quite high at low frequencies, and low at higher frequencies (mostly at periodicity until half a year). In the period of 2006-2009, probably as a consequence of the major financial crisis, the variance of futures contracts became higher, where the effect is clearer at medium and high scales, suggesting we were facing medium and long term shocks in futures markets.

Wavelet coherence depicted in figure 3 confirms the findings of the fundamental difference between electricity spot and futures returns, at least for high frequencies. This implies that in the short run both series have weak comovements. Therefore, to perform the cross-wavelet analysis we will focus on the cross-wavelet coherency.

In figure 3 we can observe the estimated wavelet coherency and phase difference between the series. The values for the significance were obtained from Monte Carlo simulations. Contours denote wavelet-squared coherency, where the thick black contour is the 5% significance level and outside the thin line is the boundary affected zone. Therefore, the cone of influence indicates the region affected by edge effects and results outside this show no statistical significance. Color code for power ranges from blue (low coherency, near zero) to red (high coherency, near one).

Looking at the pictures presented in figure 3, information on the phases shows us that the relationship among spot and futures markets was not homogeneous across scales, since arrows point right and left, down and up constantly. Moreover, the cross-wavelet coherency is high at low frequencies, but in the highest scale of all, most of the coherence results are not statistically significant since they rely below the cone of influence. The wavelet cross-coherency shows low to medium statistically significant coherence, however we are still able to observe some islands of medium power.

Figure 3: Cross-wavelet coherency and phase plots between spot base and futures base



EEXFutBase01 and EEXFutBase06 represent futures contracts for 1 and 6 months in the EEX market, respectively.

Series are correlated and in phase for lower frequencies (high scales), being higher for 6 month futures. Given that the wavelet coherence is used to identify both frequency bands and time intervals within which pairs of indices are co-varying, on the daily time scales of 4-64 days band, the 5% significance regions indicate that spot and futures contracts under analysis do not show long periods of high coherence. Still, in the majority of the cases there are long periods of higher coherence among the two series on the daily time scales d7 and d8.

Still, if long run adjustments were taken out, the short-run movements would be little correlated. Moreover, these cycles of short duration (where both series show strong statistical significant correlation and coherence among them – the red areas defined by the black contours) can possibly be related to special episodes that occurred in the history of electricity spot and futures prices in the German EEX market<sup>9</sup>. Since the finest scales capture movements in 2 to 8 weeks (because the next finest scale has a fairly large portion of energy in spot price movements as evidenced by coherence wavelet plots), we may argue that only with a considerable time span (the lowest frequencies) spillovers are transmitted between markets.

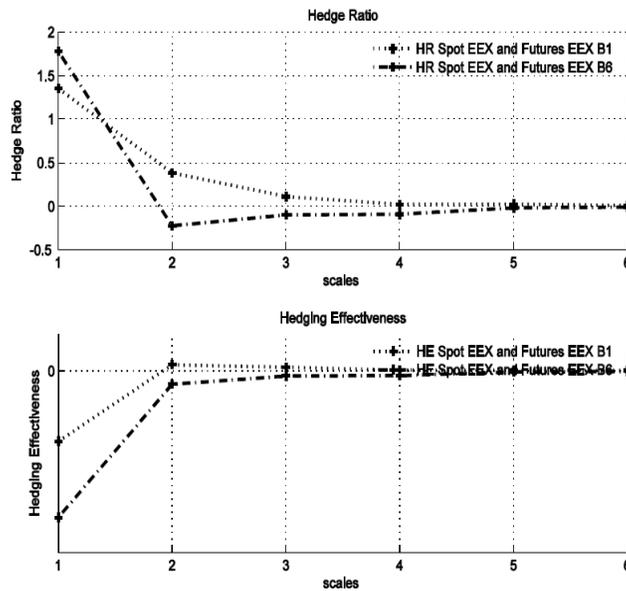
There is strong empirical evidence that time-varying cointegration relationships exist among the two markets. The time variation pattern documented in this study may carry some important implications for hedging. The instability in various aspects of market comovements may

<sup>9</sup> We leave this association for a future work.

imply serious limitations to the investor's ability to exploit potential benefits from hedging with futures contracts in electricity markets. Much variation in the contemporaneous relationships among spot and futures base prices may also highlight inadequacy in assuming (short-term) relationships in both markets, which might account for the difficulty in achieving profitable active trading.

Given previous results we continue the empirical analysis by presenting the hedge results provided by the MODWT. According to figure 4, the decomposed hedge ratio decreases monotonically converging towards zero, very different from the long-horizon hedge ratio of one. The degree of hedging effectiveness approaches zero because, over long horizons, the shared permanent component may be absent and the spot and futures series remain far apart in electricity markets. This also implies that the effect of the transitory components becomes strong. As such, in the long run, spot and futures prices are not perfectly correlated in these newly markets (contrary to In and Kim, 2006, 2006a; Fernandez, 2008, for different commodities and financial assets). Electricity markets present even a much different behavior than that reported for other commodities as presented by Lien and Shrestha (2007). The main cause is due to the huge difference reported between variance in the spot and future series, as well as the low covariance/correlation between both.

Figure 4: Hedge ratio and hedging effectiveness with different wavelet domains



As in the co-integration literature on hedge ratios, the presence of both long-run and short-run components in the stock and futures markets causes the hedge ratio and the degree of hedging effectiveness to depend on the time horizon. As the wavelet time scale increases, the decomposed data stays far away the long-run trend of one. This result contradicts the findings of In and Kim (2006a) which found a hedge ratio approaching 1 as the hedging period increases, for the S&P500 Index. Therefore, over long horizons, there is no shared long-run component tying the stock and futures series together, and the two prices will not be perfectly correlated in electricity markets (opposed to Geppert, 1995, results). Pén and Sévy (2007) argue in favor of a great inefficiency for forward electricity markets for short-term horizon because they found correlation between spot and forward returns to be too low on each considered market. As such, derived optimal hedge ratios are insignificant, which favors the results we have obtained. Moreover, we have obtained optimal hedge ratios which are insignificant for all time scales, applying wavelet techniques. Shawky, Marathe and Barrett (2003) apply EGARCH and VAR estimates for the relation between spot and futures in the NYMEX market. They found that the spot equation is more significant than the futures equation. They argue that this finding is consistent with an electricity market in which spot prices are significantly more impacted by current events than futures electricity prices.

Figure 5 shows the MODWT MRA of the spot (panel a) and future returns (Base 1 month in panel b and base 6 months in panel c) using various time scales (six different wavelet details, d1-d6). By these plots we may observe that as the time scale increases from the finer time scales (d1) to the highest time scale (d6), wavelet coefficients show a smoother movement. This means that short term noise vanishes as the scale increases. However, the difference between the futures base 1 month and the futures base 6 months is visible. The last futures maturity (6 months) reveals a higher volatile behavior at all the time scales considered.

Wavelets are still an important technique in the sense that the decomposition of the data into several time scales allows detecting the frequency burst in various time scales. Still, the gains from hedging are not great, in accordance to the findings of Maharaj et al. (2008). Being wavelets a reliable technique in terms of frequency-scale decomposition, but due to the results obtained in the empirical estimation of the hedging effectiveness, we can conclude that the problem does not rely on the technique used but on the market, and its special characteristics.

Figure 5: MODWT MRA of the spot returns (panel a), futures base 1 month futures (panel b) and futures base 6 month futures (panel c) (one week hedges for both) for the German EEX electricity market. These plots are obtained from an wavelet Daubechies filter up to six time scales.

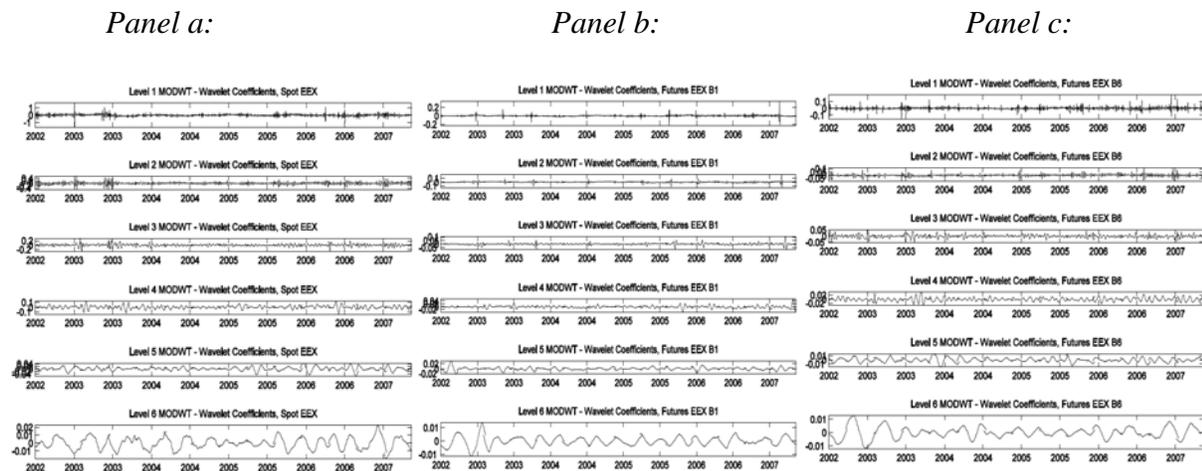


Figure 5 exposes that the first two to three high frequency components explain the higher part of the series energy, which allows us to conclude that movements in these series are mainly caused by short-term fluctuations. In fact, such a phenomenon is somewhat expected as spot and futures returns cannot be predicted in advance.

Once again, the highest volatility showed by spot returns is revealed compared to those of futures returns and this is even more noticed at different time scales. As such, the low correlation/covariance between spot and futures is even more evidenced. So, using wavelet decomposition is in fact an important technique to account for differences between return series, even for electricity markets.

In the following we estimated the hedge ratio in five different ways: the naïve one to one hedge ratio where we offset each spot contract by one futures contract; the static OLS hedge ratio where we regress the spot return over the futures return; two dynamic time varying multivariate models – Diagonal BEKK under the normality assumption and the T-Diagonal BEKK model –; finally, using wavelet decomposition to compute the hedge ratio and variance reduction.

Results for the comparisons of hedge effectiveness achieved are shown in table 2, using the calculated hedge ratios from the naïve, OLS, multivariate GARCH and wavelet strategies. Among the four different hedge strategies, the wavelet multiscale hedging effectiveness provides results which are negative or even null, regardless of the time scales, indicating that wavelet hedge strategies are not better than other strategies in terms of hedging effectiveness. The first visual inspection of table 2 leads us to conclude that taking into consideration time varying variations

seems to be important.

Table 2: Comparisons of hedge effectiveness among the four hedge strategies under analysis: OLS, naïve, multivariate GARCH and the wavelet timescale decomposition.

	Mean Return	Variance	Risk Reduction (%)
EEX Market - B1			
Spot (no hedging) (b=0)	0.00023	0.04457	-
<b>Hedging</b>			
Naive (b=1)	-0.00051	0.04598	-3.16%
OLS (b= $h_{fs}/h_f$ )	0.00046	0.04449	0.18%
Diagonal-BEKK (b <sub>t</sub> = $h_{fs,t}/h_{f,t}$ )	-0.00076	0.03950	11.38%
T-Diagonal-BEKK (b <sub>t</sub> = $h_{fs,t}/h_{f,t}$ )	-0.00121	0.03665	17.76%
Scale d1	0.00186	0.04543	-1.94%
Scale d2	0.00116	0.04449	0.17%
Scale d3	0.00097	0.04452	0.11%
Scale d4	0.00090	0.04456	0.02%
Scale d5	0.00090	0.04456	0.03%
Scale d6	0.00089	0.04456	0.01%
EEX Market - B6			
Spot (no hedging) (b=0)	0.00023	0.04457	-
<b>Hedging</b>			
Naive (b=1)	-0.00063	0.04575	-2.64%
OLS (b= $h_{fs}/h_f$ )	0.00031	0.04457	-0.01%
Diagonal-BEKK (b <sub>t</sub> = $h_{fs,t}/h_{f,t}$ )	-0.00151	0.03575	19.79%
T-Diagonal-BEKK (b <sub>t</sub> = $h_{fs,t}/h_{f,t}$ )	-0.00040	0.03981	10.67%
Scale d1	0.00216	0.04637	-4.03%
Scale d2	0.00072	0.04473	-0.37%
Scale d3	0.00081	0.04463	-0.14%
Scale d4	0.00082	0.04463	-0.13%
Scale d5	0.00087	0.04458	-0.02%
Scale d6	0.00088	0.04457	-0.01%

The table displays the risk reduction achieved by each of the hedging strategies. EEX B1 and EEX B6 stand for EEX futures base data for 1 and 6 months, respectively. Results are statistically significant.

The best hedging strategy could be defined as the one that allows the highest risk reduction and simultaneously the lowest return reduction. However, risk reduction is usually a much valid ranking criteria's, given that the average return of the portfolio depends on the underlying trend of the spots and futures returns.

Spot variance reduction is computed by comparison with the unhedged spot variance position, in the first row of each panel. Results obtained imply that the better statistical performance of the multivariate GARCH-BEKK model also implies a better hedging strategy performance. In the one week hedges, the naïve, OLS and wavelets strategies clearly obtain the worst score, favoring the multivariate GARCH models, which are those that in fact deliver the highest variance reduction, even if it is a small risk reduction.

Results provide a clear indication of the superior performance of time varying hedge ratio as compared with traditional hedge ratios and the more recent technique wavelets. Hedging is not independent from the model chosen to estimate the hedge ratio. Time varying variances models reduce the volatility of the hedge portfolio wherever naïve strategies and sometimes wavelet series decomposition led to an increase of risk. These are even more effective in terms of variance reduction using 1 month futures and the t-distribution.

Comparing to Byström (2003), Moulton (2005), Torró (2009) and Zanotti, Gabbi and Geranio (2010) we confirm the best variance reduction of the dynamic hedging method. In fact, although small in terms of variance reduction, results attained for the multivariate GARCH BEKK model both with the normality assumption and the t-student distribution are positive. Nonetheless, even using the dynamic hedging strategy, the highest risk reduction is attained with the t-distributional assumption for 1 month futures hedges (one month to maturity, one week before), being higher for the 6 month futures hedges (6 months to maturity, one week before) for the normality assumption.

As we are able to see, for the naïve strategy we have instead of variance reduction a variance increase (indicated by the negative sign), which contradicts the literature that defends unconditional hedges. The same happens for the lowest scales when resorting to wavelet analysis, and although variance reduction is confirmed for EEX base 1 month contracts in the 4 week scale

(d2) we see that hedging effectiveness diminishes at high scales (lower frequencies) for Base 1 month futures. The results for B1 strategies are bad in terms of variance reduction, but even worse were those obtained for the base 6 month futures contracts (instead of variance reduction we see variance increases), becoming negligible at high scales (64 weeks scale). Still, hedging is more effective for a 2 week period when the near month contract is used.

From the four hedging strategies used in this work the multivariate GARCH dynamic hedge shows the best results in terms of variance reduction (hedging effectiveness), given that there is a time-varying dynamic behavior between spot and futures series. Moreover, even with an analysis decomposing the series into several components, the hedge is inefficient. However, we believe that this contradictory result related to previous empirical findings using wavelets applied to financial assets and other commodities markets is not due to model specification errors. Maharaj et al. (2008) point out a possible reason for wavelets hedging inefficiency. It was attributed to their inability to match the hedge horizon to a wavelet detail series (it involves a trade-off between not having to reduce the sample, particularly for long-term hedges, and the inability of the procedure to match exactly the hedge horizon with an appropriate scale).

As evidenced by the summary statistics and wavelet variance/covariance analysis provided before we suppose that the problem does not rely on the technique being used, but on the fact that there is a weak association between spot and futures prices in the German electricity market. It is evident that these optimal hedge ratios are useful in minimizing variance but these hedges lose importance across the time horizons, as opposed to Dewally and Marriott's, (2008), results for base metal markets, where we should not forget the non-storability nature of electricity as opposed to base metal, and other commodities. Torró (2009) demonstrates that Nord Pool electricity prices are characterized by particular statistical features, particularly a low correlation between spot and futures prices, due to high volatility and kurtosis, and no storability property of the underlying which avoids the cash-and-carry relation. This means that hedging strategies could generate ineffective performances, unless more sophisticated models are applied.

The findings that static hedging strategies do not outperform dynamic ones, suggest that unconditional hedges do not outperform conditional hedges. So, there are gains including heteroskedasticity and time-varying variances in the calculation of hedge ratios, even if small ones. But multi-resolution analysis does not show improvement gains over more traditional methods in terms of hedging, at least for the German electricity market as pointed by the results attained in the empirical part. As such, a hedge may be unfavorable as the hedging horizon is

lengthened given the nature of price discovery in the spot and futures electricity relationship.

Ederington's (1979) efficient hedge ratio has been empirically found to be negatively associated with the hedge horizon, as was the basis risk. Given the present empirical findings basis risk<sup>10</sup> is found not to be negatively associated with time-to-expiry of future contracts. As such, on the expiration date, hedgers in electricity markets will not be left with price risk only and conventional hedging theory may not work efficiently. Moreover, these results point out that in electricity markets we are left with a continuous time varying basis risk which does not allow the hedge ratio to be constant over the hedging horizon.

Although hedging in electricity markets is very important given the erratic behavior of electricity spot prices, special features describing this type of markets imply low correlation between spot and futures prices, and condition effectiveness of the hedging strategy. Given the results presented, the main challenge for now will be to understand what differentiates futures and spot prices volatility behavior in electricity markets. This huge disparity in terms of volatility is causing the bad hedging effectiveness reported here, and studying these causes is the subject of an ongoing research.

However, for the period and market analyzed it has been possible to reduce the variance of electricity markets using futures hedging. As such, trading in futures allows to reduce the risk of electricity portfolios and to control the risk of adverse movements on electricity prices.

## **6. CONCLUSIONS**

We explore hedging effectiveness of electricity futures in the German (EEX) electricity market, while investigating the empirical relation between spot and futures electricity prices, resorting to a very recently used technique in finance and economics, wavelet analysis (in the present work both the continuous Morlet wavelet and the discrete MODWT technique are used) to access for this relation at different time scales. For the hedge analysis we have employed three unconditional hedging strategies, namely, OLS, naïve and multiscale wavelet analysis (which has not been applied to electricity markets previously) and a conditional hedging strategy using the multivariate GARCH model.

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<sup>10</sup> Basis risk is the unexpected fluctuations in the prices of spot and futures that is a product of influences ranging from seasonality to supply disruptions.

Our results point out that hedge ratio and effectiveness of the hedging strategies decrease as the wavelet time scale increases, whereas this time variation in optimal hedge ratios can be explained by deterministic seasonality, time-to-maturity effects and volatility differences between futures and spot returns in this type of markets.

We found that at high scales (low frequencies) both series show a strong and significant relation, not homogeneous across scales. At higher frequencies both series show a weak comovement, independently of the contract maturity (1 or 6 months), leading to the conclusion that spot and futures electricity markets do not show a feedback relationship. According to the cost-of-carry model, this could imply that the two markets are inefficient, not acting as perfect substitutes.

Overall, wavelet variances show that the futures market is much less volatile than the spot market regardless of the time scale pointing for the lack of spot liquidity. Spot and futures markets are then found to be fundamentally different in electricity markets given that the long-horizon hedge ratio does not converge to its long run equilibrium of one.

Finally, our empirical results allow us to say that the hedging effectiveness provided by the wavelet analysis is limited in electricity markets, being the dynamic multivariate GARCH hedge ratio the one to provide higher gains in terms of hedging effectiveness, even if small. Given the fundamental hypothesis that the various time scale decompositions of the series will provide improvements relative to hedging effectiveness, we show here that this was not really the case. This fact could be explained by the distinguishing volatile behavior of electricity spot and futures prices. As such, even with an analysis decomposing the series into several components, the hedge is inefficient. This contradictory result related to previous empirical findings using wavelets applied to financial assets and other commodities markets is not due to model specification errors. We believe that the problem does not rely on the technique being used but on the fact that there exists a weak relation between spot and futures prices.

Future research in these new markets should explore the magnitude of the forward risk premium in electricity prices that can change significantly over time. More research into the behavior of futures market and its relation with the spot price is needed to better understand the effectiveness of hedging in this market. Moreover, the delay in responses found may be an indication of arbitrage opportunities between the two markets, a statement that deserves a more careful analysis.

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