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Re-Examining Bitcoin's Price–Volume Relationship: A Time-Varying Spectral Analysis

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Abstract: This study employs continuous wavelet transforms to model the relationship between Bitcoin volume and prices across time and frequency space using daily data for the period between 17 September 2014 and 10 April 2023. The results show that Bitcoin price and volume have a long-term relationship at low frequency cycles mostly during the period after 2019. A statistically insignificant relationship between the price and volume of Bitcoin is observed prior to 2019 which coincides with a time of limited regulatory oversight of Bitcoin markets globally. Positive correlation is observed in the aftermath of this period, with stronger correlation recorded during and post the period of the Covid-19 pandemic. Furthermore, the findings reveal that fluc-tuations in the Bitcoin volume tends to affect the price at higher frequency synchronizations (short-term); whereas, at lower frequencies (long-term), a feedback loop is observed, whereby the price changes lead to alterations in the volume.

Keywords: Bitcoin price; volume; wavelet coherence; market efficiency

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1. Introduction

Upon its inception, Bitcoin was conceived as a digital currency stored in a digital wallet that utilises digital passwords or signatures to enable decentralised transactions on a blockchain ledger network without relying on a trusted third party. However, Bitcoin has since evolved beyond its original purpose as a transactional currency and has been increasingly embraced as the 'new gold' which can be used by investors and fund managers seeking to diversify their portfolios or engage in speculation (Baek and Elbeck 2015). On one hand, Bitcoin's underlying architecture, such as its decentralised structure and limited supply of tokens, are particularly appealing to risk-averse investors as they offer a hedge against inflation, geopolitical risk, government instability and currency risk (Aysan et al. 2019; Baur and Dimpfl 2021; Blau et al. 2021; Choi and Shin 2022). On the other hand, Bitcoin's high price volatility and bubble-like features have also attracted speculators, who view Bitcoin's periodic bursts in price increases as opportunities to earn substantial returns by buying low and selling high (Blau 2018; Bedi and Nashier 2020).

The appeal of Bitcoin as an investment instrument is closely linked to its price movements, which have experienced significant surges and crashes since its inception. However, unlike traditional financial assets, whose value is based on tangible assets, economic performance, or the value of a firm or industry (Kristoufek 2015), Bitcoin's fundamental value is rooted in people's trust in the high level of security provided by the hashing algorithm used to create digital signatures and facilitate 'proof-of-work' that authorise and verify transactions on the blockchain network (Ciaian et al. 2016; Marella et al. 2020). As a result, Bitcoin's 'fair' or 'intrinsic' price cannot be measured using traditional valuation metrics based on earnings, cash flows, or dividends. Instead, researchers have focused on market-related determinants of Bitcoin's price discovery, such as investor sentiments, attention, macroeconomic news announcements, market depth, and trade volume (Entrop et al. 2020; Ibikunle et al. 2020; Gurrib and Kamalov 2022).

Of all the market-related determinants that contribute to Bitcoin's price discovery, trade volume is often regarded as a comprehensive factor that provides concrete data regarding the number of buyers and sellers, the quantity of Bitcoin being exchanged, and the frequency of trades (Entrop et al. 2020). This information collectively governs Bitcoin's demand and supply dynamics and yields insights into the informational efficiency of Bitcoin markets. For example, in efficient markets with a symmetrical distribution of information across market participants, a change in investor sentiments, which can arise in reaction to macroeconomic news, could make investors more optimistic (pessimistic) towards the Bitcoin market which will first be signalled by increased (decreased) trade volume before the price adjusts itself to a new higher (lower) equilibrium, i.e., positive co-movement from volume to prices (Eom 2021). However, in inefficient markets where asymmetric information prevails among market participants, the co-movement between Bitcoin volume and price can become distorted. In such cases, a large coalition of 'smart money' investors may send false demand and/or supply signals to the market through their collective influence, leading to the manipulation of prices and volume which are not based on true market fundamentals (Szetela et al. 2021). Ultimately, such behaviours will be reflected by an insignificant or negative price–volume relationship.

In essence, a positive co-movement between Bitcoin volume and prices, with the former causing the latter, would indicate informational efficiency. Conversely, an inverse or insignificant co-movement or reverse causality would reflect market inefficiencies. The Bitcoin market is regarded as informationally efficient when new information is rapidly incorporated into asset prices, rendering it ineffective for predictive purposes (Phiri 2022). Therefore, a change in the Bitcoin price is accompanied by a corresponding change in trading volume. On the other hand, informational inefficiency suggests that new price information is not incorporated in trading volume which in turn can lead to investors earning large returns. Beyond the direction and sign of causality, two other empirical factors are crucial to understanding Bitcoin's volume–price relationship. Firstly, the relationship may exhibit time variation due to structural changes brought on by tighter government regulation (Borri and Shakhnov 2020), Black Swan events such as the COVID-19 pandemic (Phiri 2022), or geopolitical tensions such as the ongoing Ukraine–Russia war (Khalfaoui et al. 2023; Theiri et al. 2023). Secondly, the relationship may show cyclical variation due to asymmetric behaviour among different types of investors who base their decisions on various time horizons, which are reflected in different frequency cyclical synchronizations between price and volume (Phiri 2022).

The objective of our study is to investigate the relationship between Bitcoin volume and prices across time and frequency space using daily data for the period between 17 September 2014 and 10 April 2023. The hypotheses tested include the following:

- There is no causal relationship between Bitcoin price and volume.
- The relationship between Bitcoin price and volume is symmetric across time and frequency.

Our study contributes to the literature from a methodological perspective. The existing literature has mainly utilized conventional econometric tools such as Ordinary Least Squares (OLS), Vector Autoregressive (VAR), Vector Error Correction Model (VECM), and Autoregressive Distributed Lag (ARDL) models (Wang et al. 2016, 2019; Sovbetov 2018; Aalborg et al. 2019; Szetela et al. 2021; Dubey 2022; Yarovaya and Zięba 2022), in a linear setup and therefore fail to capture variations in time and frequency of the data. While some authors have attempted to address this limitation by using nonlinear methods such as quantile causality tests to account for location asymmetries (Balcilar et al. 2017; Bouri et al. 2019; Hau et al. 2021), as well as nonlinear dependence and cross-correlation multifractionality techniques to distinguish the price–volume relationship between bull and bear markets (Katsiampa et al. 2018; Zhang et al. 2018a, 2018b; El Alaoui et al. 2019), these models still fall short of being able to comprehensively investigate all possible dimensions of the Bitcoin volume–price relationship in a singular framework. We hypothesise that this may be the reason why previous studies have produced inconsistent empirical evidence.

In our research, we employ continuous wavelet transforms as signal extraction tools to model the relationship between Bitcoin volume and prices in a scale-by-scale fashion across various time periods. This unique approach enables us to comprehensively examine the sign, magnitude, time variation, cyclical variation, and causal effects of the relationship under a single framework. Notably, the wavelet coherence spectrum coefficients estimated from complex wavelet transforms via convolution are insensitive to the selected time window and free from any potential regression errors. This stands in contrast to previous studies that have used estimation techniques, whose results are dependent on the selected time period and may contain regression errors. As a result, our study's findings are more robust and 'permanent', and this represents the main empirical contribution of our research to the literature.

The results of our study reveal no statistically significant relationship between the price and volume of Bitcoin prior to 2019. This period coincides with a time of limited regulatory oversight of Bitcoin markets globally. However, a positive correlation between the two series emerges in the aftermath of this period. Specifically, we find that fluctuations in the Bitcoin volume tend to affect price at higher frequency (over the short-term) synchronizations; whereas, at lower frequencies (over the long-term), a feedback loop is observed, whereby price changes lead to alterations in trade volumes.Notably, this pattern is consistent across both bull and bear markets.

Overall, our study highlights the existence of long-term informational inefficiencies in Bitcoin markets via price–volume dynamics suggesting that the market may be prone to informational inefficiencies, particularly due to the overreaction of uninformed investors. We interpret the persistent nature of these reverse causality dynamics at lower frequencies as evidence of momentum trading effects in Bitcoin markets. In other words, price movements trigger changes in trade volume, which then further reinforces the initial movement of price change in the same direction during both bull and bear markets. Our results hold even when we use alternative measures of Bitcoin currencies.

The rest of the study is outlined as follows. The literature review is presented next. The methodology is discussed in Section 3. The results are presented in Section 4 whilst the study is concluded in Section 5.

2. Literature Review

This section presents the literature review which includes a theoretical and an empirical literature review. The purpose of the literature review is to present a summary of the previous research on the relationship between volume and asset prices (in particular cryptocurrency prices) and to identify gaps in the literature.

2.1. Theoretical Literature

The theoretical literature consists of various theories that explain the relationship between trading volume and asset prices. We selected the theories based on their applicability to the study and grouped them according to different themes that emerged from the analysis. The price–volume nexus is dependent on a number of factors as alluded to by Karpoff (1987). In our review of the literature, we found that the price–volume nexus is dependent on (1) heterogeneity among investors Epps (1975), (2) the order in which information is obtained by investors Copeland (1976) and Jennings et al. (1981), (3) interpretation of information by investors Harris and Raviv (1993), and (4) reasons for trading assets Wang (1994) and Llorente et al. (2002).

Epps (1975) developed a theoretical framework that shows the link between transaction volume and the prices of bonds. The model is based on the assumption that all trades occur between investors regarded as "bulls" (buyers) and "bears" (sellers). The model predicts that there is a positive relationship between transaction volume and asset prices. Furthermore, during periods when asset prices are on a higher trajectory, the ratio of transaction volume to price changes is higher.

Copeland (1976) developed the "sequential arrival of information" model where investors receive information in chronological order. The model is based on the assumptions that there are optimistic, pessimistic and uninformed investors and no short selling is possible. The availability of information enhances trading volume (demand) for the optimistic investor and decreases trading for the pessimistic investor; however, the volume of trade for the optimistic investor is greater due to the lack of short sales. Using simulations, Copeland (1976) showed that the largest changes in the price of an asset are associated with the maximum trading volume and since the number of optimistic investors is distributed symmetrically with a mean of 0.5, a positive relationship between price and trading volume is predicted.

As alluded to by Karpoff (1987), the "sequential arrival of information" model has some shortcomings which include the assumptions of lack of short selling and the inability of traders to obtain information from the market price and other more informed traders. Furthermore, the finding that the trading volume is at its greatest when all investors are in agreement is also questionable. As such, Jennings et al. (1981) extended Copeland (1976)'s model and incorporated short selling and margin requirements. Due to higher transaction costs, short positions are assumed to cost more compared to long positions. Therefore, for a given price change, the trading volume for an investor with a short position is less than that of one with a long position. The model predicts a positive correlation between volume and price given that volume is higher (lower) when the price is higher (lower).

Harris and Raviv (1993) constructed a model where the price–volume nexus is dependent on speculative trading caused by differences in the interpretation of information. Information is referred to as "signals" which include earnings announcements, macroeconomic news and news regarding political stability. The model assumes that traders agree on favourable and unfavourable information; however, they are in disagreement about the extent to which the information is valuable. The model implies that the greater the disagreements amongst speculators the larger the observed trading volume which in turn leads to higher prices of assets. He and Wang (1995) developed a "multiperiod" model where stock trading is dependent on differences in the information received by traders. Trading from exogenous information such as public announcements and private information results in large price changes compared to that of existing information.

Wang (1994) and Llorente et al. (2002) developed models of investor heterogeneity and its effect on the volume–price nexus. Wang (1994)'s model assumes that investors have different information and investment opportunities, and thus asset trading is based on the availability of public and private information. Public information includes published information on an asset's share price and realised dividends, while private information on future returns of an asset is held by informed investors. The model shows that due to information asymmetry when informed investors sell an asset, a decline in the price is expected in order to encourage uninformed investors to purchase the asset. Therefore, trading volume is positively related to absolute price changes. In a similar vein, Llorente et al. (2002) constructed a model based on investment for hedging and speculative reasons. Furthermore, the model assumed that investors lack foresight and therefore private information is shortlived. The model predicts that trading based on hedging reasons leads to a decline in the price of an asset which in turn causes low returns in the current period and higher future expected returns. Trading based on speculative reasons results in a decline in the price of an asset as future returns are expected to be lower. The model also predicts that returns from hedging trades will be reversed while those of speculative reasons tend to be prolonged.

2.2. Empirical Literature

This section presents a survey of the existing empirical literature. The review of the empirical literature will identify the gaps in the literature and outline the contribution of our study. We reviewed a total number of 20 studies that examined the linkages between trading volume and cryptocurrency prices. The review of the literature focused on the more recent studies, of which the majority investigated the price–volume nexus in the Bitcoin market. One distinguishing feature of studies reviewed in the empirical literature is the estimation technique used. In particular, some studies employ linear techniques while others utilise nonlinear methods.

Studies that employed linear regression techniques include Sovbetov (2018) (ARDL); Wang et al. (2016); Szetela et al. (2021) (VECM); Blau (2018); Naeem et al. (2020); Sapuric et al. (2022) (GARCH models) and Aalborg et al. (2019) (OLS regressions). Sovbetov (2018) found that there is both a long-run and short-run relationship between trading volume and cryptocurrency (Bitcoin, Ethereum, Litecoin and Monero) prices. However, the effect is larger in the long run. Wang et al. (2016) showed that in the Bitcoin market, trading volume has a positive effect on price in the long run. However, in the short run, the effect is minimal. Gemici and Polat (2019) also confirmed the long-run relationship between volume and price in the Bitcoin market. Contrary to the above studies, Szetela et al. (2021) found no long-run and short-run relationship between the strength of the price trend and trading volume in both bull and bear markets. Aalborg et al. (2019) also showed that trading volume has a minimal effect on daily Bitcoin returns and returns have an insignificant effect on trading volume.

Blau (2018) found that speculative trading was not a significant contributor to Bitcoin price and volatility to the price or volatility. Naeem et al. (2020) found evidence of asymmetric dependency between the returns and trading volume in cryptocurrencies (Bitcoin, Ethereum and Litecoin). Furthermore, a positive asymmetric relationship was detected for Bitcoin and Litecoin and a negative for Ethereum, and extreme high returns for all the cryptocurrencies are associated with greater trading volumes and vice versa for extreme lower returns. Sapuric et al. (2022) reported a positive relationship between returns and volume in Bitcoin before the Mt. Gox hack and its shutdown in February 2014. This can be explained by the rise in the price of the asset due to higher demand.

The majority of the studies employed causality and correlation tests. The causality tests included the Granger causality tests (Dubey 2022; Wang et al. 2019; Yarovaya and Zięba 2022), the Toda–Yamamoto causality test (Sahoo et al. 2019; Gemici and Polat 2019), the quantile causality tests (Balcilar et al. 2017; Bouri et al. 2019; Hau et al. 2021) and the frequency connectedness approach (Fousekis and Tzaferi 2021). Dubey (2022) reported that trading volume was a significant determinant of Bitcoin price in the long run and not in the short run. Wang et al. (2019) found a negative correlation between the variables which contradicted the mixture of distribution hypothesis. However, the authors reported support for the sequential information hypothesis due to the significant lead–lag relationship. Causal analysis showed that causality runs from trading volume to returns volatility. Yarovaya and Zięba (2022) found evidence of bidirectional causality between trading volume and 30 cryptocurrency returns, especially at high frequencies.

Sahoo et al. (2019) employed the linear and nonlinear Toda and Yamamoto Granger causality test to examine the price-trading volume nexus in the Bitcoin market and reported that the linear results suggest no relationship between the variables. However, the nonlinear results showed evidence of bidirectional causality between price and trading volume. Gemici and Polat (2019) found evidence of asymmetric causality in Bitcoin with unidirectional causality from negative price shocks to negative trading volume and positive shocks from trading volume to positive shocks in price. Balcilar et al. (2017) also found that there is no causality between trading volume and returns in Bitcoin markets using the linear Granger causality test. However, using the nonparametric quantile-in-causality (nonlinear) test, the authors reported the presence of nonlinearities in the return-volume nexus. Furthermore, the results suggested that causality is only observed during the normal periods of the market and not in bear and bull periods.

Bouri et al. (2019) utilised the copula-quantile causality test to investigate the relationship between cryptocurrency (Bitcoin, Ripple, Ethereum, Litecoin, Nem, Dash, and Stellar) trading volume and price. They found evidence of causality from trading volume to price for both positive and negative returns indicated by high and low quantiles. Hau et al. (2021) employed the quantile-on-quantile regressions to investigate the effect of Bitcoin transaction activity (measured by the trading volume) on returns. The authors found high transaction volume is associated with higher Bitcoin returns during bull markets and lower returns during bear markets. Furthermore, the results showed evidence of asymmetry due to the stronger relationship in the upper and lower tails of the distribution. Fousekis and Tzaferi (2021) reported bidirectional causality between returns and volume for Bitcoin, Ethereum, Ripple, and Litecoin using the frequency connectedness approach. Furthermore, the results suggest that there are larger total and directional spillovers from returns to volume at the lower frequency band compared to spillovers from volume to returns.

Correlation tests employed include multifractal detrended cross-correlation analysis (MF-DCCA) (Zhang et al. 2018b; El Alaoui et al. 2019) and other variants of correlation methods (Katsiampa et al. 2018; Zhang et al. 2018a). El Alaoui et al. (2019) found that there is a nonlinear relationship between price and volume nexus in the Bitcoin market. Furthermore, there is evidence of multifractionality in the price–volume nexus. Zhang et al. (2018b) also found evidence of nonlinear dependency and cross-correlation multifractionality in the Bitcoin return-volume relationship. Zhang et al. (2018a) showed that there is a power-law correlation between price and volume for eight cryptocurrencies (Bitcoin, Ethereum, Dash, Litecoin, NEM, Stellar, Monero and Ripple). Katsiampa et al. (2018) reported an asymmetric relationship between return and volume for cryptocurrencies due to differences in the effects of positive and negative returns. Furthermore, the extreme correlation between trading volumes and returns declines towards the distribution tails.

Other studies in the literature have shown that Machine Learning techniques can be used to predict the Bitcoin price (see McNally et al. 2018; Ji et al. 2019; Dutta et al. 2020; Chen et al. 2020; Jaquart et al. 2021; Wang and Hausken 2022). Most of these studies use Bitcoin transaction volume (number of transactions on the blockchain) as an explanatory variable for price and therefore, reverse causality is not explored. In this study, we use trading volume (number of Bitcoins that are traded on cryptocurrency exchanges) as an explanatory variable. Furthermore, our study also examines the direction of causality between the variables.

In summary, the results from the empirical studies suggest there is a relationship between trading volume and price. However, a significant number of studies highlight the asymmetries or nonlinear effects in the nexus. Furthermore, evidence suggests that causality runs mostly from trading volume to prices. This study makes a valuable contribution to the existing literature from a methodological standpoint. Previous studies in the literature primarily focused on time domain analysis when employing time series techniques. However, there has been an increasing interest in exploring multiscale relationships in the fields of economics and finance (Delfin-Vidal and Romero-Meléndez 2016). To address this, time–frequency analysis has been utilised to examine the behaviour of variables across a broad range of time scales. Moreover, by employing time–frequency analysis, it becomes possible to investigate the relationships between variables at different scales (Nguyen and He 2015).

In our study, we utilise the wavelet coherence technique, which allows us to decompose the time series of Bitcoin price and volume into both the time and frequency domains. As a result, our research aims to shed light on how the relationship between these variables changes over time and frequency. The wavelet transform technique offers several advantages, including higher levels of estimation efficiency and more robust estimations, even in the presence of modelling errors (Ramsey 2002).

3. Materials and Methods

The majority of econometric models employed in the related literature have relied on linear regression estimators, which can only provide information regarding the sign (positive or negative) or magnitude (weak or strong correlation) of the co-movement between two time series. At best, cointegration and causality models—including VAR, VECM, and ARDL—have been utilised in the literature to differentiate between short-term and long-term cointegration effects, and provide insights into the causal relationships between Bitcoin's price and volume. Additionally, some nonlinear models have been used to capture location asymmetries (quantile regression models) and multifractal nonlinearity across different time scales (MF-DCCA). Nonetheless, the current methods used by researchers do not provide an inclusive framework that can simultaneously address asymmetries arising from time and frequency variation in the data whilst simultaneously accounting for lead–lag relationships between the series.

Wavelet analysis can be considered a potential solution to the deficiencies presented by traditional estimators. Morlet et al. (1982a, 1982b) introduced wavelets as a set of mathematical functions that can decompose a signal in a scale-by-scale manner. These tools have been widely employed to investigate the time–frequency properties of geological data such as cyclones and temperature data (Lau and Weng 1995). Torrence and Compo (1998) introduced the concept of wavelet coherence to describe the co-movement between two decomposed time series in time–frequency space by using convolution operations. However, it was Aguiar-Conraria and Soares (2011) who popularised the use of wavelet coherence among economists and social scientists. A detailed discussion of the applications and uses of complex wavelet tools in economics is presented in Aguiar-Conraria and Soares (2014).

3.1. Wavelet Coherence

In our study, we utilise the two-step procedure described in Aguiar-Conraria and Soares (2014) to investigate the relationship between Bitcoin trade volume (x) and Bitcoin prices (y) in time–frequency space. Firstly, we first convolute the individual time series with a set of complex-valued 'daughter wavelets' generated by a common 'mother' wavelet. The convolution process generates the wavelet coefficients that are responsible for the amplitude and phased dynamics in time–frequency space.

The daughter wavelets for each series are defined as:

$$W_y(s,\tau) = \int_{-\infty}^{\infty} y(t)\frac{1}{\sqrt{s}}\psi * \left(\frac{t-\tau}{s}\right)dt \tag{1}$$

$$W_x(s,\tau) = \int_{-\infty}^{\infty} x(t)\frac{1}{\sqrt{s}}\psi * \left(\frac{t-\tau}{s}\right)dt \tag{2}$$

where * is the conjugate of the complex number, τ and s are the translation and dilation parameters responsible for amplitude and phase dynamics in time–frequency space; whilst ψ is the mother morlet wavelet defined as:

$$\psi(t) = \pi^{-\frac{1}{4}}\exp(i\omega t)\exp\left(-\frac{1}{2}t^2\right) \tag{3}$$

where ω_0 is set at 2π to ensure optimal joint time–frequency resolution. Secondly, we extract the wavelet power spectrum (WPS) of the y(t) and (x(t) series (i.e., $W_{xx} = |W_x|^2$ and $W_{yy} = |W_y|^2$,) as well as their cross-wavelet power spectrum (CWPS) $(WPS)_{xy} = W_{xy} = |W_{xy}|$, from which the wavelet coherence, is computed as:

$$R_{y.x}(s) = \frac{|\ S(W_{x,y})\ |}{[(S\ |\ W_x\ |^2)(S\ |\ W_y\ |^2)]^{\frac{1}{2}}} \tag{4}$$

where S is a smoothing operator in both time and scale. The phase-difference dynamics are determined as:

$$\phi_{x,y} = Arctan^{-1}\left(\frac{I\{W_x\}}{\Re\{W_x\}}\right). \tag{5}$$

where $\pi < \phi_{x,y} < -\pi$ and provides information on (i) whether the pair of series are in-phase (positive) or antiphase (negative) synchronized and (ii) whether x leads y or vice versa.

3.2. Wavelet Local Bivariate Correlation (WLBC)

Denote the vector X = $\{x_{1t}, x_{2t}\}$ as bivariate 2 by 2 time series and further denote $W_{jt} = \{w_{1jt}, w_{2jt}\}$ as the wavelet coefficient at scale λ_j, obtained from estimating the maximum overlap discrete wavelet transform (MODWT) to each x_{it} process/series at each wavelet scale $(\lambda_j) = 1, \ldots, J$, with J being the maximum level of wavelet transform decomposition). The WMC, $\varphi_x(\lambda_j)$, obtained at each wavelet scale, can be defined as a single set of multiscale correlations which is computed as the square root of the regression coefficient of determination for the linear combination of variables for which such a coefficient of determination is a maximum, i.e.,

$$\varphi_x(\lambda_j) = \sqrt{R^2} \tag{6}$$

$$= Corr\left(\theta(t-s)^{\frac{1}{2}} w_{ijt},\ \theta\left(t-s\right)^{\frac{1}{2}}\ \hat{w}_{ijt}\right) s = 1,\ \ldots,\ T. \tag{7}$$

where $\theta(x)$ is the 'Gaussian' moving average weight function satisfying the condition $\int_{-\infty}^{\infty} \theta\ (x)dx = 1$, w_{ijt} is chosen so that it maximises $\varphi_x(\lambda_j)$ and $\hat{w}_{ijt}$ is the corresponding vector of fitted values. To construct confidence intervals, we take a sample of the WLMC $(\hat{\varphi}_{X,s}\lambda_j)$ and then specify the equation below:

$$\hat{z}_j \sim <\ F\aleph(z_j,\ \left(\frac{T}{2^j} - 3\right)^{-1}) \tag{8}$$

where z_j = arctanh $(\varphi_x(\lambda_j))$, $\hat{z}_j$ = arctanh$(\hat{\varphi}_x(\lambda_j))$ and $F\aleph$ is the folded normal distribution. Since $(\hat{\varphi}_{X,s}\lambda_j)$ is the correlation between observations from two Gaussian variates of which T/2j are serially uncorrelated, and sgn(arctanh(.)) = sgn(.), applying the Fisher's transformation to ξ_j such that abs$(\xi_j) = \hat{z}_j$, the confidence intervals are obtained as:

$$CI_{1-\alpha}\ (\varphi_{X,s}(\lambda_j)) = \tanh[\hat{z}_{j,s} \pm \frac{\phi^{-1}_{1-\frac{\alpha}{2}}}{\sqrt{\frac{T}{2^j} - 3}}, \tag{9}$$

where ϕ_p^{-1} is the 100p% standard normal distribution used to compute the confidence intervals.

4. Data and Results

4.1. Data

We use daily price and volume data for Bitcoin which are sourced from Yahoo Finance (https://finance.yahoo.com/crypto/) between 17 September 2014 and 10 April 2023. The data was accessed on 14 April 2023. Price refers to the value at which Bitcoin is traded on a particular day, while volume is the number of Bitcoins traded on various cryptocurrency exchanges. The summary statistics of the variables are presented in Table 1. The average Bitcoin price over the chosen sample is USD 13,199.57 while that of volume is just over 16.6 billion. The Jarque–Bera test suggests that both variables are not normally distributed as the null of normality is rejected. Furthermore, both variables have leptokurtic distributions as indicated by the kurtosis statistics that are greater than 3. As expected, there is a greater probability of extreme values in the volume variable.

Table 1. Descriptive statistics.

	Bitcoin Price	Bitcoin Volume
Mean	13199.57	1.66×10^{10}
Median	7364.94	9.04×10^{9}
Maximum	67,566.83	3.51×10^{11}
Minimum	178.10	5,914,570
Std. dev.	16,043.44	1.98×10^{10}
Skewness	1.48	2.71
Kurtosis	4.19	30.42
Jarque–Bera	1327.61	101,855.3
Probability	0.00	0.00
Observations	3128	3128

4.2. Wavelet Power Spectrum

The wavelet power spectrum (WPS) measures the variance distribution of a variable around each time period and frequency (Verona 2016). It indicates the relative contribution of a particular frequency to the total variance of a time series at each point in time. The WPS is shown graphically in Figure 1 (Bitcoin) and Figure 2 (volume) with time measured on the horizontal axis and the frequency cycles or periods in days shown on the vertical axis. The power levels range from 0 (blue shade) to 0.3 (orange shade). Power or volatility levels that are statistically significant at 5% are shown by the white lines surrounding the colour contours. Significant Bitcoin price volatility is observed at higher frequency cycles (128–512). During the period prior to 2020, the Bitcoin price was characterised by low volatility which is confirmed by the flat trend in Figure 1. An uptick in volatility was observed from the year 2020 due to the COVID-19 pandemic with statistically significant volatility at relatively higher frequency cycles (from 64 days) compared to the period prior to 2020. As shown in Figure 1, the Bitcoin price surged in 2020 possibly due to investors seeking a safer haven from the macroeconomic instability brought about by the pandemic. Furthermore, there was a surge in demand for Bitcoin by institutional investors. The greatest volatility is observed in 2021 as shown by the darker shaded area. This is in line with the findings of Özdemir (2022) who showed evidence of greater Bitcoin price volatility during the last quarter of 2020 and 2021. Figure 2 highlights the low volatility of the volume variable for the entire sample. Similar to Bitcoin, statistically significant volatility is observed at low-frequency cycles prior to 2020. However, there is evidence of significant volatility at relatively higher frequency cycles from 2020.

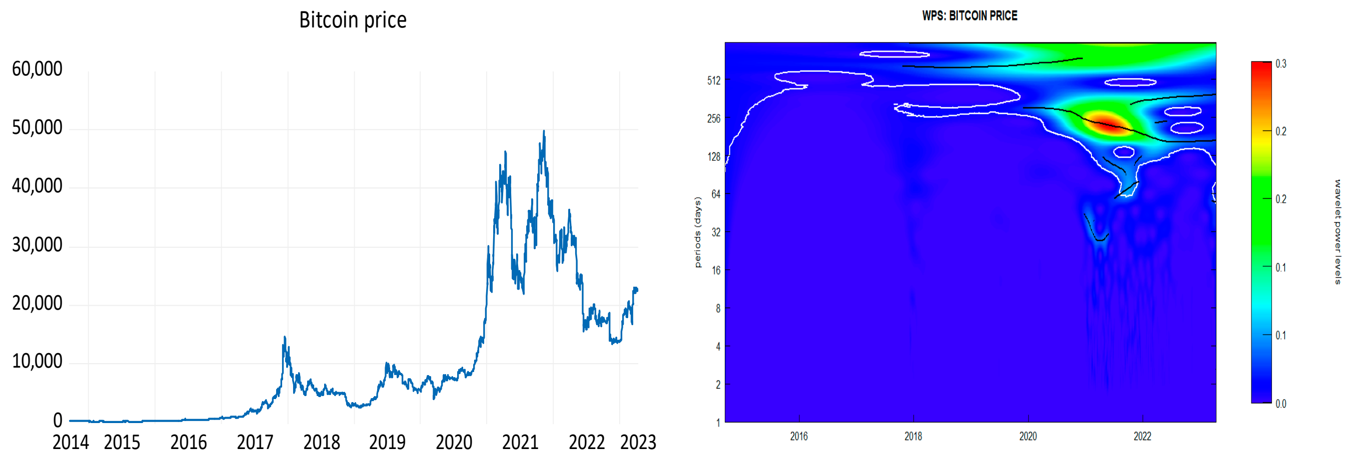

Figure 1. Times series and WPS plot for Bitcoin prices.

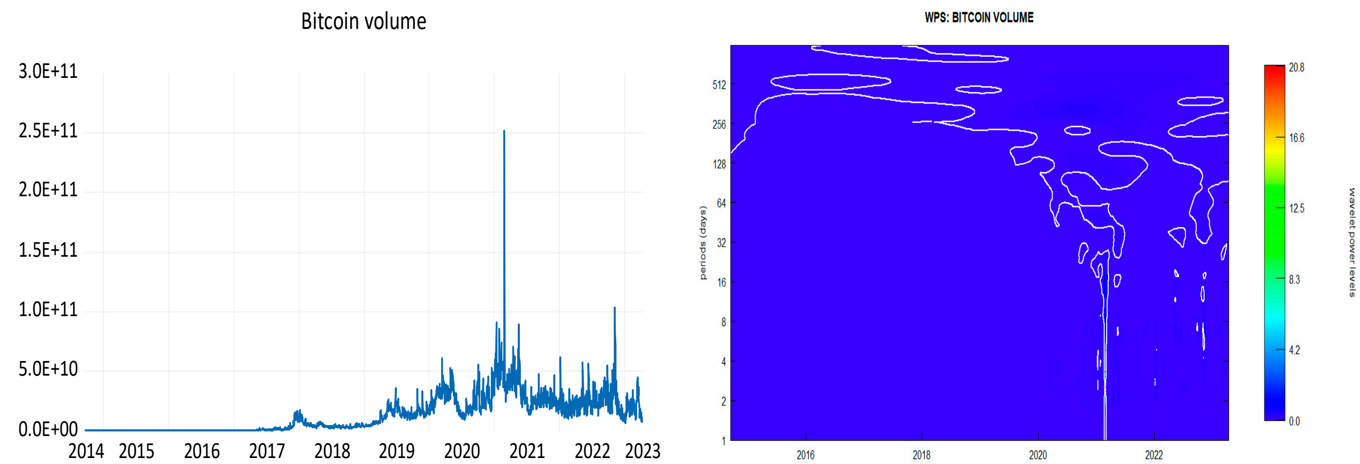

Figure 2. Times series and WPS plot for Bitcoin volume.

4.3. Wavelet Coherence

Wavelet coherence is a measure of co-movement between two variables. The measure indicates the strength and direction of the co-movement between variables. Figure 3 shows the co-movement between the Bitcoin price and volume. The white contour indicates statistically significant coherence. Arrows pointing in the north-east (↗) and south-east (↘) directions indicate positive co-movement while those pointing to the north-west (↖) and south-west (↙) indicate negative coherence. Arrows pointing to the north-east and south-west show causality from volume to Bitcoin price and vice versa for arrows pointing in the south-east and north-west directions. North-east pointing arrows show an in-phase relationship (positive from volume to Bitcoin price) and those pointing in the south-west indicate an antiphase (negative from volume to Bitcoin price).

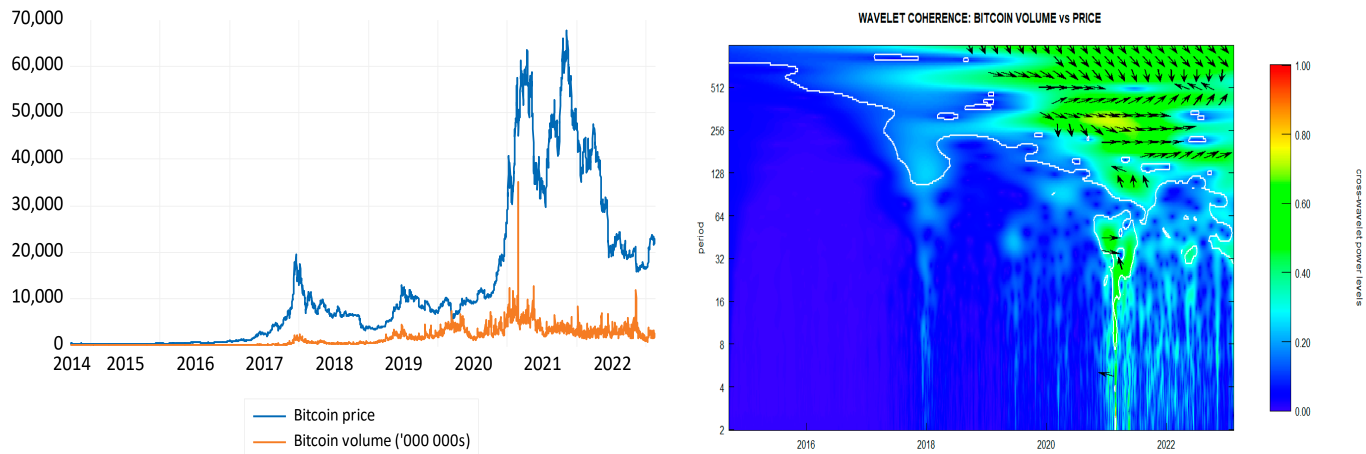

Figure 3. Wavelet coherence.

The results show that Bitcoin price and volume have a long-term relationship at low-frequency cycles (128–512 days) mostly during the period after 2019. A statistically insignificant relationship between the price and volume of Bitcoin is observed prior to 2019 especially at high frequency (over the short term). This period coincides with a time of limited regulatory oversight of Bitcoin markets globally. Stronger coherence is observed during and post the period of the COVID-19 pandemic. Furthermore, there is evidence of significant coherence at a much higher frequency in 2021 which coincides with the period of higher volatility in both variables The finding is in line with that of Kristoufek (2015) who also found mostly long-term connectedness between Bitcoin price and volume. Furthermore, Sovbetov (2018), Gemici and Polat (2019) also found evidence of a long-run relationship between Bitcoin price and volume using time series techniques. The result suggests that at high frequency (over the short-term), there is evidence that Bitcoin volume is a predictor of the price as indicated by the South-west pointing ar-rows at high frequency. This indicates an in-phase relationship over a short period during 2021 which coincided with the period of greater volatility in the Bitcoin price. At lower frequencies (over the long-term), a feedback loop is observed, whereby price changes lead to alterations in the volume. Notably, this pattern is consistent across both bull and bear markets. Similar to our study, Fousekis and Tzaferi (2021) also found evidence of larger spillover effects from Bitcoin returns to volume at lower fre-quency (long-term). The study, therefore rejects the hypothesis that there is no causal-ity between Bitcoin price and volume. Furthermore, the hypothesis that the relation-ship between the variables is symmetric across frequency and time is also rejected.

4.4. Wavelet Local Bivariate Correlation

The wavelet local bivariate correlation (WLBC) is used to determine the strength of correlations between variables over time and frequency domains (Polanco-Martínez et al. 2020). The WLBC analysis is also able to indicate the contribution of the variables to the observed correlation. In Figure 4a the strength of the correlation across frequency and time is shown, while Figure 4b indicates the contribution of the variables to the correlation. In Figure 4a the darker shaded areas represent the higher correlation levels. In Figure 4b the contributions of Bitcoin price and volume to the observed correlation are shown by the black and pink colours, respectively. The results shown graphically in Figure 4a indicate that the degree of correlation between Bitcoin price and volume varies with time. The strongest correlation is observed in the long-term (at low frequency) in the first 1500 days of the sample period (2014 to 2018) and between 2500 and 3000 (2020 to 2023) as shown by the dark-shaded colour. There is evidence of a strong correlation in the medium term (frequency cycle 64–128) during the period 2014 to 2018. Figure 4b shows the dominant variable that is the largest contributor to the observed correlations between the variables. Bitcoin price is the more dominant variable in the entire sample as shown by the larger dark-shaded blocks.

(a)

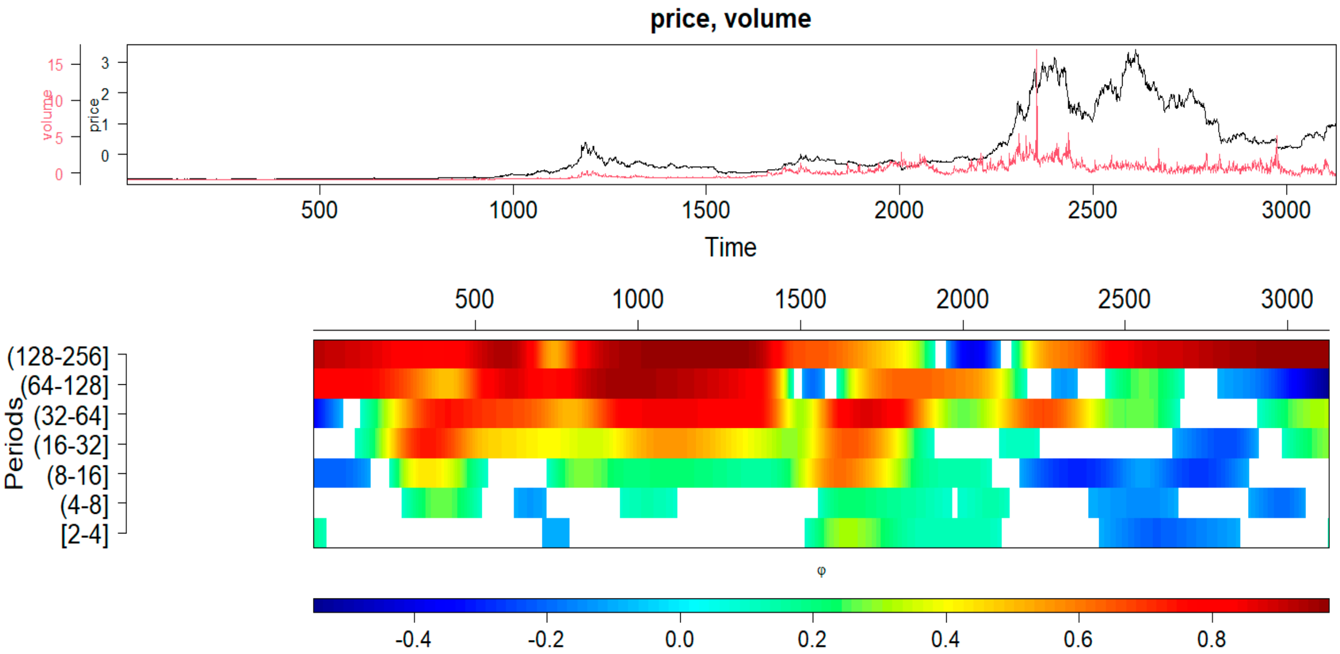

Figure 4. *Cont.*

(**b**)

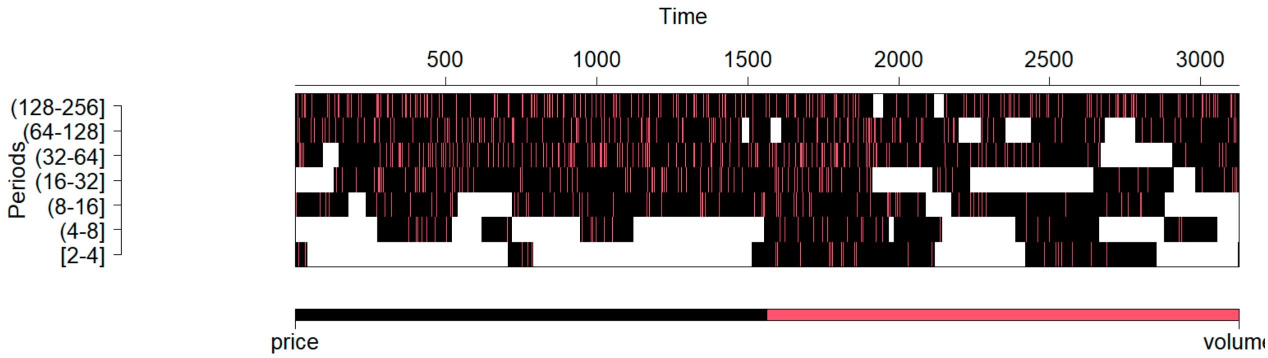

Figure 4. Wavelet local bivariate correlation.

4.5. Discussion of Results

Overall, our empirical analysis contributes to literature in the following ways. Firstly, the study showed that the relationship between Bitcoin price and volume changes over time and frequency. Prior to 2019 the nexus between the variables was largely insignificant especially at higher frequency. However, during the period of the COVID-19 pandemic a significant relationship was observed mostly over the long-term. The findings are in contrast to those of conventional time series techniques used in literature which show the relationship between the variables over time. Secondly, the study showed that the causality between the variables varies with time. For the most part, the observed causality is from Bitcoin price to volume, however, in 2021 during the period of excessive volatility in Bitcoin price, there is evidence of causality being in the opposite direction. This is in contrast to other techniques which show causality over an entire sample. Thirdly, our study used a sample which includes the period prior to and during the COVID-19 pandemic. During the period of the pandemic, higher levels of volatility in the Bitcoin price were observed, therefore, the findings from the study can be used to make comparisons between periods of high and low volatility.

The findings of the study have implications for both investors and regulators. Overall, the study found that Bitcoin price is a determinant of volume in the long term which highlights the existence of long-term informational inefficiencies in Bitcoin markets via price–volume dynamics. This implies that the markets may be prone to informational inefficiencies, particularly due to the overreaction of uninformed investors. In such cases, a large coalition of 'smart money' investors may send false demand and/or supply signals to the market through their collective influence, leading to the manipulation of prices and volume which are not based on true market fundamentals.

The finding that Bitcoin price drives volume suggests evidence of herding behaviour in Bitcoin markets. This could mean that investors often buy or sell assets based on the actions of other investors, rather than on their own assessment of the asset's fundamentals as alluded to by Youssef (2022). Herding behaviour can lead to high levels of volatility in asset prices, which can in turn lead to bubbles and crashes. For instance, if the price of Bitcoin starts to rise, investors may be more likely to buy the asset, even if they do not believe that it is undervalued. This can drive up the price even further, leading to a bubble. Conversely, if the price of Bitcoin starts to fall, investors may be more likely to sell the asset, even if they believe that it is undervalued. This can drive down the price even further, leading to a crash.

The finding that Bitcoin price drives volume makes it more difficult for regulators to manage the market. This is because regulators need to be able to predict how investors will react to changes in price. However, if investors are simply following the herd, it is difficult to predict how they will react. Overall, the findings of the study suggest that Bitcoin markets are volatile and unpredictable. This makes it difficult for investors to make informed decisions and for regulators to manage the market.

5. Conclusions

The purpose of this paper was to investigate the relationship between Bitcoin price and volume using daily data for the period between 17 September 2014 and 10 April 2023. The continuous wavelet analysis was employed to model the relationship between Bitcoin volume and prices in a scale-by-scale fashion across various time periods. The theoretical literature underpinning the study suggests that there is a positive relationship between the variables with causality running from volume to price. However, this does not take into account the possibility of price manipulation by large institutional investors in the event of inefficient markets. Such manipulation may distort the price–volume nexus.

The results show that Bitcoin price and volume have a long-term relationship at low frequency cycles mostly during the period after 2019. A statistically insignificant relationship between the price and volume of Bitcoin is observed prior to 2019 which coincides with a time of limited regulatory oversight of Bitcoin markets globally. Positive correlation is observed in the aftermath of this period, with stronger correlation recorded during and post the period of the COVID-19 pandemic. Specifically, we find that fluctuations in Bitcoin volume tend to affect price at higher frequency synchroni-zations; whereas, at lower frequencies, a feedback loop is observed, whereby price changes lead to alterations in trade volumes. Notably, this pattern is consistent across both bull and bear markets. The results have profound implications for stakeholders in Bitcoin markets including investors. Firstly, the causality from Bitcoin price to volume is an indication of the possible long-term inefficiency of the Bitcoin market. The Bitcoin market may be prone to informational inefficiencies, particularly due to the overreaction of uninformed investors. Secondly, the findings suggest possible price manipulation by large investors. Chen et al. (2019) and Gandal et al. (2018) found evidence of Bitcoin price manipulation by Bitcoin exchange Mt. Gox, which supports the calls for tighter supervision of the Bitcoin market to protect investors.

Our study has a few delimitations that should be taken into account. Firstly, the study only considered Bitcoin due to its popularity compared to other cryptocurrencies. Therefore, the findings of our study should not be generalised to all cryptocurrencies. Secondly, the study did not consider the effect of other variables and external events that impact Bitcoin price and volume. However, it should be noted that the wavelet coherence method is able to produce robust estimates for bivariate analysis. Areas of future research include estimating the price and volume nexus for other cryptocurrencies to determine the strength and direction of the relationship.

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References

Aalborg, Halvor Aarhus, Peter Molnár, and Jon Erik de Vries. 2019. What Can Explain the Price, Volatility and Trading Volume of Bitcoin? *Finance Research Letters* 29: 255–65. [CrossRef]

Aguiar-Conraria, Luís, and Maria Joana Soares. 2011. *The Continuous Wavelet Transform: A Primer*. Braga: NIPE-Universidade do Minho.

Aguiar-Conraria, Luís, and Maria Joana Soares. 2014. The continuous wavelet transform: Moving beyond uni-and bivariate analysis. *Journal of Economic Surveys* 28: 344–75. [CrossRef]

Aysan, Ahmet Faruk, Ender Demir, Giray Gozgor, and Chi Keung Marco Lau. 2019. Effects of the Geopolitical Risks on Bitcoin Returns and Volatility. *Research in International Business and Finance* 47: 511–18. [CrossRef]

Baek, Chung, and Matt Elbeck. 2015. Bitcoins as an Investment or Speculative Vehicle? A First Look. *Applied Economics Letters* 22: 30–34. [CrossRef]

Balcilar, Mehmet, Elie Bouri, Rangan Gupta, and David Roubaud. 2017. Can Volume Predict Bitcoin Returns and Volatility? A Quantiles-Based Approach. *Economic Modelling* 64: 74–81. [CrossRef]

Baur, Dirk G., and Thomas Dimpfl. 2021. The Volatility of Bitcoin and Its Role as a Medium of Exchange and a Store of Value. *Empirical Economics* 61: 2663–83. [CrossRef] [PubMed]

Bedi, Prateek, and Tripti Nashier. 2020. On the Investment Credentials of Bitcoin: A Cross-Currency Perspective. *Research in International Business and Finance* 51: 101087. [CrossRef]

Blau, Benjamin M. 2018. Price Dynamics and Speculative Trading in Bitcoin. *Research in International Business and Finance* 43: 15–21. [CrossRef]

Blau, Benjamin M., Todd G. Griffith, and Ryan J. Whitby. 2021. Inflation and Bitcoin: A Descriptive Time-Series Analysis. *Economics Letters* 203: 109848. [CrossRef]

Borri, Nicola, and Kirill Shakhnov. 2020. Regulation Spillovers across Cryptocurrency Markets. *Finance Research Letters* 36: 101333. [CrossRef]

Bouri, Elie, Chi Keung Marco Lau, Brian Lucey, and David Roubaud. 2019. Trading Volume and the Predictability of Return and Volatility in the Cryptocurrency Market. *Finance Research Letters* 29: 340–46. [CrossRef]

Chen, Weili, Jun Wu, Zibin Zheng, Chua Chen, and Yuren Zhou. 2019. Market Manipulation of Bitcoin: Evidence from Mining the Mt. Gox Transaction Network. Paper presented at the IEEE INFOCOM 2019-IEEE Conference on Computer Communications, Paris, France, April 29–May 2.

Chen, Zheshi, Chunhong Li, and Wenjun Sun. 2020. Bitcoin Price Prediction Using Machine Learning: An Approach to Sample Dimension Engineering. *Journal of Computational and Applied Mathematics* 365: 112395. [CrossRef]

Choi, Sangyup, and Junhyeok Shin. 2022. Bitcoin: An Inflation Hedge but Not a Safe Haven. *Finance Research Letters* 46: 102379. [CrossRef] [PubMed]

Ciaian, Pavel, Miroslava Rajcaniova, and d'Artis Kancs. 2016. The Economics of Bitcoin Price Formation. *Applied Economics* 48: 1799–15. [CrossRef]

Copeland, Thomas E. 1976. A Model of Asset Trading under the Assumption of Sequential Information Arrival. *The Journal of Finance* 31: 1149–68. [CrossRef]

Delfin-Vidal, Rafael, and Guillermo Romero-Meléndez. 2016. *The Fractal Nature of Bitcoin: Evidence from Wavelet Power Spectra*. Cham: Springer.

Dubey, Priti. 2022. Short-Run and Long-Run Determinants of Bitcoin Returns: Transnational Evidence. *Review of Behavioral Finance* 14: 533–44. [CrossRef]

Dutta, Aniruddha, Saket Kumar, and Meheli Basu. 2020. A Gated Recurrent Unit Approach to Bitcoin Price Prediction. *Journal of Risk and Financial Management* 13: 23. [CrossRef]

El Alaoui, Marwane, Elie Bouri, and David Roubaud. 2019. Bitcoin Price–Volume: A Multifractal Cross-Correlation Approach. *Finance Research Letters* 31: 374–81. [CrossRef]

Entrop, Oliver, Bart Frijns, and Marco Seruset. 2020. The Determinants of Price Discovery on Bitcoin Markets. *Journal of Futures Markets* 40: 816–37. [CrossRef]

Eom, Yunsung. 2021. Kimchi Premium and Speculative Trading in Bitcoin. *Finance Research Letters* 38: 101505. [CrossRef]

Epps, Thomas W. 1975. Security Price Changes and Transaction Volumes: Theory and Evidence. *The American Economic Review* 65: 586–97.

Fousekis, Panos, and Dimitra Tzaferi. 2021. Returns and Volume: Frequency Connectedness in Cryptocurrency Markets. *Economic Modelling* 95: 13–20. [CrossRef]

Gandal, Neil, J. T. Hamrick, Tyler Moore, and Tali Oberman. 2018. Price Manipulation in the Bitcoin Ecosystem. *Journal of Monetary Economics* 95: 86–96. [CrossRef]

Gemici, Eray, and Müslüm Polat. 2019. Relationship between Price and Volume in the Bitcoin Market. *The Journal of Risk Finance* 20: 435–44. [CrossRef]

Gurrib, Ikhlaas, and Firuz Kamalov. 2022. Predicting Bitcoin Price Movements Using Sentiment Analysis: A Machine Learning Approach. *Studies in Economics and Finance* 39: 347–64. [CrossRef]

Harris, Milton, and Artur Raviv. 1993. Differences of Opinion Make a Horse Race. *The Review of Financial Studies* 6: 473–506. [CrossRef]

Hau, Liya, Huiming Zhu, Muhammad Shahbaz, and Wuqin Sun. 2021. Does Transaction Activity Predict Bitcoin Returns? Evidence from Quantile-on-Quantile Analysis. *The North American Journal of Economics and Finance* 55: 101297. [CrossRef]

He, Hua, and Jiang Wang. 1995. Differential Information and Dynamic Behavior of Stock Trading Volume. *The Review of Financial Studies* 8: 919–72. [CrossRef]

Ibikunle, Gbenga, Frank McGroarty, and Khaladdin Rzayev. 2020. More Heat Than Light: Investor Attention and Bitcoin Price Discovery. *International Review of Financial Analysis* 69: 101459. [CrossRef]

Jaquart, Patrick, David Dann, and Christof Weinhardt. 2021. Short-Term Bitcoin Market Prediction Via Machine Learning. *The journal of Finance and Data Science* 7: 45–66. [CrossRef]

Jennings, Robert H., Laura T. Starks, and John C. Fellingham. 1981. An Equilibrium Model of Asset Trading with Sequential Information Arrival. *The Journal of Finance* 36: 143–61. [CrossRef]

Ji, Suhwan, Jongmin Kim, and Hyeonseung Im. 2019. A Comparative Study of Bitcoin Price Prediction Using Deep Learning. *Mathematics* 7: 898. [CrossRef]

Karpoff, Jonathan M. 1987. The Relation between Price Changes and Trading Volume: A Survey. *Journal of Financial and Quantitative Analysis* 22: 109–26. [CrossRef]

Katsiampa, Paraskevi, Konstantinos Gkillas, and François Longin. 2018. Cryptocurrency Market Activity During Extremely Volatile Periods. Available online: https://ssrn.com/abstract=3220781 (accessed on 20 January 2023).

Khalfaoui, Rabeh, Giray Gozgor, and John W. Goodell. 2023. Impact of Russia-Ukraine War Attention on Cryptocurrency: Evidence from Quantile Dependence Analysis. *Finance Research Letters* 52: 103365. [CrossRef]

Kristoufek, Ladislav. 2015. What Are the Main Drivers of the Bitcoin Price? Evidence from Wavelet Coherence Analysis. *PLoS ONE* 10: e0123923. [CrossRef]

Lau, K.-M, and Hengyi Weng. 1995. Climate signal detection using wavelet transform: How to make a time series sing. *Bulletin of the American Meteorological Society* 76: 2391–402. [CrossRef]

Llorente, Guillermo, Roni Michaely, Gideon Saar, and Jiang Wang. 2002. Dynamic Volume-Return Relation of Individual Stocks. *The Review of Financial Studies* 15: 1005–47. [CrossRef]

Marella, Venkata, Bikesh Upreti, Jani Merikivi, and Virpi Kristiina Tuunainen. 2020. Understanding the Creation of Trust in Cryptocurrencies: The Case of Bitcoin. *Electronic Markets* 30: 259–71. [CrossRef]

McNally, Sean, Jason Roche, and Simon Caton. 2018. Predicting the Price of Bitcoin Using Machine Learning. Paper presented at the 2018 26th Euromicro International Conference on Parallel, Distributed and Network-Based Processing (PDP), Cambridge, UK, March 21–23; pp. 339–43.

Morlet, Jetal, Georges Arens, Eliane Fourgeau, and Dominique Glard. 1982a. Wave propagation and sampling theory; Part II, Sampling theory and complex waves. *Geophysics* 47: 222–36. [CrossRef]

Morlet, Jetal, Georges Arens, Eliane Fourgeau, and Dominique Glard. 1982b. Wave propagation and sampling theory—Part I: Complex signal and scattering in multilayered media. *Geophysics* 47: 203–21. [CrossRef]

Naeem, Muhammad, Elie Bouri, Gideon Boako, and David Roubaud. 2020. Tail Dependence in the Return-Volume of Leading Cryptocurrencies. *Finance Research Letters* 36: 101326. [CrossRef]

Nguyen, Tung, and Tian-Xiao He. 2015. Wavelet Analysis and Applications in Economics and Finance. *Research & Reviews: Journal of Statistics and Mathematical Sciences* 1: 22–37.

Özdemir, Onur. 2022. Cue the Volatility Spillover in the Cryptocurrency Markets During the COVID-19 Pandemic: Evidence from Dcc-Garch and Wavelet Analysis. *Financial Innovation* 8: 1–38.

Phiri, Andrew. 2022. Can Wavelets Produce a Clearer Picture of Weak-Form Market Efficiency in Bitcoin? *Eurasian Economic Review* 12: 373–86. [CrossRef]

Polanco-Martínez, Josué M., Javier Fernández-Macho, and Martín Medina-Elizalde. 2020. Dynamic Wavelet Correlation Analysis for Multivariate Climate Time Series. *Scientific Reports* 10: 21277. [CrossRef]

Ramsey, James B. 2002. Wavelets in Economics and Finance: Past and Future. *Studies in Nonlinear Dynamics & Econometrics* 6: 1–27.

Sahoo, Pradipta Kumar, Dinabandhu Sethi, and Debashis Acharya. 2019. Is Bitcoin a near Stock? Linear and Non-Linear Causal Evidence from a Price–Volume Relationship. *International Journal of Managerial Finance* 15: 533–45. [CrossRef]

Sapuric, Svetlana, Angelika Kokkinaki, and Ifigenia Georgiou. 2022. The Relationship between Bitcoin Returns, Volatility and Volume: Asymmetric Garch Modeling. *Journal of Enterprise Information Management* 35: 1506–21. [CrossRef]

Sovbetov, Yhlas. 2018. Factors Influencing Cryptocurrency Prices: Evidence from Bitcoin, Ethereum, Dash, Litcoin, and Monero. *Journal of Economics and Financial Analysis* 2: 1–27.

Szetela, Beata, Grzegorz Mentel, Yuriy Bilan, and Urszula Mentel. 2021. The Relationship between Trend and Volume on the Bitcoin Market. *Eurasian Economic Review* 11: 25–42. [CrossRef]

Theiri, Saliha, Ramzi Nekhili, and Jahangir Sultan. 2023. Cryptocurrency Liquidity During the Russia–Ukraine War: The Case of Bitcoin and Ethereum. *The Journal of Risk Finance* 24: 59–71. [CrossRef]

Torrence, Christopher, and Gilbert P. Compo. 1998. A practical guide to wavelet analysis. *Bulletin of the American Meteorological Society* 79: 61–78. [CrossRef]

Verona, Fabio. 2016. Time–Frequency Characterization of the US Financial Cycle. *Economics Letters* 144: 75–79. [CrossRef]

Wang, Guizhou, and Kjell Hausken. 2022. A Bitcoin Price Prediction Model Assuming Oscillatory Growth and Lengthening Cycles. *Cogent Economics & Finance* 10: 2087287.

Wang, Jiang. 1994. A Model of Competitive Stock Trading Volume. *Journal of Political Economy* 102: 127–68. [CrossRef]

Wang, Junpeng, Yubo Xue, and Minghao Liu. 2016. An Analysis of Bitcoin Price Based on Vec Model. Paper presented at the 2016 International Conference on Economics and Management Innovations, Wuhan, China, July 9–10; Amsterdam: Atlantis Press, pp. 180–86.

Wang, Pengfei, Wei Zhang, Xiao Li, and Dehua Shen. 2019. Trading Volume and Return Volatility of Bitcoin Market: Evidence for the Sequential Information Arrival Hypothesis. *Journal of Economic Interaction and Coordination* 14: 377–418. [CrossRef]

Yarovaya, Larisa, and Damian Zięba. 2022. Intraday Volume-Return Nexus in Cryptocurrency Markets: Novel Evidence from Cryptocurrency Classification. *Research in International Business and Finance* 60: 101592. [CrossRef]

Youssef, Mouna. 2022. What Drives Herding Behavior in the Cryptocurrency Market? *Journal of Behavioral Finance* 23: 230–39. [CrossRef]

Zhang, Wei, Pengfei Wang, Xiao Li, and Dehua Shen. 2018a. Multifractal Detrended Cross-Correlation Analysis of the Return-Volume Relationship of Bitcoin Market. *Complexity* 2018: 8691420. [CrossRef]

Zhang, Wei, Pengfei Wang, Xiao Li, and Dehua Shen. 2018b. Some Stylized Facts of the Cryptocurrency Market. *Applied Economics* 50: 5950–65. [CrossRef]

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