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Dependence Structure and Time–Frequency Impact of Exchange Rates on Crude Oil and Stock Markets of BRICS Countries: Markov-Switching-Based Wavelet Analysis

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Abstract: This paper used the Markov-switching (MS)-based wavelet analysis technique to study the dependence structure and the time–frequency impact of exchange rates on crude oil prices (West Texas Intermediate (WTI)) and stock returns. Daily data from 1 January 2005 to 1 March 2020 were collected for exchange rates, crude oil prices, and the BRICS stock market returns. The findings indicate that crude oil prices display higher volatility compared to stock returns and exchange rates. Furthermore, the wavelet analysis reveals consistent changes in the co-movement patterns of both volatility regimes, albeit with some variations in the time periods and frequency domains. The time–frequency dependence between Brazilian, Indian, and Chinese stock markets and crude oil is significantly influenced by exchange rates, which play a pivotal role in their co-movement in the medium term. The findings reveal that these three countries share economic interests, have strong economic ties and interdependencies, and may be motivated to cooperate during crisis periods. However, when it comes to Russia and South Africa (SA), exchange rates do not exhibit a long-term impact on the co-movement in time–frequency. Therefore, we recommend investors to look for investment opportunities that are less correlated with the co-moving markets.

Keywords: dependence structure; Markov-switching; wavelet; BRICS; crude oil prices; stock markets; foreign exchange rates

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1. Introduction

The outbreak of the global financial crisis (GFC) and the European debt crisis (EDC) brought a main issue that needed further inquiry in financial economic literature, such as the role of specific financial markets in the propagation of risks. The theory of financial risk management shows tail dependence as an important and useful tool to determine whether two markets co-move successfully or crash together. Following the GFC and EDC, researchers and scientists became more aware of the need to develop theoretical and empirical understanding in response to the risk linked from the market dependence. Basher et al. (2012) used a structural vector auto-regression model to examine the dynamic relationship between exchange rates, stock markets, and the oil price variables. Chkir et al. (2020) used vine copulas to examine the multivariate dependence between oil prices, exchange rates, and equity markets in oil-exporting and oil-importing countries. Roubaud and Arouri (2018) employed a multivariate Markov-Switching Vector Autoregressive (MS-VAR) model to add to the existing body of research regarding the interplay among oil prices, stock markets, and exchange rates while considering the impact of uncertain economic policies.

Tiwari et al. (2019a, 2019b) and Tiwari et al. (2020) are among the prior studies that examine the dependence structure and the systemic risk, respectively, between the return series of oil prices and the BRICS equity market indices and also between oil prices and exchange rates. Using the wavelet coherence approach, He et al. (2021) examined the causal relationship between Turkish stock market returns (XU100) and foreign exchange rates (USD/TRY and EUR/TRY). Aloui et al. (2013) examined the conditional dependence structure between crude oil prices and US dollar exchange rates.

Oil is an energetic commodity that is strategic for all the economies of the world. It motivates and interconnects with virtually every important sector of the world economy. For the past few decades, oil prices have extensively increased and decreased with varying desires over relatively short periods of time. In recent years, oil markets have been financially quoted as a result of increased exposure to different sets of markets. Various financial instruments, including exchange-traded funds, options, futures, and index funds, contribute to facilitating this process. In another view, investors use the oil asset to improve returns, diversify portfolios, and hedge against inflation since this oil is taken as a resource stock and a store of value. All these characteristics brought oil markets nearer to stock markets, and global forces have amplified their connectedness (Mensi et al. 2017).

The volatility of energy commodity prices, particularly oil, significantly influences the performance of macroeconomic variables (Delgado et al. 2018). The oil price is considered an eminent indicator of movements in the exchange rate in the worldwide economy (Amano and van Norden 1998). Krugman (1983) is one of the prior researchers who suggested the theoretical literature on the crude oil–exchange rate relationship. Several studies based on empirical evidence have investigated the connection between oil prices and exchange rates in both developed and emerging economies during a previous time frame (Basher and Sadorsky 2016; Zhang et al. 2016; Huang et al. 2017; Jain and Biswal 2016).

The fundamental question remains to identify the link between commodities (crude oil) and stock market relationships, taking into account the exchange rate as the control variable for portfolio optimization in time localization. Arfaoui and Rejeb (2017) used a simultaneous equation system to identify direct and indirect associations and examine a global perspective on relationships among oil, gold, the US dollar, and stock prices from 1995 to 2015. The results indicate some significant interactions between the interested variables. Certainly, there is a negative link between oil and stock prices; therefore, the oil price is affected positively and significantly by the stock markets. International investors' attention is drawn to commodity markets not only as a financial risk hedge but also as a safe haven to hedge against economic risks. Nevertheless, it is taken as an alternative investment, with a greater sense of certainty observed during periods of financial market turmoil (Baur and McDermott 2010).

The dependence structure and source of co-movement between energy commodities and equity markets raise concerns about finding a mechanism for the spread of shock between two markets in distress or calm periods. The challenge faced in this research field is to identify the link between commodities (crude oil) and stock markets in a relationship and the influencing effect of the exchange rate as the controlling variable. This raises the need to show the importance of time localization in assessing correlation and coherence in association with the exchange rate as a control variable. This implies rebalancing hedging and portfolio weights according to the states of the economy (calm and turbulent periods) in the time scale and frequency domain. Authors like (Tiwari et al. 2020; Tiwari et al. 2019b; Ahmad et al. 2018) have studied the dependence structure and systemic risk of oil and equity markets and advanced some important findings showing that shocks and the transmission of risk bring instability to the market and to the financial system in general. From this perception, what major role is played by exchange rates in the transmission of shocks between crude oil and stock markets?

The implications of the wavelet approach in economics are used in order to clarify some relationships between several macroeconomic variables. On the one hand, a wavelet gives an isolated area where the co-movement exists in timescale and frequency and persists. It allows the description of the local behavior of heterogeneous market participants. Certainly, some market participants have an investment prospect of several minutes or hours to several days, weeks, months, and several years (e.g., short-term movements, medium-term movements, and long-term movements of the stock markets). Yousefi et al.

(2005) said that wavelet analysis gives a clear understanding of the spillover effects across commodity markets and international stock markets and reveals the prospective existence of contagion. On the other hand, the wavelet is useful for portfolio diversification and risk management. Truly, identifying the timescales where the relationship is lower may ensure the profits of portfolio diversification for investors who are looking for alternative investment opportunities (Benhmad 2013).

We are motivated to demonstrate the significance of the relationship pattern and the impact of exchange rates on the simultaneous co-movement of stock markets and crude oil. We specifically focus on examining this connection in terms of time scales and frequency domains. The objective of this study relies on the impact of the exchange rate as a control variable to capture the multiscale features distinguished in periodicity and timescale of dependence between the stock and crude oil markets during regimes of low and high volatility, showing the ability of wavelet analysis to explain hidden patterns such as the GFC crisis.

Several studies demonstrated the usefulness of wavelet analysis, showing the outperformance of the wavelet method compared to other traditional methods (Altun and Tatlidil 2016; Ismail et al. 2016; Tan et al. 2010; Conejo et al. 2005).

To the best of our knowledge, no paper to date has examined the dependence structure and the time–frequency impact of exchange rates on crude oil and stock returns of BRICS countries under different market conditions (lower- and higher-volatility regimes). The volatility in oil prices has the potential to impact the stock market, and this influence can be either positive or negative, depending on the fluctuations in exchange rates. Our study differs from prior research in that we analyze the impact of exchange rates on the interdependence of crude oil and the BRICS stock markets at different time intervals during the sample period. The main contribution of this study relies on the related literature, including highlighting the usefulness of the wavelet methodology. The investigation of the relationship between crude oil and the stock market in BRICS countries incorporates the use of exchange rate time series as a control variable. This analysis reveals the dynamic nature of the interconnections between these variables, both in terms of their evolution over time and their variation across different frequency.

This paper differs from and adds to the existing literature on crude oil–stock market co-movements in four aspects. First, it analyzes the utilization of crude oil as a standard measure within oil markets, symbolizing oil extracted in the United States, and explores the connections between the BRICS stock market and the economies of various states during periods of both stability and turmoil, while considering exchange rates as the controlling factors. Second, we further employ a novel method of partial wavelets and multiple wavelets to identify the effects of exchange rates on the relationships between stock market indices in the BRICS and crude oil over time and in the frequency domain. Third, to identify isolated shocks from crude oil to stock markets in the frequency period, by identifying the leading (lagging) variables between the stock market and crude oil, justifying the fact that various stock markets can be affected differently by oil price changes. Fourth, and finally, we identify the heterogeneity of stock indices sensitive to the fluctuations of oil prices, which has significant implications for portfolio risk assessment and asset allocations. Hence, exploring the increasing interactions between crude oil prices and the stock markets of the BRICS countries, by considering the exchange rate as a control variable, this paper may be one of the first to investigate the dependence structure and the time–frequency impact of exchange rates on the co-movement between crude oil and the stock indices. This study differs from other studies that examine the relationships between the three variables (stock, oil price, and exchange rates) (Roubaud and Arouri 2018; Delgado et al. 2018; Basher et al. 2012; Chkir et al. 2020).

The structure of this paper is divided into several sections. In Section 2, the literature review is presented, and Section 3 outlines the methodology used in the study. Section 4 focuses on the results obtained and their interpretation. Lastly, Section 5 offers conclusions and recommendations for policy.

2. Literature Review

The financial markets have become exceedingly volatile in the past decade, especially during the GFC in 2008, the European debt crisis in 2009, and COVID-19, which dropped the stock markets globally. This has drawn much attention from researchers and scientists trying to measure the transmission risks and look at how to control their spread across markets. The European Central Bank (ECB) (ECB 2011) shows evidence of threats from financial systemic risk to the function of a financial system. The treat is considered to be one causing many participants to suffer serious losses in the market and is rapidly transmitted into the financial system (Benoit et al. 2017).

Market dependency is one of the main channels through which risks are transmitted from one market to another. Many studies investigated the dependence structure between the energy commodity crude oil and the stock markets, and diversified findings were obtained. Pastpipatkul et al. (2015) used C-vine copula and D-vine copula to examine the co-movement and dependence between the oil market, the gold market, and the stock market. This method enabled the capturing of correlation and dependence. The findings show that the C-vine copula has a better structure than the D-vine copula. Furthermore, there is a positive relationship between the London Stock Exchange and the other markets; however, when the London Stock Exchange, the Dow Jones Industrial Average, and Brent oil were used as the conditions, complex results were obtained. Finally, the result shows that gold could be a safe haven in these portfolios. Ji et al. (2020) examined the dynamic dependence and risk spillover between different types of oil shocks and BRICS stock returns. The results showed, after using the structural VAR and the time-varying copula-GARCH-based CoVaR approach, an indication of the dependence between oil shocks and BRICS stock returns. These results presented different behaviors reliant on the shock from the oil market and a substantial risk spillover from the oil-specific demand shock to the stock returns of BRICS countries. In their study, Wu et al. (2020) used partial- and multiple-wavelet coherence analyses to investigate the relationship between international stock markets while also considering the influence of crude oil from a time domain perspective. The study revealed that crude oil plays a significant role in driving co-movement between international stock markets, particularly in the medium and long term. However, the impact of crude oil on co-movement in oil-importing or oil-exporting countries was found to be comparatively lower, suggesting the presence of other influencing factors. Lastly, the study suggests that the stock market of the Gulf Cooperation Council has the potential to outperform the stock markets of oil-importing countries in the long term. Soni et al. (2023) used wavelet-based quantile and wavelet-based Granger causality to investigate the causal relationship and causality between economic policy uncertainty (EPU) and markets. According to the findings, when negotiating oil deals in the short and medium term, Indian crude oil buyers do not need to take into account Indian EPU. Nonetheless, persistent uncertainty can make securing lower-cost oil deals challenging. EPU causes unfavorable instabilities because macroeconomic decisions have a significant impact on the stock market. They also note that gold is a reliable indicator of inflation caused by uncertainty and a measure of economic imbalances, demonstrating a safe-haven characteristic.

According to Mensi et al. (2017), the stock markets of the BRICS countries increased quickly in terms of size and volume of investment, and they attracted investors' attention from both domestic and international markets. Then, the factors changing in the global economy—for example, the international price movements of crude oil—may be a transmission channel for the fluctuations in the world's economic and financial conditions spreading to the BRICS stock markets. The variational mode decomposition (VMD) method and copula functions were used to examine bear, normal, and bull markets across various time frames. The results reveal that oil and all stock markets exhibit tail dependence. Furthermore, when considering the different time horizons, it was found that short-term and long-term horizons exhibit average dependence among the studied markets, while asymmetric market risk spillovers also exist. Mensi et al. (2020) investigated the relationship between crude oil and two stock markets: The Dow Jones Islamic World Index and the conventional Dow Jones Market Index. They used a copula approach to examine dependence and regime-switching. The results of the study suggest that the U.S. Islamic stock market serves as both a hedge and a safe haven against fluctuations in oil prices, while the conventional Dow Jones market only acts as a hedge. The researchers also found that the tail dependence between the Islamic stock and oil markets is lower when the Islamic stock market is bullish and the oil market is bearish. In contrast, the dependence between the conventional stock and oil markets is smallest when the conventional market is bearish and the oil market is bullish. Tiwari et al. (2019b) employed quantile coherency alongside NCoVaR and NCoVaR-Gc to evaluate the interdependence between BRICS equity markets and oil prices, as well as to gauge the level of systemic risk present. The results show a significant long-term dependence and coherency between oil shifts and the Brazilian, Russian, and SA stock markets. Tiwari et al. (2020) examined the dependence and systemic risk between oil and stock market indices in G7 economies. Using Markov–copula models, the results show evidence that oil price dynamics contribute substantially more to the G7 stock market returns during turbulent times than during calm periods.

Laeven et al. (2016), Acharya et al. (2012), and Jiang and Yoon (2020) are among the researchers who have contributed empirically to the literature that deals with studies of the dependence structure of markets and the co-movement between crude oil and the stock markets. An extreme Granger causality analysis model was used to uncover the causal relationships between crude oil and BRICS stock markets by decomposing the data into three cumulative components. The empirical results revealed that the effect of oil price changes on the stock markets is stronger under extreme circumstances than under normal circumstances, causing oil price changes to have an asymmetric effect on extreme stock price movements (Wang et al. 2020). Mensi et al. (2018) studied the co-movements between the stock market index of BRICS countries and the energy commodity prices (crude oil, Brent, and gold prices). Using the wavelet approach, the results indicate that the BRICS index returns co-move with the WTI crude oil price at low frequencies. Besides that, there is a strong level of co-movement especially captured in the GFC. They also found no evidence of a relationship between the gold price and the BRICS stock markets, showing that gold could be considered a hedge for the BRICS against extreme market movements. Mensi (2019) used a wavelet approach and a VaR measure to investigate the dynamic co-movements and portfolio risk management between crude oil and the sectorial stock markets of Saudi Arabia. The findings show substantial co-movements between crude oil and stock sectoral markets over time and across frequencies, which were more significant during the GFC. The analysis of risk influenced by the cross-market co-movement is a significant task to investigate in the crisis period. The substantial indication of co-movement between the Saudi stock market and crude oil is affected by the rising oil price movements, with a higher risk of low frequency. The negative impact of oil prices on the stock markets was investigated by Kilian and Park (2009), while Narayan and Narayan (2010) investigated the positive shocks of oil on stock markets. Wei and Guo (2017) used structural VAR to examine the effects of China's stock market and oil price shocks, and the findings showed the instability of the relationship between both markets for the full sample. Kang et al. (2015) used structural VAR to investigate the shocks between the oil price and the US stock market. The results exhibited positive shocks related to the oil market and aggregate demand and negative effects linked to a specific aggregate demand. Aloui et al. (2012) studied the effects of oil price shocks on the stock market in developing countries. Based on the empirical analysis of the conditional multifactor pricing model and long-term correlation, the results indicate that oil price risk is meaningfully priced in developing markets and that oil has an asymmetric impact with respect to market phases. Ogiri et al. (2013) used the vector error correction model (VECM) and vector auto-regressive (VAR) to examine the link between oil price and stock market performance in Nigeria, and the findings showed a substantial relationship between oil prices and the stock market, indicating that the variations are important factors to explain the oil price movement. The relationship between crude oil (WTI) and the stock markets of G7 countries was investigated by Khalfaoui et al. (2015) using a wavelet-based multivariate generalized autoregressive conditional heteroskedasticity (MGARCH) method. The findings showed that there were considerable volatility spillovers between the two markets and that the pairs had time-varying relationships across various markets. Nevertheless, the results of wavelet coherence designate the leading crude oil market. An indication of low co-movement, with the exception of South Africa and Egypt, was found in the study of Gourène and Mendy (2018). They investigated the co-movement between the most active African stock markets and the oil prices of the Organization of Petroleum Exporting Countries. Yıldırım et al. (2018) examined the dynamic link between global crude oil prices and stock prices for BRICS economies using the MS-VAR model. The findings indicated that the response of the stock market to the oil price shock fluctuates over the regimes for all countries and is precisely positive and statistically substantial in the high regime of volatility, with the exception of China. Consequently, these results propose that the rising oil prices may be assessed as a demand-side shock in these countries.

Some important studies make a number of contributions to the literature by investigating the dependence structure and the co-movement between crude oil and foreign exchange rates. Wen et al. (2020) investigated the spillover effects of crude oil prices and exchange rates across seven major oil-exporting and oil-importing countries from 2000 to 2018. Their findings revealed the presence of both upside and downside risk spillovers, which were found to be more pronounced from exchange rates to crude oil markets as opposed to the other way around. The researchers also noted that the oil-exporting countries experienced stronger risk spillovers compared to the oil-importing countries. Additionally, they identified a significant extreme risk dependence between the two markets following the financial crisis of 2008–2009. The long-term correlation between exchange rates and oil prices, along with their influencing factors, was evaluated by Yang et al. (2018) using dynamic conditional correlation–mixed data sampling (DCC-MIDAS) on the exchange rates of Japan, Canada, Germany, and Eurozone-based USD and crude oil data. According to Ferreiro (2020), the issue in the literature lies in how the stock market responds to a distressed scenario caused by oil prices in the local currency. To address this, he proposed creating an oil-related scenario that takes into account the source of risk. In contrast, the European stock market reacts differently to the same oil-related scenario, depending on the cause of the risk. The study showed that when oil prices in Euros decline due to a drop in oil prices in USD or when oil prices in Euros rise due to a decrease in the value of the Euro against the USD, Eurostoxx experiences higher losses. To investigate the conditional dependence structure between crude oil prices and US dollar exchange rates, Aloui et al. (2013) employed a copula–GARCH methodology. Their analysis revealed considerable and symmetrical dependence in nearly all of the oil–exchange rate pairs examined during the period from 2000 to 2011. The depreciation of the US dollar has been linked to a rise in oil prices, and even when employing different GARCH-type specifications and during the crisis period, the key findings remain consistent. In their study, Wu et al. (2012) employed dynamic copula-based GARCH models to analyze the relationship between the oil price and the US dollar exchange rate. They also tested the effectiveness of these models using an asset allocation strategy and assessed their economic value. Despite detecting a notable signal in the oil market, this information did not enhance the profits of the asset allocation decision. Ma and Yang (2020) analyzed the correlation between oil and the exchange rate by conducting partial and multiple-wavelet coherence analyses. They found that the co-movement is largely driven by the monetary facilitation policies of the Federal Reserve System (FED). By excluding the effects of the FED's monetary policy, they discovered that the Euro is highly dependent on crude oil price changes, whereas the Japanese yen is the least dependent. The British pound shows moderate dependence, while the Chinese yuan exhibits a strong co-movement only over a long period, indicating a low degree of integration with global markets. Tiwari et al. (2019a) used quantile coherency methods and the NCoVaR-Gc test to investigate the systemic risk and the dependence structure between the exchange rates of BRICS-based USD and crude oil prices. The results showed evidence in the long-run dynamics revealing substantial negative dependence between oil prices and the Indian, South African, and Brazilian currencies.

Studies on market connections are as follows: Mensi et al. (2023a) explored the relationship between crude oil prices and stock market returns in three developed economies (Canada, Japan, and the USA) and five emerging economies (the BRICS economies). The bivariate and multivariate wavelet approaches were used, and the findings demonstrate that there is time–frequency co-movements between the examined markets, especially at medium and low frequencies. Additionally, the research indicates that the co-movements intensify during global financial crises and the COVID-19 pandemic, supporting the recoupling hypothesis. The risk analysis shows that co-movements are persistent, and there is a dependence on portfolio risk in the BRICS economies and across markets during turmoil periods. Mensi et al. (2023b) investigated how African stock markets and oil are linked in three different market conditions: bearish, normal, and bullish. Using the quantile connectedness approach, the findings reveal that during bearish market conditions, spillovers are more significant than in calm and bullish market conditions, and oil plays a vital role in transmitting these spillovers to African markets. Additionally, the findings reveal that certain African nations, such as Ghana, Kenya, Nigeria, and South Africa, are net receivers of spillovers, while others, including Tunisia, Egypt, Morocco, and Mauritius, are net transmitters of lower-quantile spillovers. The spillover effects were highest in the initial phase of the COVID-19 pandemic in early 2020. The examination of portfolios demonstrates that an appropriately weighted portfolio is the most efficient approach to minimizing downside risks in all markets, whereas a hedged portfolio provides the most reliable risk mitigation. Wu et al. (2023) add to the previous literature on the relationship between commodities and financial markets at different time scales, with a focus on the impact of economic policy uncertainty (EPU). The findings show strong evidence of short-term information, volatility, and risk transmission between commodity and financial markets, with these markets serving as primary shock transmitters and receivers, especially during times of crisis. They also find that EPU has a substantial impact on market interactions in both static and switching regimes. Cagli and Mandaci (2023) investigated the relationship between uncertainty in cryptocurrency, stock, currency, and commodity markets, using the innovative cryptocurrency uncertainty indices and global implied volatility indices to gauge uncertainty proxies for the energy, precious metals, currency, and stock markets. The findings reveal a minimal degree of uncertainty connectedness between cryptocurrency and other markets. Consequently, the results suggest that there are opportunities for long-term diversification and that the dynamics of the cryptocurrency markets are distinct.

The relationship that exists between stock markets and foreign exchange rates plays an important role in the transmission of risk. Michelis and Ning (2010) used a symmetric Joe Clayton (SJC) copula model to examine the dependence structure between real Canadian stock returns and real USD/CAD exchange rate returns with monthly data from 1995 to 2006. The findings show substantial tail dependence between both markets and a static and dynamic asymmetry, with more dependence in the left tail than in the right tail. Warshaw (2020) investigated the volatility spillovers between foreign exchange markets and European equity during the 2003–2019 period. They found causal relationships in realized volatility across the frequency domain; asymmetric and bidirectional volatility spillovers across the frequency domain; and a substantial spread of shocks from equity to foreign exchange markets at low, mid-range, and high frequencies. In the global financial crisis, volatility spillovers were principally unidirectional. In their study, He et al. (2021) used the wavelet coherence approach to investigate the causal link between Turkish stock market returns (XU100) and foreign exchange rates. The results indicate that the exchange rate experienced significant volatility during the banking crisis in 2000, the economic crisis in 2001, and the exchange rate crisis in 2018. Similarly, the stock market demonstrated significant volatility during the banking crisis, economic crisis, and global crisis in 2008. Additionally, a negative correlation between the Turkish stock market and foreign exchange rates was identified across various frequency domains. Lin et al. (2021) looked into risk spillovers and hedge strategies between global crude oil markets and stock markets. A GARCH framework with multivariate long memory and asymmetry was used. In the short term, the results indicate that there are linear risk spillovers from the US stock markets to the WTI oil market. The linear risk spillover effect from the oil market to the US stock market, on the other hand, can only exist in the long term. Finally, dynamic hedge effectiveness demonstrates that the method employed appears to be an acceptable and viable method of carrying out hedge strategies between global crude oil markets and stock markets. Wu et al. (2021) investigated the relationship between foreign exchange and general financial markets, with a particular focus on the extreme effects of spillover from foreign exchange to general financial markets. To investigate this relationship in the G7 countries, the authors employed asymmetric time-varying copula models and copula-based CoVaR approaches. The results of the copula estimations suggest that there is an asymmetric tail dependence and a positive (or negative) correlation between the Canadian currency and the stock markets in Japan and the United States. Moreover, the study found evidence of both downside and upside spillovers in most G7 countries, with downside spillovers being more prevalent than upside spillovers, especially in the stock market.

Among studies that investigate the relationship between oil prices, stock markets, and exchange rates, none of them considered the exchange rate as a control variable in driving the dependence between stock and oil. For example, Chkir et al. (2020) used vine copulas to examine the multivariate dependence between oil prices, exchange rates, and equity markets in oil-exporting and oil-importing countries. Except for the Japanese Yen and British Pound exchange rates, the findings show that the dependence between oil and exchange rates is substantially negative over different time periods, implying that oil may be a poor hedge against currency fluctuations. Basher et al. (2012) used a structural vector auto-regression model to examine the dynamic relationship between exchange rates, stock markets, and oil price variables. It is indicated that in the short term, positive shocks to oil prices, specifically, tend to reduce emerging market stock prices and US dollar exchange rates. A positive shock to oil production sinks oil prices, whereas a positive shock to real economic activity raises oil prices, and rising stock prices in emerging markets raise oil prices and US dollar exchange rates. A positive shock to oil production sinks oil prices, whereas a positive shock to real economic activity raises oil prices, and rising stock prices in emerging markets raise oil prices. Aloui and Aïssa (2016) used the vine copula approach to examine the dynamic relationship between energy, stock, and currency markets using a sample of more than ten years of daily return observations of WTI crude oil, the Dow Jones Industrial Average stock index, and trade-weighted US dollar index returns. The findings indicate that these variables have a substantial and symmetric relationship. Taking different sample periods reveals that the dynamic of the return link is changing over time. Moreover, the findings also suggest that the financial crisis has had a significant impact on the dependency structure. Kayalar et al. (2017) mentioned that oil price variations have fluctuating effects on the economies and financial indicators of the global markets. They investigate the dependence structure of some developing countries among crude oil prices, stock market indices, and crude oil and exchange rates through copula models. The results show substantial effects of the global crisis, and the stock markets and exchange rates of the country's largest oil exporter demonstrate higher oil price dependence, while emerging oil importer markets are less susceptible to price instabilities. Roubaud and Arouri (2018) made a contribution to the existing literature on the relationships among oil prices, stock markets, and exchange rates by using a multivariate MS-VAR model that takes into account the impact of uncertain economic policy. The findings suggest that there are significant interconnections between the three variables and that there may be non-linear relationships between them. However, the relationships between the variables differ across different periods, with more pronounced effects during times of crisis and high volatility. As a result, oil appears to have a dynamic role in the transmission of price shocks to both the stock and exchange rate markets. According to Raheem and Ayodeji (2016), the impact of fluctuations in the exchange rate, oil prices, and the Nigerian stock market revealed that oil and stock are not co-integrated.

Among the studies that showed the usefulness of wavelet compared to other traditional methods, we quote the following: Altun and Tatlidil (2016) employed a wavelet-based GARCH-Extreme Value Theory (EVT) approach to predict daily value-at-risk using data from the ISE-100, the selected stock exchange in Turkey (International Security Exchange), alongside real data from the S&P-500 index and the Nikkei-225. Through backtesting, the empirical findings demonstrate that the wavelet-based GARCH-EVT model exhibits superior performance at higher quantiles. Conejo et al. (2005) introduced an innovative approach to predicting day-ahead electricity prices in mainland Spain's electric energy market. Based on the wavelet transform with ARIMA models using the inverse wavelet transform for the predicted constituent series, they were able to generate precise forecasts for the original price series. The findings revealed that the wavelet technique consistently outperformed the direct application of ARIMA models throughout the study period. These results highlight the effectiveness and practicality of the proposed wavelet, showcasing its potential utility in forecasting day-ahead electricity prices. In a study conducted by Ismail et al. (2016), the performance of two models, namely the GARCH(1,1) model and the newly suggested MODWT-GARCH(1,1) model, was compared using daily returns from four African stock market indices. While both models demonstrated a good fit to the returns data, the forecast generated by the GARCH(1,1) model was found to underestimate the observed returns. On the other hand, the newly proposed MODWT-GARCH(1,1) model accurately predicted the observed returns, presenting a more reliable forecasting value. Tan et al. (2010) investigated the FTSE-Bursa Malaysia Emas Sharia'h Index (FBEMAS) using a novel price forecasting method based on the wavelet transform combined with ARIMA and GARCH models. The findings reveal that the wavelet transform produces constitutive series that are more accurately predicted than the other forecast methods.

From the above literature conducted, we observe that several studies used different techniques to investigate the relationships between stock market indexes and oil prices; some studies investigated the relationships between stock market indexes and exchange rates, the exchange rate versus oil prices, or the interrelationships between the three variables in the spread of spillover risk. Among the studies mentioned above, none of them take into account the implication of the exchange rate on the co-movement between the stock markets of the BRICS and the crude oil in the states of the economies. To see the importance of this challenge, this study will extend the paper of Roubaud and Arouri (2018), who just investigated the interrelationships between oil prices, exchange rates, and stock markets. As outlined in the introduction section, our approach will deviate in four distinct aspects. We contribute to the existing literature by examining the dependence structure and the time-frequency impact of the exchange rates on the co-movement between crude oil and the stock markets of the BRICS economies in time scale and frequency domain. This study uses Markov-switching-based wavelet analysis similar to Wei and Yanfeng (2017). We applied the wavelet technique to remove the noise from oil price, stock market, and exchange rate fluctuations. Wavelet analysis is particularly advantageous when working with non-stationary signals that display varying frequencies over time, such as financial market data. This method not only reduces noise effectively but also portrays the non-stationary nature of the signal accurately. For more details, see (Aguiar-Conraria et al. 2008; Roueff and Von Sachs 2011). Unlike other techniques, this approach enables the detection of co-movements between the variables at various scales, which can cater to the interests of investors looking for both short-term and long-term trends. The benefits of wavelet methods in comparison to conventional approaches usually vary based on the distinct application and the properties of the data under analysis. The ability to explain hidden patterns such as the GFC crisis. This is due to their capability to capture temporal and spectral information, as well as spatial information, which makes them valuable in examining data that exhibit non-stationary and irregular features, such as biomedical signals and financial data. See for example Jammazi (2012).

3. Methodology

3.1. Markov Regime-Switching

Let consider a hidden regime that follows a Markov chain process as described by (Guidolin 2011):

$$y_t = u_{st} + \vartheta y_{t-1} + \varepsilon_s \tag{1}$$

$$y_t = u_{st} + \alpha \sum\nolimits_{t=1}^{3} x_t + z_t \gamma_s + \varepsilon_s \tag{2}$$

The Markov-Switching Autoregressive Model is given by Equation (1) and the Markov-Switching Dynamic Regression Model is given by Equation (2). These models have a fixed transition probability and a time-varying transition probability that is responsive to the changing form of the transition probability from one state to another.

In the above equations, y_t is a vector of observations, x_t is a vector for unknowns, α is the constant coefficient of the states, γ_s is the state-dependent coefficient for the control variables, z_t is a vector of exogenous variables, and ε_s is the state-dependent variance that follows $N(0,\delta_{St})^2$ independently identically distributed random variable (iid).

Following Hamilton (1989), Markov-switching models, along with a variety of alternate variations, have been widely used to study many types of economic and financial time series. According to Bollen et al. (2000), a regime-switching model with an independent shift in the mean and variance displays a closer fit and a more accurate variance forecast than a range of other models. Regime-switching models excel at capturing the time series behavior of a wide range of financial variables.

This model will be applied to model potential changes within the dependence structure of crude oil, stock markets, and the currency exchange rates of the BRICS economies. It was initially introduced by Hamilton (1989). S_t is a state variable of a Markov chain, standing for different hidden regimes of the time-dependent variable. The variable S_t follows a first-order Markov chain, which has a probability transition matrix P, such that each element $P_{i,j}$: $P(S_t = j \backslash S_{t-1} = i)$ gives the probability of being in state j at time t, knowing that at time $t-1$, the state was i, and the sum of each column of P is equal to 1. In the case of two regimes, the state variables take two different values, $S_t = \{1,2\}$, states k = 1 and k = 2, which are also called regimes of high risk and low risk, respectively, and they are given by:

$$P = \begin{pmatrix} p_{1,1} & p_{2,1} \\ p_{1,2} & p_{2,2} \end{pmatrix} = \begin{pmatrix} p & 1-q \\ 1-p & q \end{pmatrix} \tag{3}$$

3.2. Wavelet Multivariate Markov-Switching Model

The selected models are parsimonious and were used before based on constant shock and volatility transmissions. Recently, multivariate Markov-switching models, which are time-varying and state-dependent, were used to solve this problem. The major advantage of the procedure of Markov-switching, frequently supported in the literature, is its ability to consider features such as temporal asymmetries, nonlinear phenomena, and the persistence of macroeconomic time series. These features are important in the analysis of the dynamic relationship between energy commodity prices, currency exchange rates, and stock market returns. To model the transition probabilities, we define them as follows:

$$P_{ij}(t) = \frac{\exp\left(\beta_0^{ij} + \sum_{l=1}^{L} \beta_l^{ij} \omega_t\left(\frac{t}{T}\right)\right)}{1 + \exp\left(\beta_0^{ij} + \sum_{l=1}^{L} \beta_l^{ij} \omega_t\left(\frac{t}{T}\right)\right)} \tag{4}$$

$$i = 1, \ldots, k \text{ and } j = 1, \ldots, k-1$$

where $\omega(.)$ and l represent the wavelet and each pair of scale and location, respectively.

3.3. Wavelet Model

This study used Markov-switching-based wavelet analysis. It is important to separate the states of economies in terms of time scale and frequency. This method is able to first separate the regimes via MS and then apply wavelet analysis to detect the localization of isolated shocks. The wavelet transform overwhelmed the restrictions of the Fourier transform. Nevertheless, the wavelet transform is capable of showing both information in wavenumber and in spatial terms, which are prerequisites for a good understanding of financial time series analysis.

A significant departure from conventional assumptions involves taking into account data with correlated noise. Johnstone and Silverman (1997) extensively examined this matter. While correlated noise can pose challenges for several smoothing methods, the extension is simple in the case of wavelets. The wavelets must be square-integrable and satisfy the admissible condition: $\int_{-\infty}^{\infty} \frac{|\psi(t)|}{|f|} df < \infty$, where $\Psi(f)$ is the Fourier transform of ψ. The admissibility condition is equivalent to requiring $\int_{-\infty}^{\infty} \psi(t) df = 0$. Hence, ψ needs to oscillate along the t-axis, exhibiting wave-like behavior, thereby providing justification for the use of the term wavelet. For more details about the wavelet assumptions, see (Silverman 1999; Antoniadis et al. 1994). However, applying wavelets to a much broader range of statistical problems is critical. One of the most significant advances in statistics in recent decades has been the development and widespread use of generalized linear models (see, for example, McCullagh and Nelder 1989).

The wavelet is a useful method and an estimator that uses signal processing, presenting a single chance to investigate co-movements amongst economic series in the time–frequency dimension. Ferrer et al. (2016) showed that wavelets offer a better understanding of potential interdependence at scales other than one or two scales. The wavelet function is a square integral function containing a real value and a zero as an average value. Its value is given by:

$$\int_{-\infty}^{\infty} \omega(t) dt = 0 \tag{5}$$

ω fluctuates through the time axis like a wave. The wavelet method described the heterogeneity behavior of market participants, which is expressed in minutes or hours, days or months, and years as short-term, medium-term, and long-term movements, respectively.

The wavelet transform implements the energy conservation of the chosen time series. This property accepts the power spectrum analysis with total variance σ_x^2 given by the following expression:

$$\sigma_x^2 = \frac{1}{C_\omega} \int_{-\infty}^{+100} \int_{-\infty}^{\infty} |W_x(u,s)|^2 \frac{duds}{s^2} \tag{6}$$

3.4. Cross-Wavelet Transform

The Cross-Wavelet Transform (XWT) was introduced before by Torrence and Compo (1998). The extensions to wavelet analysis, such as filtering, coherence, power Hovmöller, and cross-wavelet spectra, are described. The expression of the cross-wavelet transforms for both signals x(t) and y(t) is given by:

$$W_{x,y}(u,s) = W_x(u,s) W_y{}^*(u,s) \tag{7}$$

where u, s, and * represent the position, the scale, and the complex conjugate, respectively.

The XWT demonstrates the region in the timescale space where the time series has a higher conjoint power. This timescale space shows the local covariance between both signals at each scale.

3.5. Wavelet Transform Coherence

The wavelet transform coherence (WTC) associates the cross-spectrum technology with the linear correlation and differentiates itself by combining the correlation in the joint of two-time series in the time–frequency domain. The measurement of the wavelet coherence is built on the cross-wavelet transform and the wavelet power spectrum of each time series. Therefore, the equation of the wavelet coherence is given by:

$$R(x,y) = \frac{|s(s^{-1}W_{xy}(u,s))|}{S(s^{-1}|W_x(u,s)|^{1/2})S(S^{-1}|W_y(u,s)|^{1/2})} \tag{8}$$

where S is a smoothing simultaneous process in time and frequency.

3.6. Wavelet-Squared Coherence

The wavelet-squared coherence is used to examine the joint behavior of the time scale and frequency scale. Nevertheless, the cross-wavelet transform among two-time series is merely the multiplication of the first complex wavelet transform with the complex conjugate of the second, as demonstrated in the study of Bloomfield et al. (2004).

Torrence and Webster (1999) assessed the squared absolute number of the smoothed cross-wavelet power spectrum of every time series selected as the expression of the wavelet coherence. As a result, the coefficient representing the squared wavelet is given by:

$$R^2(u,s) = \frac{|(s^{-1}W_{xy}(u,s))|^2}{S(s^{-1}|W_x(u,s)|^2)S(S^{-1}|W_y(u,s)|^2)} \tag{9}$$

where s represents the smoothing parameter. The squared wavelet coherence is in the interval range of $0 \leq R^2(u,s) \leq 1$. When the coefficient tends to zero, there is a weak interdependence; when it is higher, there is a strong interdependence. This coefficient is efficient in identifying the contagion effect of the variables in the study.

3.7. The Phase Patterns

The differencing phase allows assessing the dependence and causality links between the market variables, as it is difficult to detect whether the dependence is positive or negative with the squared wavelet coherence. According to the theory of Mother, the complex wavelet transform can be separated into two parts: the real and the imaginary. The following expression depicts the phase difference between $x(t)$ and $y(t)$, as shown in the study of Bloomfield et al. (2004):

$$\vartheta_{xy} = \arctan\left(\frac{\tau\{S(s^{-1})W_{xy}(u,s)\}}{R\{S\left(S^{-1}\right)W_{xy}(u,s)\}}\right) \tag{10}$$

with $\omega_{xy} \in [\pi, \pi]$, where τ and R represent the imaginary and the real components, respectively, of the smooth cross-wavelet transform. The estimation of the phase pattern based suddenly on the time lag between x(t) and y(t), as mentioned in the studies of (Aguiar-Conraria and Soares 2011; Voiculescu and Usoskin 2012), is expressed as $(\Delta t)_{xy} = \frac{\omega_{xy}}{2\pi f}$, where $2\pi f$ indicates the angular frequency with respect to the timescale s. The time lag is expressed by:

$$(\Delta t)_{xy} = \frac{\omega_{xy}S}{2\pi} \tag{11}$$

In the plot of wavelet coherence, the phase patterns are the direction of the arrows indicating the causality link between two variables (Tiwari et al. 2013).

3.8. Partial Wavelet Coherence

Wavelet coherence is a measure of linear correlations and provides a substantial map when a correlation is real (Gurley and Kareem 1999; Gurley et al. 2003). In this regard, a pure correlation cannot be found between international oil markets and international stock markets because they are influenced by other factors. The application of partial wavelet coherence (PWC) helps to eliminate the influence of time series foreign exchange rates on the wavelet coherence between stock markets in BRICS countries and WTI. Following Ng and Chan (2012), PWC is expressed as follows:

$$R_p^2(x,y,z) = \frac{|R(x,y) - R(y,z) \times R(x,y)^*|^2}{[1 - R(x,y)]^2[1 - R(y,z)]^2} \quad (12)$$

where $R_p^2(x,y,z)$ ranges from 0 to 1 and has the same interpretation as $R^2(x,y)$. Particularly, a low R_p^2 region was seen where a high R^2 region indicated that time series y does not have a significant effect on x. Instead, time series z dominates the variance of x. Both y and z have a significant influence on x, when there is no difference between R_p^2 and R^2. In this paper, x and y denote the stock market returns of BRICS and WTI, while z indicates the foreign exchange rate.

3.9. Multiple-Wavelet Coherence

Multiple-wavelet coherence (MWC) examines the coherence of multiple independents on a dependent, that is, of y and x on z, in this case. Therefore, the application of MWC can be expressed as follows:

$$R_m^2(x,y,z) = \frac{R^2(x,y) + R^2(y,z) - 2R_e[R(y,x) \times R(y,z) \times R(x,z)]}{1 - R^2(x,z)} \quad (13)$$

The above equation calculates the proportional wavelet power of the two independent variables x and y to explain the dependent time series z in the time–frequency domain. The variables x, y, and z are the same as noted in the previous section on partial wavelet coherence. The significance levels are found by using the Monte Carlo method. Because MWC is sensitive to dependent time series z, exceptionally high coherence can be produced when x and y are dependent. The MWC is used as the robust test at this point.

4. Empirical Results and Discussion

4.1. Data

In this paper, we empirically study the dependence structure and time–frequency impact of exchange rates on crude oil and stock markets of BRICS[1] countries sampled by data availability for the crude oil (WTI) stock markets of BRICS countries and their currency exchange rates. The data for crude oil and stock markets were collected from Thompson Reuter's database, and the currency exchange rate data were collected from Yahoo Finance. All sample data range from 1 January 2005 to 1 March 2020, including the global financial crisis, the European debt crisis, and the COVID-19 pandemic. The following formula is used to convert all series to log returns:

$$r_t = \ln\left(\frac{p_t}{p_{t-1}}\right) \times 100 \quad (14)$$

Regime-switching-based wavelet analysis is used in this paper. This model is useful for assessing the dependency of markets on the state of the economy and detecting financial market regime shifts. The wavelet analysis is used to detect the impact of exchange rates on the co-movement between stock returns and crude oil in the frequency domain and over a specific time period. It is a novel model with precise results that is able to capture the co-movement in the frequency domain and time period simultaneously. The model will be executed in four steps: first, the Markov-switching; second, the continuous wavelet transform; third, the multiple-wavelet coherence; and finally, the partial wavelet coherence will be applied in both regimes.

4.2. Baseline Results

The descriptive statistics for equities returns, oil prices, and foreign exchange rates are reported in Table 1. The standard deviation (Std. Dev) represents the volatility; it is higher in crude oil, confirming it to be more volatile than the stock market returns and the foreign exchange rates, as revealed in the study of Tiwari et al. (2019a). The higher fluctuations in oil prices are followed by USDBRL, Bovespa, and MOEX, suggesting the dispersion of volatility across markets. The returns of Brazil, Russia, India, and South Africa are negatively skewed, but China's are not, whereas the crude oil and currency exchange rates are positively skewed, suggesting that the stock market indices' distributions have longer left-hand tails and the exchange rate and oil have longer-right tails. The markets with negative skewed parameters imply that investors may expect frequent small gains and a few large losses.

Table 1. Descriptive statistics of energy commodity prices (crude oil), the stock market indices of BRICS countries, and their respective USD foreign exchange rates against domestic currencies.

	Bovespa	MOEX	Sensex	Hang Seng	FTSE/JSE	USDBRL	USDRUB	USDINR	USDCNY	USDZAR	WTI
Mean	0.00038	0.00033	0.00041	0.00016	0.00034	1.16×10^{-6}	0.00026	0.000124	-5.74×10^{-5}	0.00029	−0.00027
Median	0.00064	0.00027	0.00071	0.00062	0.00084	−0.00014	7.53×10^{-5}	0.000000	0.00000	0.00015	−0.00056
Max	0.15860	0.23060	0.15990	0.13407	0.09057	1.34176	0.029645	0.036858	0.01816	0.00854	2.87619
Min	−0.15994	−0.19800	−0.14102	−0.13582	−0.10450	−0.01724	0.00000	−0.035497	−0.02032	0.00000	−0.26448
Std.Dev.	0.01864	0.01818	0.01410	0.01465	0.01330	0.02084	0.00093	0.004570	0.00167	0.00047	0.06227
Skewn.	−0.05612	−0.48403	−0.18572	0.03329	−0.23337	64.2827	16.4219	0.151300	0.06107	5.95263	23.7715
Kurtosis	13.5775	23.8257	14.9932	11.8945	8.3205	4139.218	405.3729	9.471384	20.4410	64.9609	1098.93
J-B	19,362.85	75,212.07	24,913.46	13,690.36	4936.111	2.96×10^{9}	28,202.79	7262.625	52,639.65	68,8860.3	2.08×10^{8}

Notes: Test statistics are tabulated in Table 1 for equity returns of BRICS countries (Brazil, Russia, India, China, and South Africa). Statistics for foreign exchange rates for Brazil (USDBRL), Russia (USDRUB), India (USDINR), China (USDCNY), and South Africa (USDZAR). Statistics for crude oil prices are named West Texas Intermediate (WTI); Jarque–Bera test (J-B).

The positive skewness of both the exchange rate and crude oil suggests a potential favorable situation for exporters, oil-producing countries, and energy-related companies. However, importers, consumers, and industries reliant on oil may face challenges due to increased costs. It is important to note that market conditions and various factors can influence the relationship between exchange rates, crude oil prices. The positive skewness in the exchange rate may attract foreign investors, anticipating currency appreciation. The kurtosis level is greater than 3 in all markets, indicating that their distributions are more leptokurtic (fat tails). This shows that large daily changes occur more frequently in the stock market index, crude oil prices, and foreign exchange rate markets. It is evident that the daily return series for all equity is leptokurtic, indicating that significant fluctuations in the daily prices are much more common than estimated by the normal distribution. The null hypothesis of normal distribution at the 1% significance level is rejected, as indicated by J-B tests, which is consistent with the statistics for skewness and kurtosis. The averages of the stock markets and the foreign exchange rates are positive, except for China (USDCNY) and WTI, which are negative. This implies that the USD appreciated against Brazilian, Russian, Indian, and South African currencies over our sample period on average. The descriptive statistics form the foundation of financial analysis by providing concise summaries, visual representations, and key insights into the behavior, variability, and risk associated with financial data.

4.3. Markov-Switching Model Results

Table 2 presents state 1 (high-risk regime) and state 2 (low-risk regime). The dependent estimate parameters in state 1 show positive and significant effects for Russia and South Africa, implying that these countries have an important impact on the market in the context of high-risk conditions, suggesting that investors perceive these countries as having higher levels of risk compared to others. A negative and significant effect for China could suggest that market participants perceive China as a source of risk or uncertainty. This perception might lead to reduced investor confidence, heightened market volatility, or increased risk aversion, which could potentially impact various aspects of the market. In regime 2, we observe a positive and significant dependence in Russia, India, and South Africa, indicating that market movements in these countries can have important effects on global markets. For the currency exchange rates of the BRICS countries, the state parameters of USDBRL, USDRUB, USDINR, USDCNY, and USDZAR in regime 1 are −0.0946, 0.0063, −0.0018, −0.0043, and 0.00724, respectively, while in regime 2, they are −0.0141, 0.0598, 0.0419, −0.0079, and 0.0140, respectively. The results show that the market is characterized by low returns in currency exchange rates in regime 1, making Brazilian, Indian, and Chinese currencies less attractive to investors seeking higher returns, leading to increased risk aversion among investors. They may prefer to hold more stable or safer currencies, such as those from countries with stronger economies or higher returns, which could further exacerbate the depreciation pressure on the currency with low returns. The high returns in Russia and South Africa can contribute to increased exchange rate volatility. In this case, the currency markets can be influenced by various factors, such as economic indicators, political developments, and market sentiment. The higher returns in these countries can attract speculative traders and amplify market fluctuations, leading to increased volatility in currency exchange rates. In regime 2, we observe low returns in Brazil and China and high returns in Russia, India, and South Africa. The transition probabilities were examined for both regimes, revealing that regime 1 was more persistent for the foreign exchange rates of Russia, India, and China. Russia has higher and significant volatility in regime 2, India has lower and insignificant volatility in regime 1, and finally China has higher and insignificant volatility in regime 2. Conversely, regime 2 persists for foreign exchange rates of Brazil and South Africa with higher volatility in regime 1. The findings show that higher transition probabilities indicate a higher propensity for the system to shift from the current regime to a different regime in the next time period. This implies that the system is more likely to experience a change in regime or condition, potentially reflecting shifts in underlying economic factors or other relevant variables.

Table 2. MS for foreign exchange rate against domestic currencies of BRICS countries.

Par	USDBRL		USDRUB		USDINR		USDCNY		USDZAR	
	Regime 1	**Regime 2**	**Regime 1**	**Regime 2**	**Regime 1**	**Regime 2**	**Regime 1**	**Regime 2**	**Regime 1**	**Regime 2**
u	−0.0946	−0.0141	0.0063 ***	0.0598 ***	−0.0018	0.0419 *	−0.0043 ***	−0.0079	0.0724 ***	0.0140 ***
	(0.1758)	(0.0456)	(0.0002)	(0.0089)	(0.0065)	(0.019	(0.0012)	(0.0055)	(0.0045)	(0.0004)
α	−0.0361	0.0307	0.0695 ***	0.2969 **	−0.0684 **	0.0307	−0.0632 *	−0.0153	0.1614 ***	0.0327 **
	(0.0307)	(0.0206)	(0.0029)	(0.0452)	(0.0221)	(0.0276)	(0.0325)	(0.0241)	(0.0431)	(0.0100)
γ	0.0348	−0.0169	0.1392 ***	0.3334 ***	−0.0086	−0.0940 ***	−0.0314 *	0.016	0.1995 ***	0.0356 ***
	(0.0309)	(0.0191)	(0.0068)	(0.039	(0.0223)	(0.028	(0.0164)	(0.0243)	(0.0442)	(0.0090)
p_{1i}	0.9494	0.0192	0.9373	0.3574	0.9748	0.0484	0.9064	0.1061	0.455	0.1141
p_{2i}	0.0506	0.9808	0.0627	0.6426	0.0252	0.9516	0.0936	0.8939	0.545	0.8859

Note: This table exhibits the regime dependent with the estimate parameters noted by u; α and γ the autoregressive coefficient; the transition probabilities are (p_{1i},p_{2i}), with i = 1, 2 representing the states of the MS for the exchange rate against the domestic currencies of BRICS countries. The values in brackets are standard errors for the estimated value of the coefficient parameters. ***, **, and * indicate that the estimated value of the parameter is significant at a significance level of 1%, 5%, and 10%, respectively.

In Table 3, we present the findings of Markov-switching for the stock market index of BRICS countries and crude oil. We observe that the parameters of high-risk returns are positive and significant for Brazil, Russia, India, and South Africa, indicating that investors are being rewarded for taking on additional risk. This can lead to increased investor confidence and optimism in the market. This may attract more investors to participate in the market, thereby increasing liquidity and potentially driving stock prices higher. In low-risk markets, the Chinese stock markets generate positive high-risk returns, which signals investor confidence in the country's economy and financial markets. This suggests that investors are willing to take on greater risk in anticipation of substantial rewards, indicating positive sentiment toward China's economic prospects. We observe non-significant coefficients for crude oil in both regimes. The findings indicate the transition probabilities of regime 1 more persistent than for regime 2 for Bovespa, MOEX, Sensex, and

FTSE/JSE, while regime 2 persistent than regime 1 for Hang Seng and WTI. The regime 1 for Bovespa, MOEX, Sensex, and FTSE/JSE have lower and insignificant volatility, except for FTSE/JSE which has a lower and significant volatility. However, Hang Seng and WTI have higher and insignificant volatility.

Table 3. MS for the stock market index of BRICS countries and WTI.

Par	Bovespa		MOEX		Sensex		Hang Seng		FTSE/JSE		WTI	
	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2
u	0.0559 *	−0.1757	0.0918 ***	−0.1876	0.073 ***	−0.1050	−0.0869	0.0417 *	0.0770 ***	−0.0876	−0.0946	−0.0141
	(0.0233)	(0.2530)	(0.0192)	(0.1254)	(0.0163)	(0.0890)	(0.0886)	(0.0178)	(0.0169)	(0.0702)	(0.1758)	(0.0456)
α	−0.0133	−0.157 ***	−0.0076	0.0168	0.0703 ***	0.0259	−0.0465	0.0182	−0.0068	−0.0331	−0.0361	0.0307
	(0.0166)	(0.0568)	(0.0173)	(0.0339)	(0.0187)	(0.0336)	(0.0344)	(0.0182)	(0.0177)	(0.0317)	(0.0307)	(0.0206)
γ	−0.0108	0.0568	−0.0296	0.0191	−0.0052	−0.0346	0.0099	−0.0079	−0.0403 *	0.0179	0.0348	−0.0169
	(0.0162)	(0.0575)	(0.0187)	(0.0353)	(0.0168)	(0.0349)	(0.0315)	(0.0179)	(0.0189)	(0.0329)	(0.0309)	(0.0191)
p_{1i}	0.9942	0.0674	0.9911	0.0361	0.9899	0.0381	0.9753	0.0066	0.9906	0.0299	0.9494	0.0192
p_{2i}	0.0058	0.9326	0.0089	0.9639	0.0101	0.9619	0.0247	0.9934	0.0094	0.9701	0.0506	0.9808

Note: This table displays the regime dependent with the estimate parameters noted by u; α, and the autoregressive coefficient γ; the transition probabilities are (p_{1i},p_{2i}), with i = 1, 2 representing the states of the MS for the stock market of BRICS countries and WTI. The values in brackets are standard errors for the estimated value of the coefficient parameters. *** and * indicate that the estimated value of the parameter is significant at a significance level of 1% and 10%, respectively.

4.4. Wavelet Analysis Results

In wavelet coherency, the cone of influence is shown with a thick black line. Coherency ranges from blue (low coherency) to yellow (high coherency). The Multiple-wavelet coherency and partial wavelet coherency are the time-frequency plane's simple general equivalents of Fourier multiple coherencies and partial coherency. The partial wavelet coherency is an extension of the wavelet coherency concept. The concept of partial wavelet coherency is an extension of the concept of wavelet coherency, just like partial correlation is an extension of simple correlation. With this tool, one can move beyond univariate and bivariate wavelet analysis to higher-order variate wavelet analysis.

Figure 1a exhibits the WTC plot; two episodes appear for Brazil. The first episode is detected at a low level of co-movement at the 5% significance level in the long-term (256–1024 days) band of the scale, covering the period of 2012–2018. From 2012 to 2014, the arrows pointed straight to the right, indicating that the relationship between Bovespa and Crude oil is in-phase. This suggests that they exhibit a similar pattern of behavior over time, and their prices tend to move in the same direction or follow a similar trend. This information can be valuable for portfolio management, risk assessment, or identifying trading opportunities. The arrows pointing right and upward, indicating that Bovespa positively influences crude oil. The arrows pointing right and downward, it suggests that changes in crude oil lead or precede changes in Bovespa stock prices. This could imply that fluctuations in crude oil impact Bovespa stock prices, and investors may consider analysing crude oil data to predict potential future movements in Bovespa stock prices from 2015 to 2018 in the same area of the frequency domain. The second episode is visible in the low-frequency (8–32, short-term; 32–64, medium-term) days over the period 2018–2020. The direction of the arrows is not clear, as different directions are observed: right up, right down, and left up. By including USDBRL in MWC (Figure 1b), we notice that USDBRL increases the intensity of the co-movement between Bovespa and crude oil. This suggests a strong relationship between the Bovespa and crude oil, showing that the recorded co-movement in WTC increased for the long-term, medium-term, and short-term according to the intense yellow area in MWC from 32 to 1024 days, which appears at high frequency, over the period 2005–2019. A 5% significance level at high frequency from 128 to 1024 days is seen over the period 2005–2014 and from 32 to 128 days covering the period 2005–2010. Furthermore, interesting results are seen by removing the USDBRL in PWC (Figure 1c). We discover two significant aspects. The first aspect is seen at high frequency from the period 2005–2008 in the short- and medium-term (16–32 and 32–128 days) band of scale, showing the significant persistence of co-movement between Bovespa and crude oil. The second aspect is seen at high frequency, from 128 to 256 days over the periods 2009–2014 and 2016–2019, with an indication of the persistence of co-movement between the two variables. There is a reduced significance level in PWC, implying that USDBRL is a critical factor driving co-movement between Bovespa index for Brazil and the crude oil.

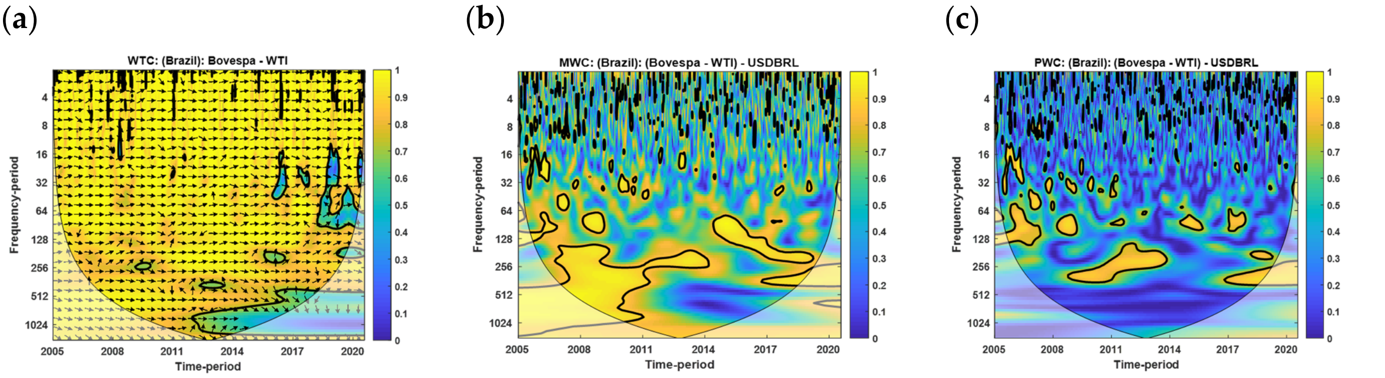

Figure 1. Represents the set of WTC, MWC, and PWC for Brazil: (**a**) represents the plot of the WTC of Bovespa-WTI; (**b**) represents the plot of the MWC of Bovespa-WTI-USDBRL; and (**c**) represents the plot the PWC of Bovespa-WTI-USDBRL.

Figure 2a exhibits the findings of Russia for WTC, which shows significance at a 5% level at high frequency. We observe four episodes. The first episode starts in the short-term (16–32 days) band of scale, covering the period from 2005 to 2006. The second episode is also visible with high frequency in the medium term (64–188 days), covering the period of the financial crisis period from 2007–2008. The arrows pointed left and upward, implying that the crude oil leads the MOEX, or, in other words, that the crude oil positively influences the MOEX index. The third episode appears at high frequency, starting from 256 to 1024 days covering the GFC and the EDC from 2007–2011. The arrows pointed right and upward, it suggests a positive correlation or an increasing trend in the financial data between MOEX index and crude oil, as MOEX increases, the crude oil tends to increase as well. The fourth episode occurs at high frequency from 256 days over the period 2017–2020. The relationship between the two variables is anti-phase, as the arrows are pointing straight down. In Figure 2b of MWC, we examine the role of USDRUB as an intermediary variable and detect a significant change in the correlation pattern, specifically a shift from 16 to 32 days, in the high-frequency data of 2005, with a 5% level of significance. The medium-term covers the periods 2005–2006, 2007–2008, and 2015–2016 at the high-frequency level. By removing USDRUB from PWC (Figure 2c), we observe a significant increase in high frequency corresponding to the co-movements between the two variables. This indicates that the USDRUB is not a key driver of co-movements between the MOEX index and crude oil in the long term.

(**a**) (**b**) (**c**)

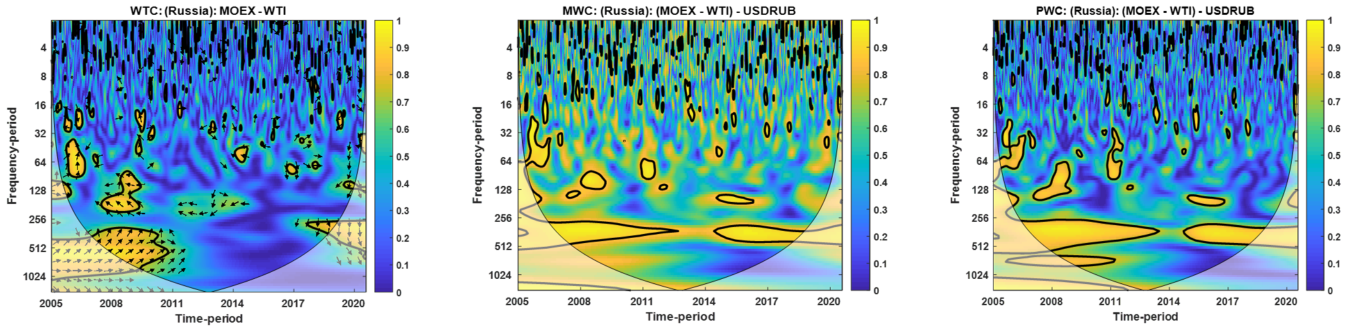

Figure 2. Represents the set of WTC, MWC, and PWC for Russia: (**a**) represents the plot of the WTC of MOEX-WTI; (**b**) represents the plot of MWC of MOEX-WTI-USDRUB; and (**c**) represents the plot of the PWC of MOEX-WTI-USDRUB.

The WTC plot in Figure 3a represents the results of India. We observe a 5% significance level in the relationship between Sensex and crude oil during the period 2005–2016 at the high-frequency level from 8 to 32 days as the first episode, and there is no clear view of the direction of co-movement.

(**a**) (**b**) (**c**)

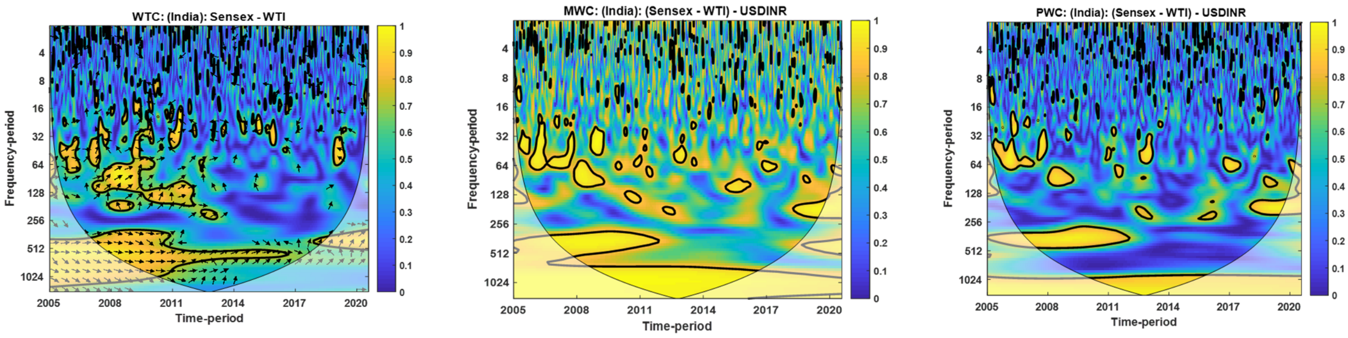

Figure 3. Represents the set of WTC, MWC, and PWC for India: (**a**) represents the plot of the WTC of Sensex-WTI; (**b**) represents the plot of the MWC of Sensex-WTI-USDINR; and (**c**) represents the plot of the PWC of Sensex-WTI-USDINR.

Figures of Regime 1

Figures 1–5. Notes: The set WTC-MWC-PWC of "the stock markets of BRICS countries and crude oil" pair with USD currency exchange rates of BRICS countries as a control variable in MWC and PWC. Figure 1 first raw: Brazil, Figure 2 second raw: Russia, Figure 3 third raw: India, Figure 4 fourth raw: China, Figure 5 fifth raw: South Africa. Legend: (1) The frequency period lines indicate the 4–1024-days solar cycle periodicity band, subdivided as short-term (4–32 days), medium-term (32–128 days), and long-term (128–1024 days). (2) The thick black contour indicates the 5% significance level. The light shadow shows the cone of influence where the edge effects might distort the picture. (3) The color code for power ranges goes from blue (low power) to yellow (high power), indicating the intensity of co-movement. (4) Arrows indicate the relative phase by their angle relative to the horizontal line; the phase difference between the two series is indicated by the arrows' position: the variables are in-phase when the arrows are pointing sharp right (positively linked) and out-of-phase when the arrows are pointing sharp left (negatively linked). In the in-phase scenario, stock markets drive crude oil when the arrows are oriented to the right and upward. Crude oil drives stock markets when the arrows are pointed to the right and downward. In the out-of-phase scenario, stock markets explain crude oil when the arrows are oriented to the left and downward. Crude oil predicts stock markets when the arrows point to the left and upward.

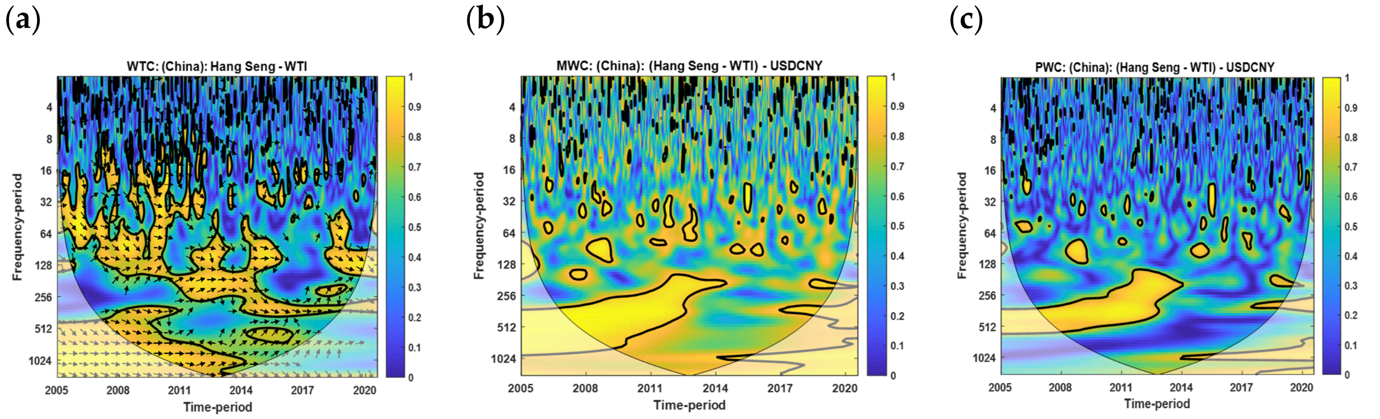

Figure 4. Represents the set of WTC, MWC, and PWC for China: (**a**) represents the plot of the WTC of Hang-Seng-WTI; (**b**) represents the plot of the MWC of Hang-Seng-WTI-USDCNY; and (**c**) represents the plot of the PWC of Hang-Seng-WTI-USDCNY.

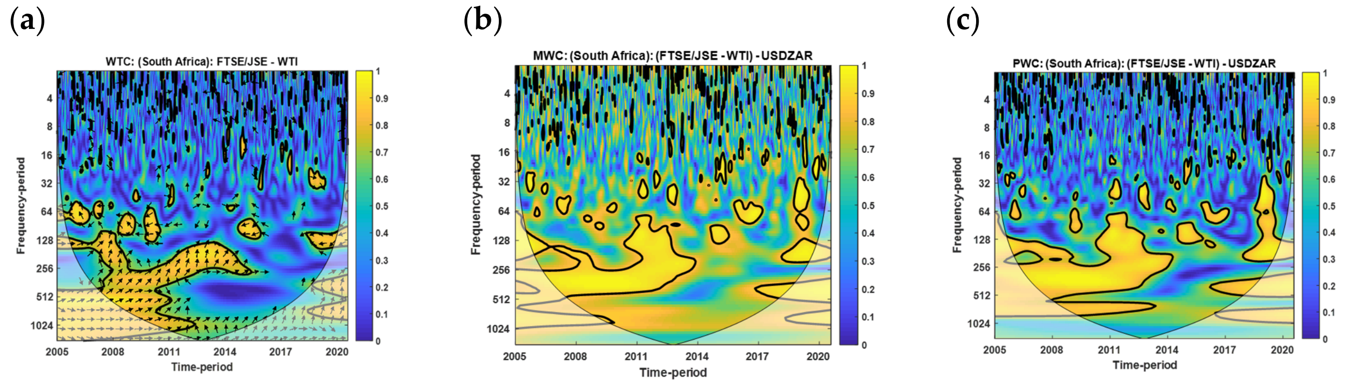

Figure 5. Represents the set of WTC, MWC, and PWC for South Africa: (**a**) represents the plot of the WTC of FTSE/JSE-WTI; (**b**) represents the plot of the MWC of FTSE/JSE-WTI-USDZAR, and (**c**) represents the plot of the PWC of FTSE/JSE-WTI-USDZAR.

The second episode is in the medium term (32–128 days) covering the period 2006–2013, with the direction of the arrows pointed right and upward, showing a positive correlation. This suggests that as the Sensex prices increases, crude oil prices tend to increase as well. In the third episode, the period covering 2007–2017 on 128–512 days with the arrows pointed straight to the right, showing a strong positive correlation or a shift towards higher frequency components between two signals. In this context, Sensex and crude oil are the signals being analysed, which suggests that there is a strong relationship or synchronization between the two signals, with Sensex leading or influencing crude oil in terms of frequency or timing. Furthermore, the insertion of USDINR in MWC regression (Figure 3b) to depict the co-movement between Sensex and crude oil showed a significant increase in high frequency during the period 2005–2020 in the medium term, representing the first episode observed in this result. The second episode is observed in high frequency over 2006–2011 in the long-term (256–512 days) band of scale. In the PWC results presented in Figure 3c, obtained after removing USDINR, we notice that the significant area persists in the high frequency in the medium and long term. Therefore, we conclude that there is persistent co-movement between Sensex and crude oil in the medium term and long term for India. We observe a reduced significance level in PWC, indicating that USDINR is the key driver of the co-movement between Sensex and crude oil in the medium term.

In Figure 4a, the plot of the WTC presents the 5% significance level in high frequency in the short term, medium term, and long term, covering all periods. As the arrows are pointing to the right and downward, there is an indication of crude oil negatively influencing the Hang Seng stock index from 2005 to 2006 in the medium term, but the remaining area has no clear direction. In the long term, the relationship between Hang Seng and crude oil is in-phase in high frequency, as the arrows pointed straight to the right. A 5% significance level is observed only at 256 days in the period 2015–2016, as the arrows pointed right and upward, showing that Hang Seng is leading crude oil. The USDCNY insertion in MWC (Figure 4b), showed evidence of the co-movement between Hang-Seng and crude oil at the 5% significance level in the high frequency. We remove USDCNY to test the co-movement between Hang-Seng and crude oil with the PWC regression (Figure 4c), we notice a significant reduced effect on the co-movement between Hang Seng and crude oil. Therefore, the regression's inclusion highlights the significant role that USDCNY plays in driving the effect of co-movement between Hang Seng and crude oil for China in the medium term.

In Figure 5a, the WTC plot shows some significant areas, with the arrows pointed to the right and upward, suggesting that the FTSE/JSE is leading crude oil in the high frequency from days 128 to 1024 over the period 2007–2016. Conversely, if there are negative factors impacting the FTSE/JSE, such as economic downturn or political instability in South Africa, the stock market may decline. However, crude oil prices might rise due to factors like increased global demand or supply disruptions. This contrasting movement again demonstrates the anti-phase relationship between the FTSE/JSE and crude oil from 64 to 128 days covering the period 2019–2020. Furthermore, we plotted MWC in Figure 5b to validate the WTC results for South Africa, including USDZAR as the control variable to depict the co-movement between FTSE/JSE and crude oil. We observe a 5% significance level in the high frequency from days 128 to 512 covering the period 2007–2014. We also notice a 5% significance level in the high frequency from the same frequency period over 2017–2020. There is a consistency of co-movement in the results after the insertion of USDZAR. Finally, in Figure 5c, The results demonstrate the persistent co-movement at high frequency from 128 to 1024 days covering the sample period. After removing USDZAR in the PWC, there is an increase of the thick black contour indicates the 5% significance level in the PWC plot, indicating that the USDZAR is not a key driver of the co-movement between FTSE/JSE and crude oil in the long term. Hence, we conclude that there is persistent co-movement of FTSE/JSE and crude oil in the long term for South Africa.

Figures A1–A5 are presented in Appendix A, and they exhibit the results of WTC, MWC, and PWC in the lower regime of volatility. Figure A1a demonstrates the WTC plot, with a 5% significance level, which shows two episodes with moderate correlations. The first episode is observed in the medium-term (32–64 days) band, covering the period 2018–2022, and the second episode is in the long-term (128–256 and 512–1024 days) band of scale, subdivided into two different periods, 2009 and 2011–2020, where the relationship between the two markets is not clearly indicated, as the arrows are not showing a consistent relationship. By including USDBRL in the MWC plot (Figure A1b), we notice three episodes at a 5% significance level linked with strong correlations. The first episode is visible in the long-term (128–512 days) band of scale covering the period 2005–2015. The second episode is in the medium term, covering the period 2014–2015, and the third episode is in the short term, visible in high-frequency levels observed in different isolated areas. By removing USDBRL from the PWC plot in Figure A1c, we notice high frequency at a 5% significance level in the relationship between Bovespa and crude oil. We observe a shift in co-movement in the frequency period and the time period compared to WTC and MWC. There is a reduced significant region in the long term and short term for PWC, implying that USDBRL is the key driver of the co-movement between the Bovespa stock index and crude oil. We finally notice a persistence of co-movement between the two variables.

Figure A2a shows the WTC plot, presenting two episodes observed at a 5% significance level in high frequency. The first episode is from 64 to 512 days, subdivided into three periods: 2007–2009, 2007–2010, and 2018–2020. The first period (2007–2009) from 64 to 128 days is significant at 5%, with the arrows pointed left and upward, suggesting that MOEX leads the changes in crude oil, and they also exhibit coherence or similarity in their patterns at this specific scale; in the second period (2007–2010) from 256 to 512 days, it indicates a positive correlation. This suggests that as the price of MOEX increases, the crude oil tends to increase as well; and the third period (2018–2020) is not clear as there is no one direction of correlations showed. The second episode is observed in 2006 in the medium term (32–64 days) at high frequency, with the arrows pointing straight upward, it indicates that the analysed financial time series or variable exhibits a prominent upward trend or positive movement at this particular scale or frequency. By including USDRUB in MWC (Figure A2b), we notice the 5% significance level in different frequency periods in high frequency from days 256 to 512 in the periods 2006–2013 and 2014–2018. There is an isolated 5% significance level of high frequency in the short term over the entire sample period. By removing USDRUB from the PWC plot (Figure A2c), we observe a persistence of co-movement between the stock and crude oil at high frequency. There is an increase in the significant region in the long term that indicates that the removed variable, USDRUB, has no influence on the co-movement between the MOEX stock index of Russia and crude oil in the long term.

Figure A3a plots the result of the WTC for India. Two episodes are observed. The first episode is from 256 to 1024 days covering the period 2007–2017 and 2018, where the relationship between the Sensex stock index and crude oil is an in-phase difference. The second episode starts from the frequency period of 16 to 256 days over the entire sample period, pronounced by the financial crisis of 2008 and the debt crisis of 2011, showing the Sensex positively influencing crude oil at high frequency. By including USDINR in the MWC plot (Figure A3b), we observe three episodes at a 5% significance level in high frequency. The first episode is visible in the long term from 2009 to 2016. The second episode from 256 to 512 days covers the period 2007–2011. The third episode from days 8 to 32 covers the entire sample period. By removing USDINR in Figure A3c, we observe a reduced significance level, implying that USDINR is a key driver of the co-movement between the Sensex stock index of India and crude oil in the medium term.

Figure A4a exhibits the WTC for China. We observe an immense significance level for the entire period in the high frequency. For example, from 16 to 64 days in 2005, we notice the direction of the co-movement between Hang Seng and crude oil, showing that crude oil is leading, and the same result is seen in 2018–2020 from the frequency period of 32–128 days, contrary to the period 2015 in the 512-day band of scale, where the Hang Seng stock index positively led the crude oil. The two variables in-phase relationship tend to move in the same direction or exhibits similar patterns of behavior from 16 to 256 days covering the entire period. Including USDCNY in Figure A4b yields the following result for MWC: there is a significance level at 5% in the high frequency with two episodes. The first starts from 128 to 512 days over 2007–2014. The second episode covers the entire sample period with a more isolated significance level from 128 to 256 days. By removing USDCNY in Figure A4c, we notice a persistence of co-movement and reduced areas of significance, implying that USDCNY is the key driver of the co-movement between Hang Seng and crude oil in the medium term.

Figure A5a shows the plot of WTC; we observe in general the 5% significance level at high frequency. Two episodes are mainly visible; the first episode begins from 128 to 1024 days over 2008–2015, indicating a positive lead of FTSE/JSE to crude oil. The second episode begins on 32–128 days at high frequency with isolated significance levels, most of which show an in-phase relationship between the two variables. By including USDZAR in MWC in Figure A5b, we observe that there is a persistence of co-movement between the FTSE/JSE and crude oil in high frequency that appears in the first episode from 64 to 512 days covering the periods 2006–2014 and 2017–2020. The second episode appears with a more isolated significance level in the 16–128 days, from 2005 to 2013, from 2015 to 2017, and finally from 2019 to 2020. Furthermore, by removing USDZAR from PWC in Figure A5c, we notice an increase in the significant area in the first episode from 64 to 512 days over 2006–2014. This indicates that in this specific period, USDZAR was not a key driver of the co-movement between the two variables. In the second episode, we observe a decrease in the significant yellow area from 16 to 64 days over the entire sample period, indicating that USDZAR is the key driver of co-movement in this period.

5. Conclusions and Policy Implications

In this study, we performed an empirical analysis focusing on the dependence structure and the time–frequency impact of exchange rates on the equities markets of BRICS economies and crude oil. Markov-switching-based wavelet analysis was used, with the data spanning the years 2005–2020. Our analyses show that the Markov-switching-based wavelet approach is a very promising approach for analyzing the dependence structure and the time–frequency impact of exchange rates on equities markets. We found that the dependent estimation parameters in the high-risk regime show positive and significant effects for Russia and South Africa, indicating that market movements in these countries can have a prominent impact on global markets during high-risk periods. Any significant developments or events in these nations could trigger increased market volatility, leading to fluctuations in stock prices, exchange rates, and other financial instruments. A significant negative effect for China could suggest that market participants perceive China as a source of risk or uncertainty. This perception might lead to reduced investor confidence, heightened market volatility, or increased risk aversion, which could potentially impact various aspects of the market. In the low-risk regime, we observe the positive and significant dependence of Russia, India, and South Africa, where changes in crude oil prices have a noticeable impact on the stock returns of companies in these countries. Regarding interdependence with the energy sector, Russia, India, and South Africa are all countries with significant energy sectors, including oil production, refining, and consumption. Positive and significant correlations indicate that when crude oil prices increase, the stock returns of energy-related companies in these countries tend to rise as well. The market is characterized by low returns in the high regime for Brazil, India, and China and high returns in Russia and South Africa. The high-risk regime took longer to shift to another state in Russia, India, and China, whereas low-risk regimes have higher transition probabilities in Brazil and South Africa, implying that the durations of these regimes are longer before changing from regime 2 to regime 1 and vice versa.

The Brazilian index Bovespa and crude oil show the co-movement at a low level of significance in the long term, covering the period of 2012–2018. There is an indication of an in-phase relationship from 2012 to 2014, implying that Bovespa positively influences crude oil. By including USDBRL in the MWC, we discover an increase in the intensity of co-movement between Bovespa and crude oil in the long term, medium term, and short term, which appears at a particular 5% significance level in high frequency over the period 2005–2019, implying a strong interaction with the global economy. Moreover, interesting outcomes are perceived after considering the non-influence of USDBRL from PWC; we notice significant and substantial persistence of co-movement between Bovespa and crude oil, a clear indication showing that USDBRL is a critical factor driving the co-movement between crude oil and the Bovespa index for Brazil in the timescale and frequency domain.

Russia's index shows a 5% significance level in the high frequency of WTC. We detect different episodes: in the short term and the medium term, covering the periods 2005–2006 and 2007–2008, respectively, and in the latest period, the financial crisis. The other episode covers the GFC and the EDC in the high-frequency (256–1024 days) band of scale from 2007 to 2011. The MOEX index positively leads crude oil in this specific timescale and frequency domain. Considering the intermediating effect of USDRUB, we notice the indication of co-movement in the high frequency covering the period 2005 at a 5% significance level. The PWC shows a significant increase in co-movements at high frequency, but after removing the influence of USDRUB, we notice that in the long term, USDRUB is not a key driver of co-movement between the MOEX index and crude oil because Russia is a great producer of oil.

The Sensex index and the crude oil have a 5% level of significance of co-movement in the high-frequency level, but without a clear direction of the leading market as seen in the WTC plot during the period 2005–2016 in the short term. The medium-term shows Sensex leading crude oil over the period 2006–2013. Moreover, by including USDINR as a control variable, the findings show an increase in the significant area of high frequency during the period 2005–2020 in the medium term. After removing USDINR from the PWC plot, we observe a persistent co-movement between the Sensex and crude oil in the medium term and long term for India, showing USDINR to be the key driver of the co-movement between Sensex and crude oil in the medium term.

The Hang Seng index and the crude oil in WTC show a 5% significance level of high frequency in the short, medium, and long term throughout the study period. We notice that crude oil negatively influenced the Hang Seng stock index from 2005 to 2006 in the medium term. In the long term, the relationship between Hang Seng and crude oil is in-phase at high frequency. At 256 days in the period 2015–2016, the Hang Seng index leads crude oil. Then, in MWC, there is evidence of co-movement at 5% of significance level at high frequency, it indicates that the observed patterns or relationships are likely to be meaningful and not due to random fluctuations. This finding suggests that there may be underlying structural or systematic relationships between the variables being analyzed. It shows the existence of persistent co-movement within the frequency period. Therefore, the regression's inclusion highlights the important role that USDCNY plays in driving the co-movement between Hang Seng and crude oil for China in the medium term.

The South African FTSE/JSE leads crude oil in the high frequency in the long term over the period 2007–2016. There is an anti-phase relationship between the FTSE and crude oil from 64 to 128 days covering the period 2019–2020. Moreover, with USDZAR as the control variable, we notice consistent co-movement between FTSE/JSE and crude oil. After removing USDZAR, we still observed persistent co-movement at high frequency in the long term over the sample period and a 5% significantly increased area, indicating that USDZAR is not a key driver of the co-movement between FTSE/JSE and crude oil in the long term.

In the lower regime of volatility, we notice high frequency at a 5% significance level in the relationship between Bovespa and crude oil. We observe a shift in co-movement in the frequency period and the time period compared to WTC and MWC. There is a reduced significant region in the long term and short term for PWC, implying that USDBRL is the key driver of the co-movement between the Bovespa stock index and crude oil. In summary, USDBRL plays a similar role in driving the co-movement in both regimes. We notice a persistence of co-movement between the MOEX stock index and crude oil in high frequency and an increase in the significant region in the long term, which shows that the removed variable, USDRUB, has no influence on the co-movement between the MOEX stock index of Russia and crude oil in the long term. The link between the Sensex stock index and crude oil is an in-phase difference, and during the financial crisis of 2008 and the debt crisis of 2011, the Sensex stock index of India influenced crude oil positively with high frequency. We notice a reduced area of significance level, implying that USDINR is a key driver of the co-movement between the Sensex stock index of India and crude oil in the medium term. The co-movement between Hang Seng and WTI shows crude oil leading, and there is an in-phase relationship from 16 to 256 days over the entire period. By including USDCNY in MWC, there is a significance level of 5% in the high frequency with two episodes. By removing USDCNY, we notice a persistence of co-movement and reduced areas of significance, implying that USDCNY is the key driver of the co-movement between Hang Seng and crude oil in the medium term. The co-evolution between FTSE/JSE and crude oil indicates a positive relationship between FTSE/JSE and crude oil. By including USDZAR in MWC, a persistent co-movement between the FTSE/JSE and energy crude oil in high frequency appears in the first episode from 64 to 512 days. Moreover, by removing

USDZAR in the first episode, there is an indication that in this specific period, USDZAR is not a key driver of the co-movement between the two variables, and in the second episode, USDZAR is the key driver of the co-movement.

Wavelet analysis is a useful tool for revealing relationships between variables that could remain unveiled. Our findings, based on coherency between the stock index and energy crude oil in time scale and frequency domain, and based on multiple and partial wavelets, are more significant. The introduction of the third variable, the exchange rates of each BRICS country, as a control variable has important implications for policy makers, governments, and investors in the area of effective risk management. Because exchange rates are the main drivers of co-movement in the stock markets and crude oil prices, policy makers should pay greater attention to changes in exchange rate prices. A stable foreign exchange rate is critical for the stock markets of BRICS countries and international crude oil prices to avoid extreme fluctuations in stock markets and crude oil. Furthermore, the foreign exchange rate is critical in determining the short-term, medium-term, and long-term co-movement of BRICS countries' stock markets and crude oil.

The empirical results discussed above are important for energy traders and investors to manage the risk of their portfolios. We have made a significant contribution to the existing literature in the co-movement or dependence structure that is seen in time scale and frequency domains. Policy makers should take initiatives that will make the stock market more efficient, which will enhance the BRICS economies, and develop the countries' infrastructure, strengthen the stock markets' capacity, and restore market participants' trust in their markets by knowing which time period and frequency domain exchange rates can drive the co-movement between stock and crude oil prices.

The difference between this paper and those of Ma and Yang (2020), Wu et al. (2020), Jain and Biswal (2016), and Arfaoui and Rejeb (2017) is that our analysis shows how exchange rates act as a control variable in the co-movement of BRICS stock markets and international oil price variables. There is scarce literature that has examined the simultaneity of oil prices, exchange rates, and stock market returns. According to Oberndorfer (2009), a stock market is always considered an economic indicator due to its close affiliation, and oil price volatility hikes are also economically harmful.

Author Contributions: Conceptualization, J.W.M.M.; methodology, J.W.M.M. and B.M.M.; software, B.M.M.; validation, J.W.M.M.; formal analysis, B.M.M.; investigation, J.W.M.M. and B.M.M.; resources, B.M.M.; data curation, B.M.M.; writing—original draft preparation, B.M.M.; writing—review and editing, J.W.M.M.; visualization, B.M.M.; supervision, J.W.M.M.; project administration, B.M.M. All authors have read and agreed to the published version of the manuscript.

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Data Availability Statement: The dataset consisting of daily stock prices and crude oil was taken from Thompson Reuter's database, and foreign exchange rates were taken from Yahoo Finance; these data are available at https://eikon.thomsonreuters.com/index.html and https://finance.yahoo.com/, respectively (accessed on the 22 April 2021 and 11 February 2021).

Conflicts of Interest: The authors declare no conflict of interest. The authors certify that they have no affiliations with or involvement in any organization or entity with any financial or non-financial interest in the subject matter or materials discussed in this manuscript.

Appendix A

Figures A1–A5. Same notes as in Figures 1–5.

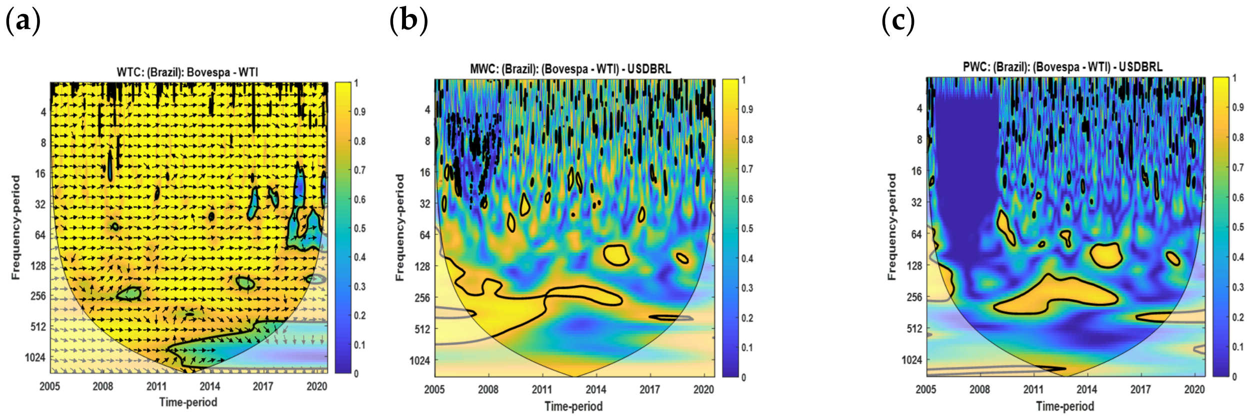

Figure A1. Represents the set of WTC, MWC, and PWC for Brazil: (**a**) represents the plot of the WTC of Bovespa-WTI; (**b**) represents the plot of the MWC of Bovespa-WTI-USDBRL; and (**c**) represents the plot of the PWC of Bovespa-WTI-USDBRL.

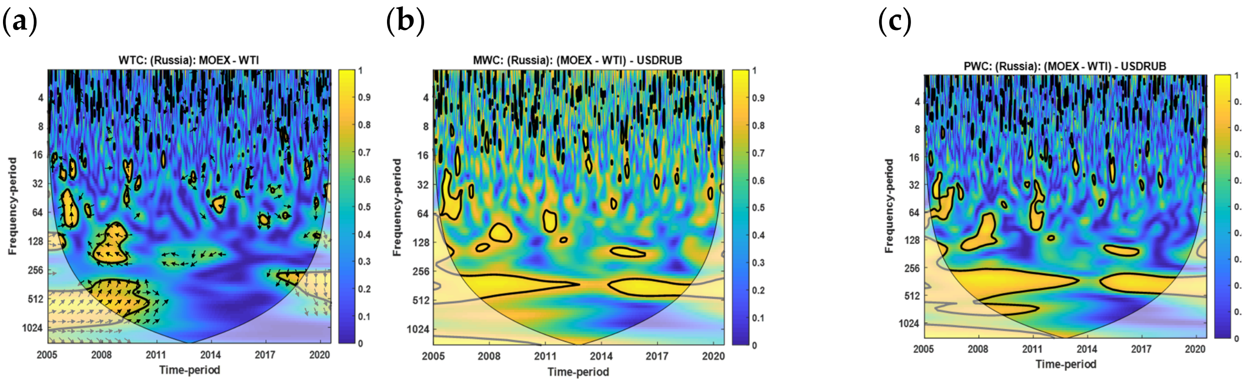

Figure A2. Represents the set of WTC, MWC, and PWC for Russia: (**a**) represents the plot of the WTC of MOEX-WTI; (**b**) represents the plot of the MWC of MOEX-WTI-USDRUB; and (**c**) represents the plot of the PWC of MOEX-WTI-USDRUB.

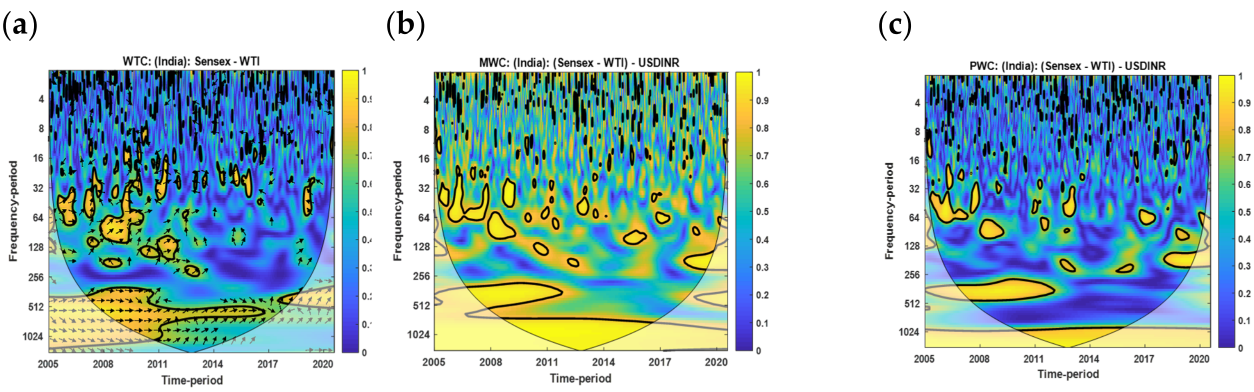

Figure A3. Represents the set of WTC, MWC, and PWC for India: (**a**) represents the plot of the WTC of Sensex-WTI; (**b**) represents the plot of the MWC of Sensex-WTI-USDINR; and (**c**) represents the plot of the PWC of Sensex-WTI-USDINR.

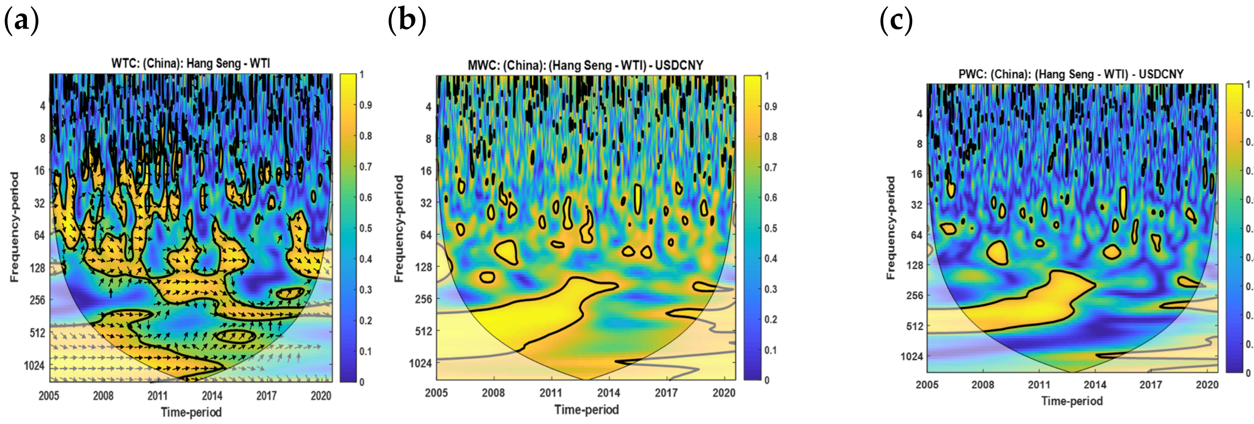

Figure A4. Represents the set of WTC, MWC, and PWC for China: (**a**) represents the plot of the WTC of Hang-Seng-WTI; (**b**) represents the plot of the MWC of Hang-Seng-WTI-USDCNY; and (**c**) represents the plot of the PWC of Hang-Seng-WTI-USDCNY.

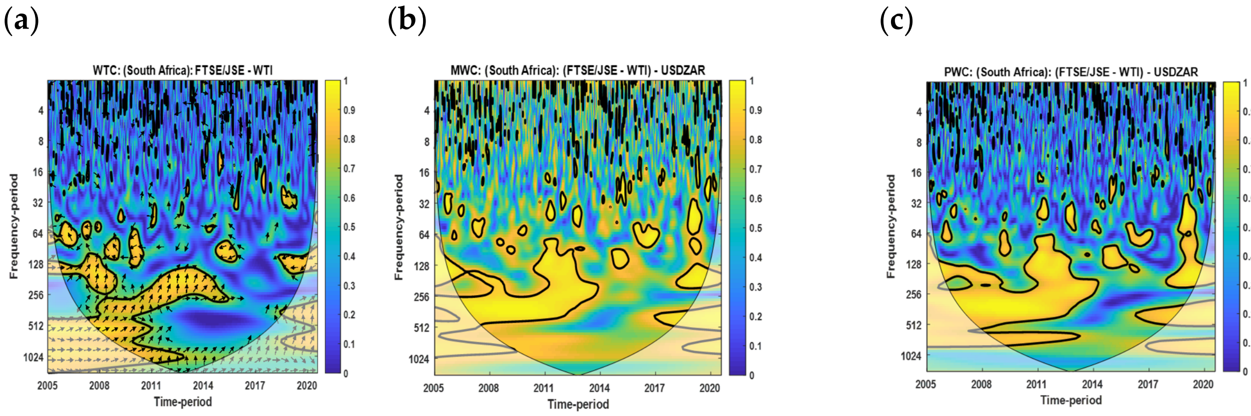

Figure A5. Represents the set of WTC, MWC, and PWC for South Africa: (**a**) represents the plot of the WTC of FTSE/JSE-WTI; (**b**) represents the plot of the MWC of FTSE/JSE-WTI-USDZAR, and (**c**) represents the plot of the PWC of FTSE/JSE-WTI-USDZAR.

Note

[1] Brazil, Russia, India, China, and South Africa

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