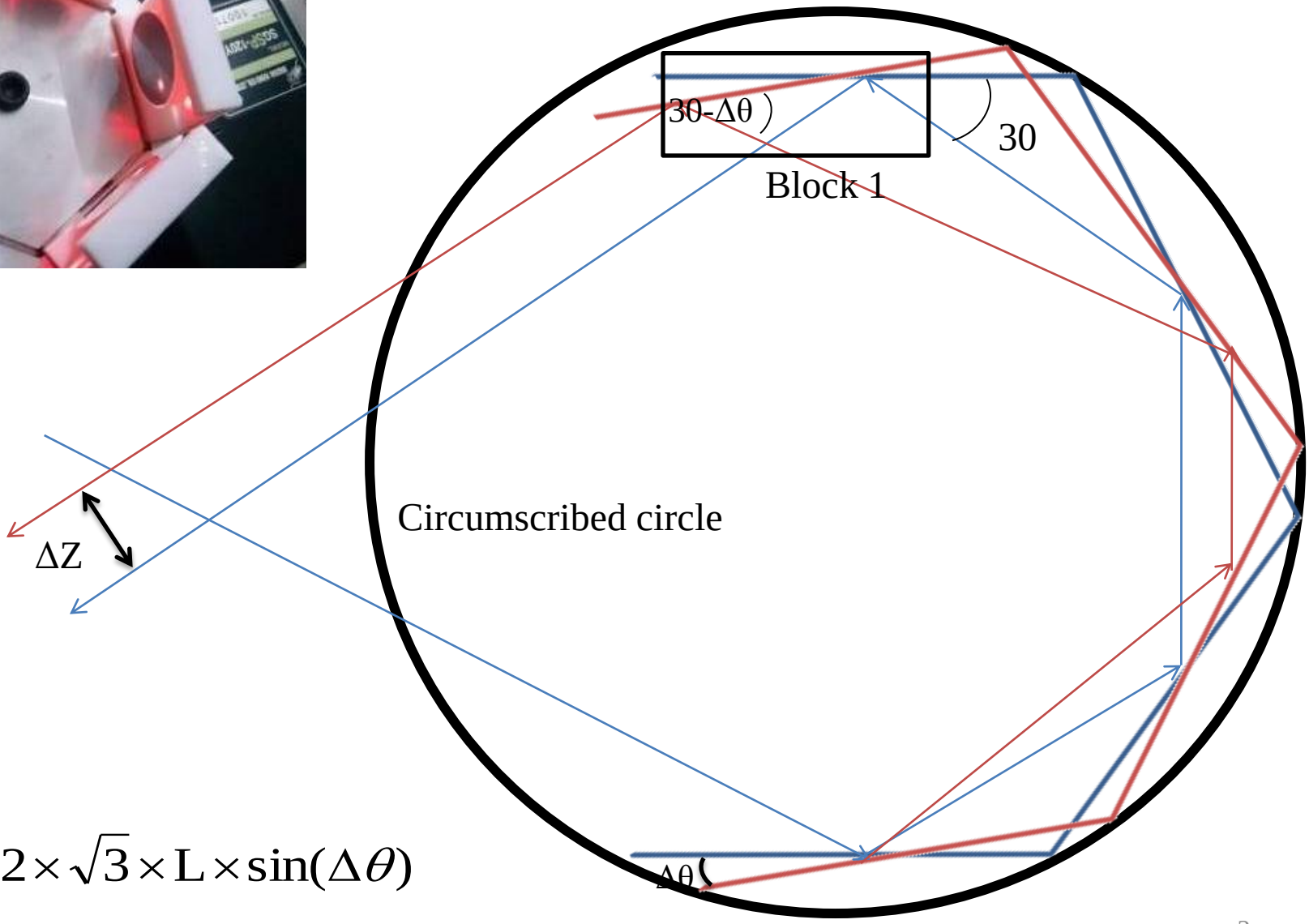
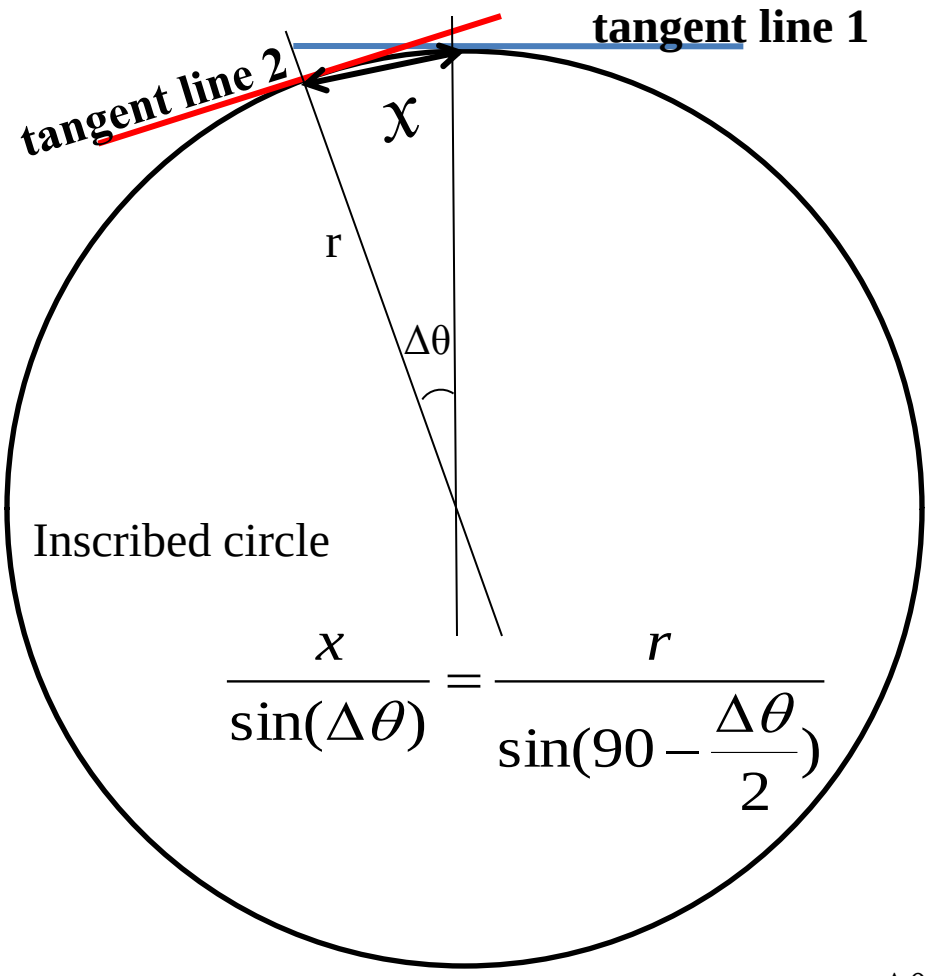
Hexagonal Mirror

When the hexagonal mirror was rotated by a small angle(Δθ), the laser beam was parallel shifted(ΔZ).

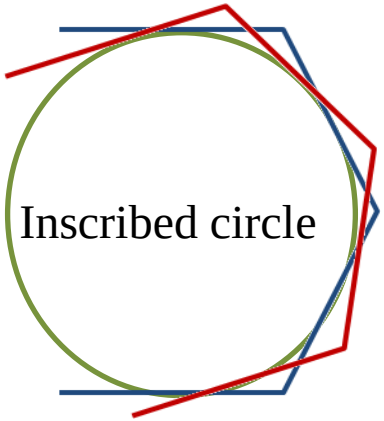


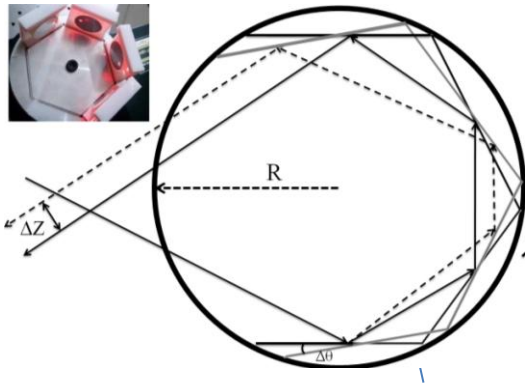
$$\Delta z = 2 \times \sqrt{3} \times \mathrm{L} \times \sin(\Delta\theta)$$

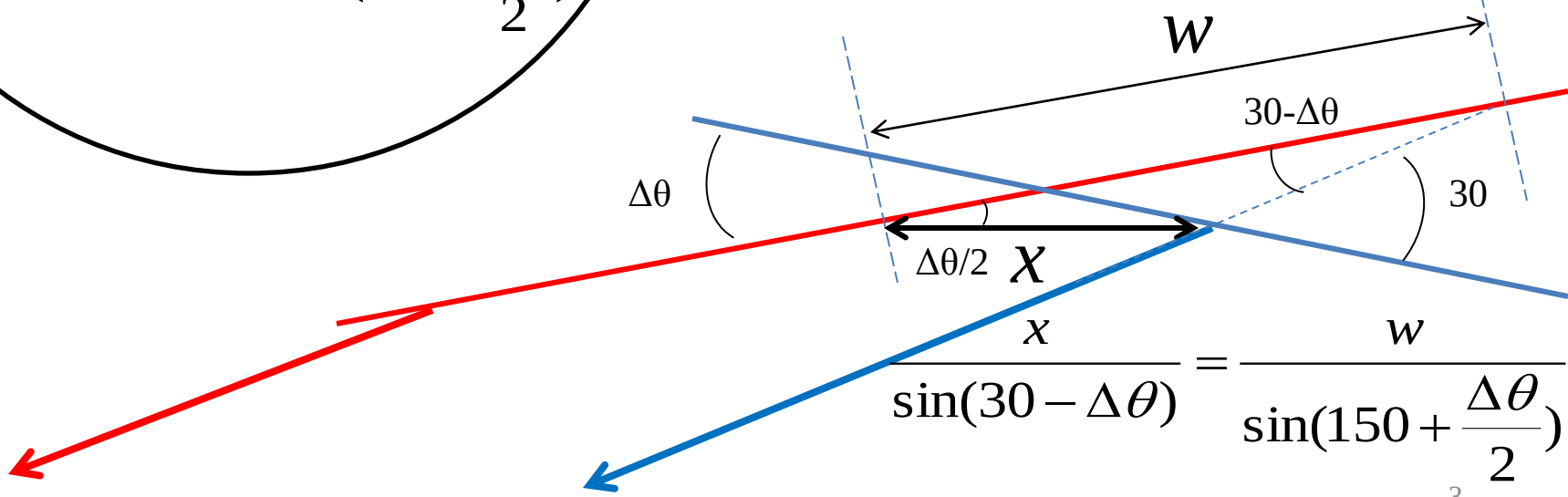


$$\frac{x}{\sin(\Delta\theta)} = \frac{r}{\sin(90 - \frac{\Delta\theta}{2})}$$

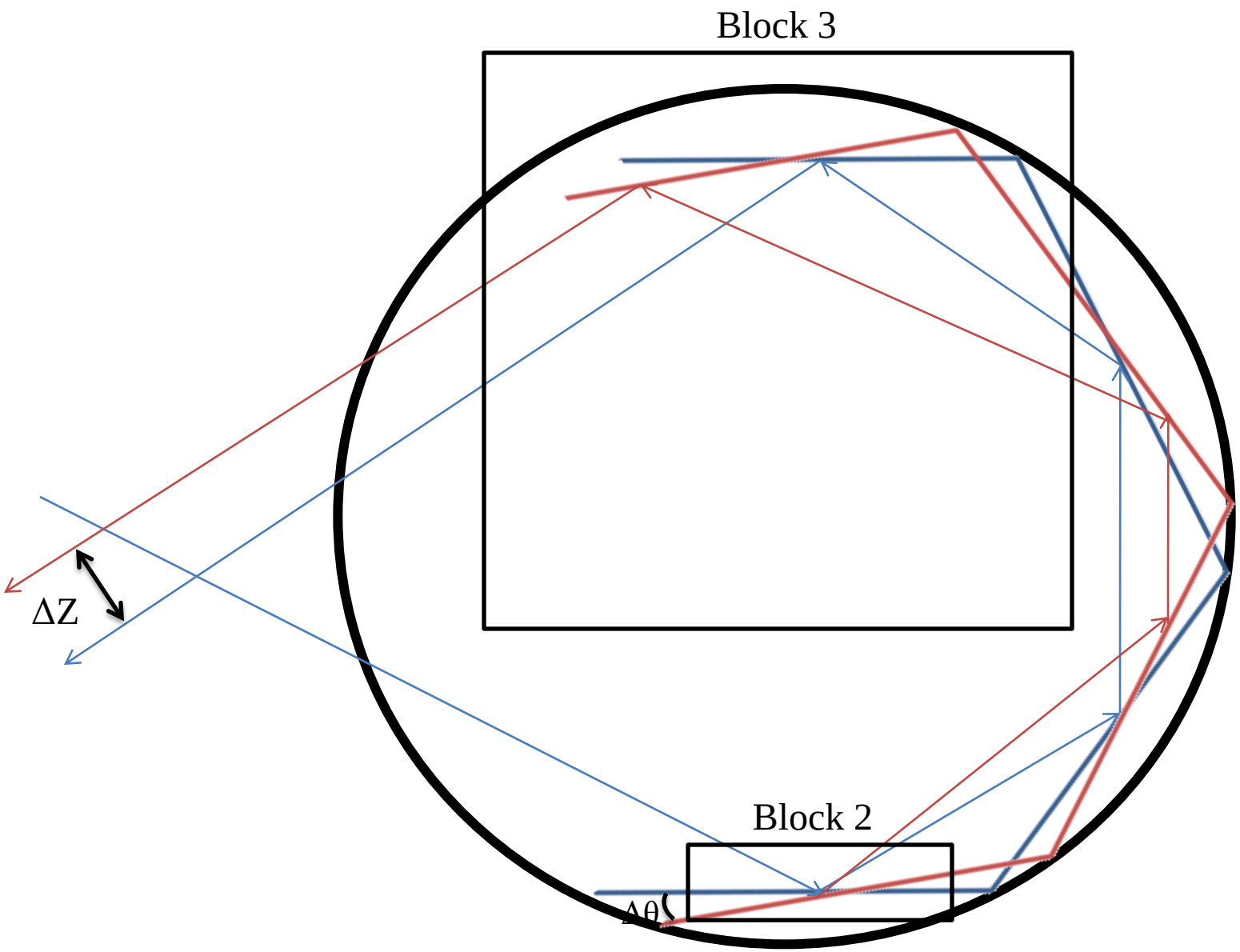
x=the distance of two tangent line
According to sine rule, we can find *w*.



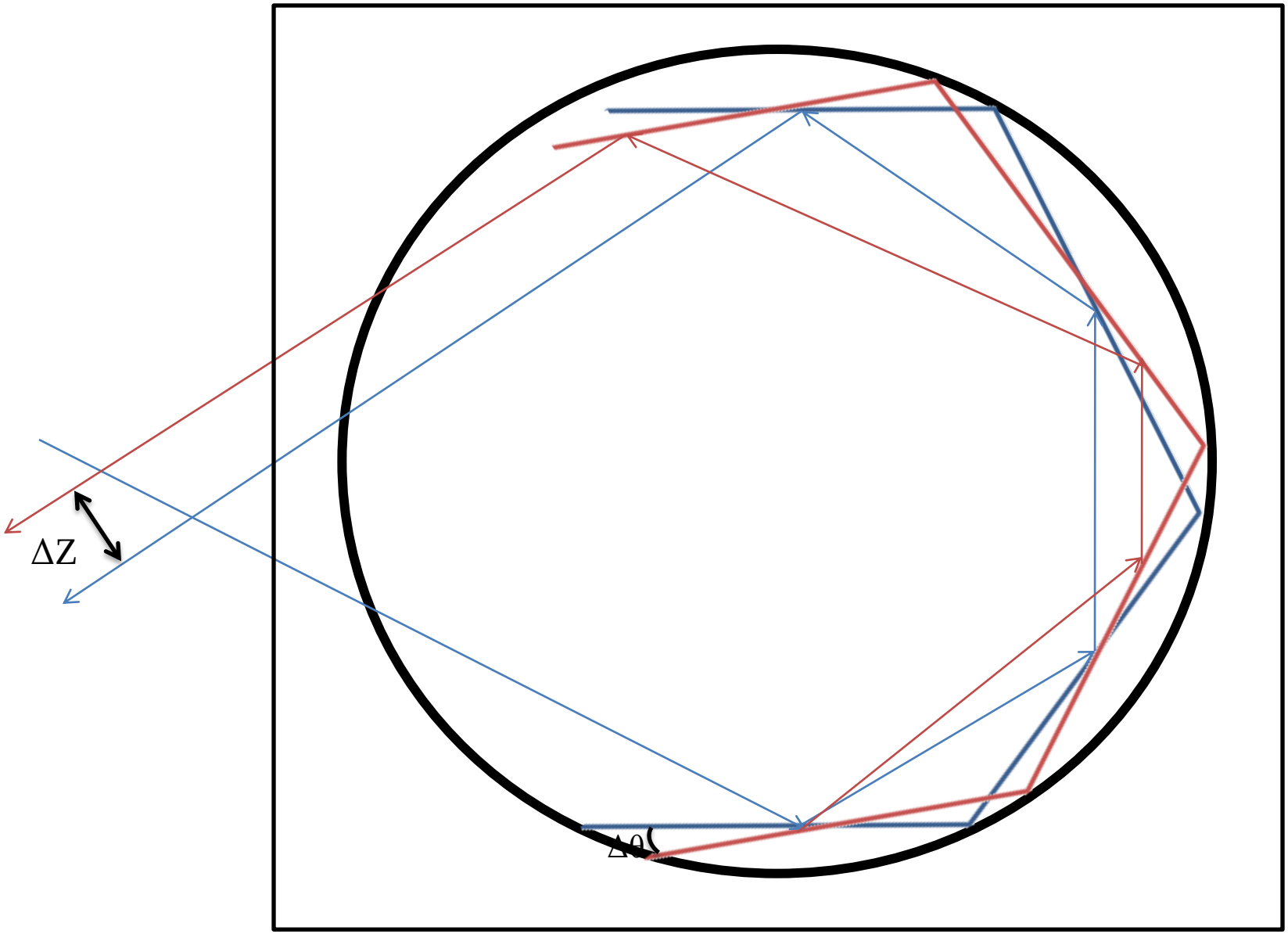




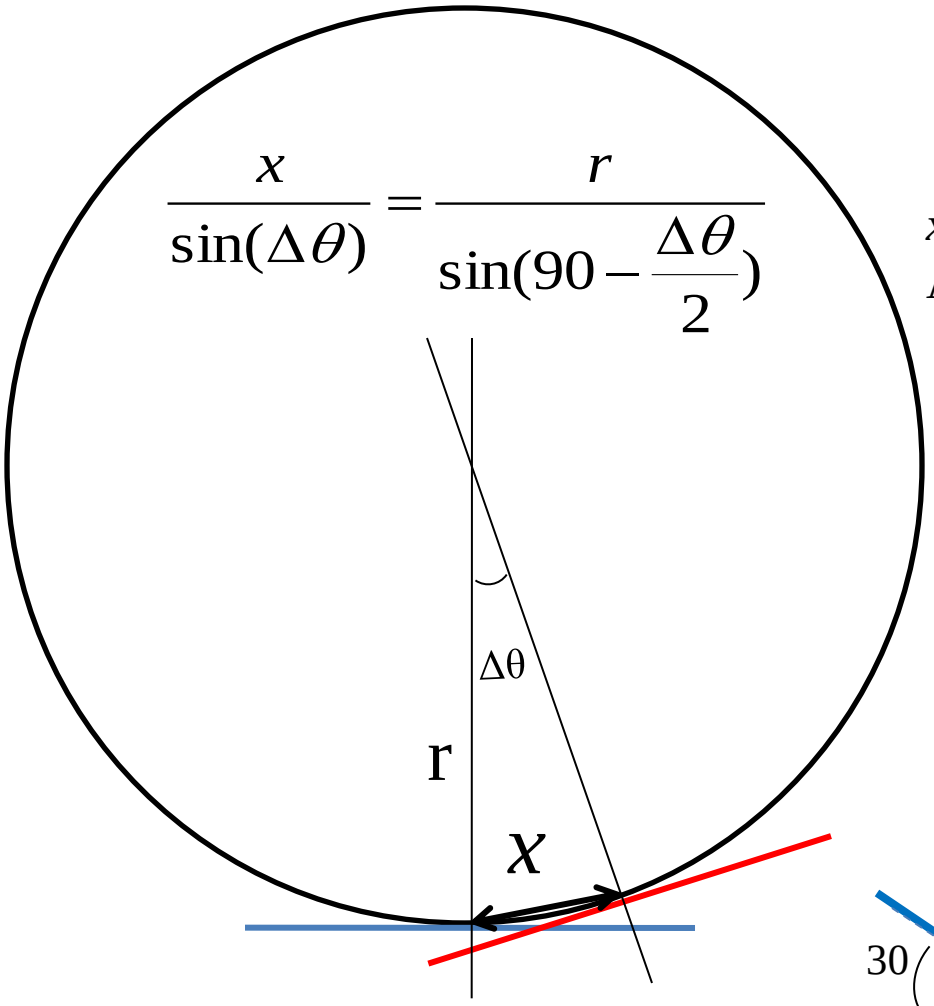
$$\frac{x}{\sin(30 - \Delta\theta)} = \frac{w}{\sin(150 + \frac{\Delta\theta}{2})}$$


Block 3
ΔZ
Block 2
Δθ

Block 4



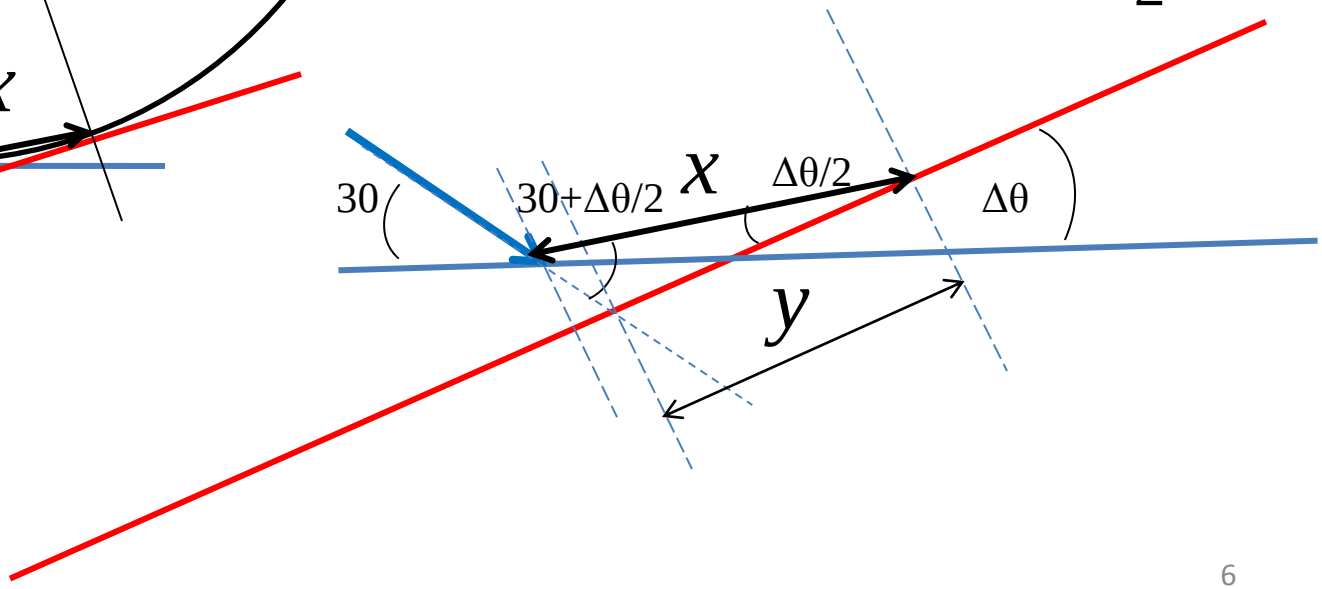
$$\frac{x}{\sin(\Delta\theta)} = \frac{r}{\sin(90 - \frac{\Delta\theta}{2})}$$



x=the distance of two tangent line
According to sine rule, we can find *y*.

r: The radius of inscribed circle.
r will mention in the next page.

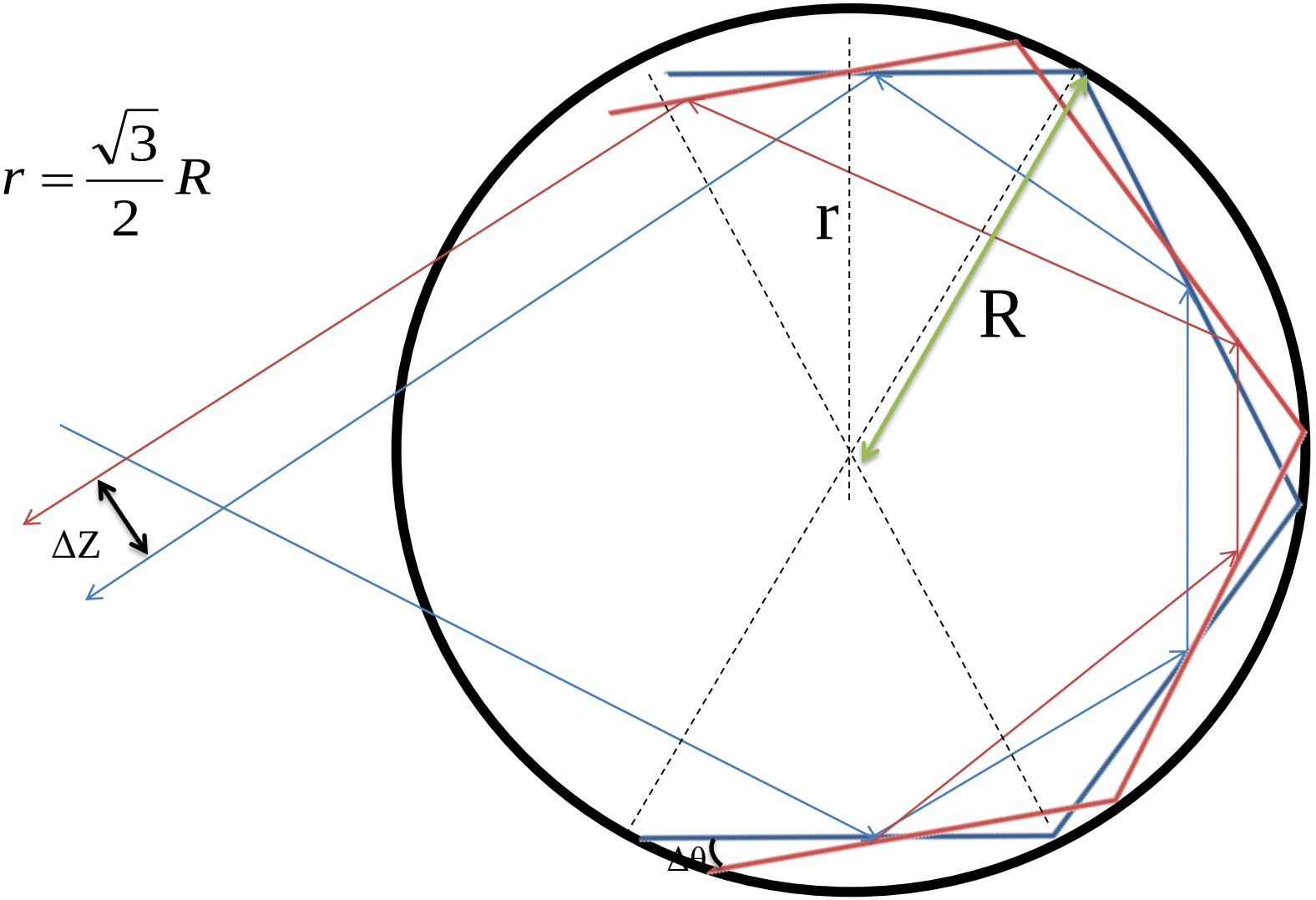
$$\frac{x}{\sin(150 - \Delta\theta)} = \frac{y}{\sin(30 + \frac{\Delta\theta}{2})}$$



Block 3: R is the radius of circumscribed circle.

R is equal to the side length of hexagonal mirror (L).

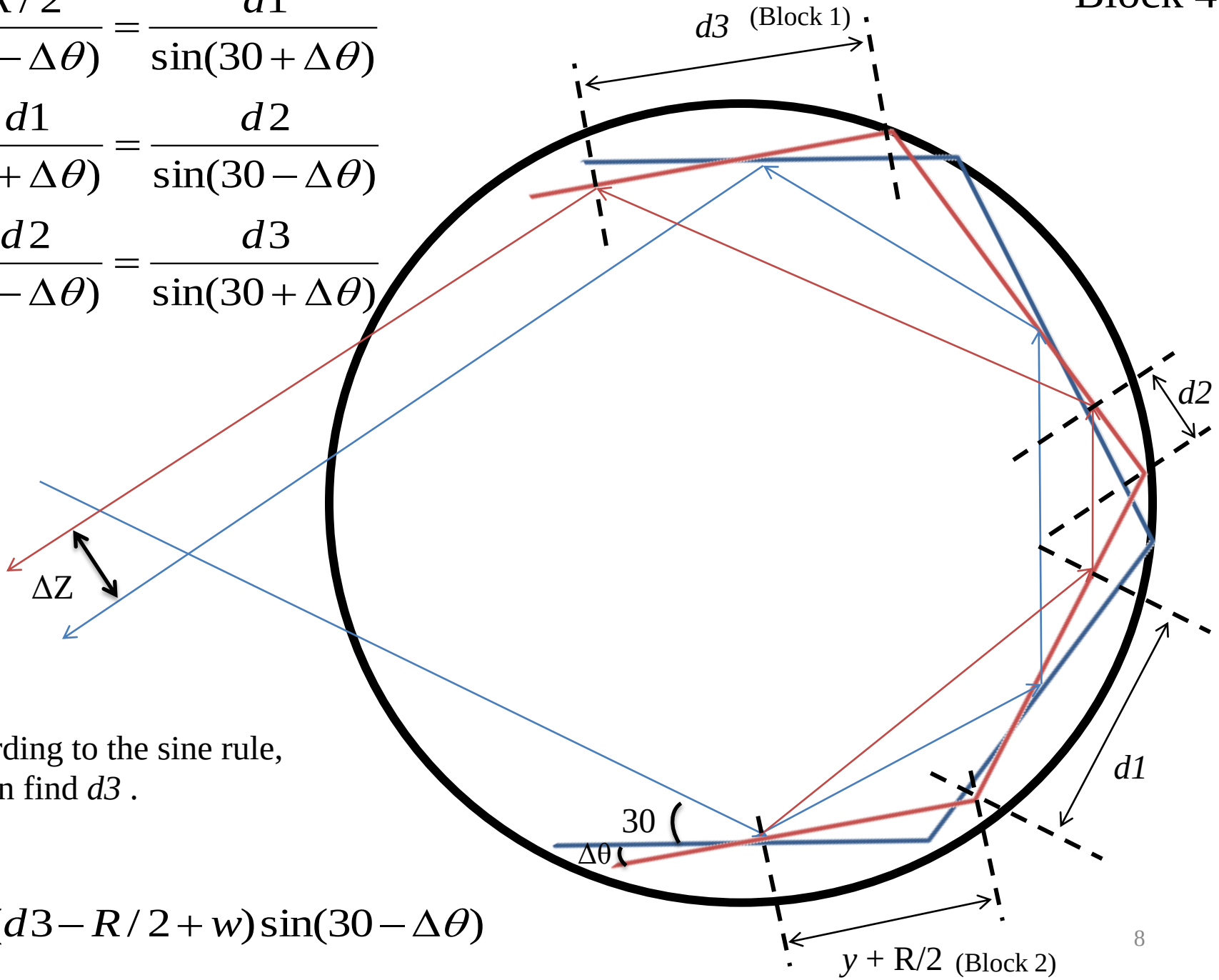
$$r = \frac{\sqrt{3}}{2} R$$



$$\frac{y+R/2}{\sin(30-\Delta\theta)}=\frac{d1}{\sin(30+\Delta\theta)}$$

$$\frac{R-d1}{\sin(30+\Delta\theta)}=\frac{d2}{\sin(30-\Delta\theta)}$$

$$\frac{R-d2}{\sin(30-\Delta\theta)}=\frac{d3}{\sin(30+\Delta\theta)}$$



According to the sine rule, we can find *d3* .

$$\Delta z=(d3-R/2+w)\sin(30-\Delta\theta)$$

$$r = \frac{\sqrt{3}}{2} R$$

$$\frac{x}{\sin(\Delta\theta)} = \frac{r}{\sin(90 - \frac{\Delta\theta}{2})}$$

$$\frac{x}{\sin(30 - \Delta\theta)} = \frac{w}{\sin(150 + \frac{\Delta\theta}{2})}$$

$$\frac{x}{\sin(150 - \Delta\theta)} = \frac{y}{\sin(30 + \frac{\Delta\theta}{2})}$$

$$\frac{y + R/2}{\sin(30 - \Delta\theta)} = \frac{d1}{\sin(30 + \Delta\theta)}$$

$$\frac{\mathrm{R} - d1}{\sin(30 + \Delta\theta)} = \frac{d2}{\sin(30 - \Delta\theta)}$$

$$\frac{\mathrm{R} - d2}{\sin(30 - \Delta\theta)} = \frac{d3}{\sin(30 + \Delta\theta)}$$

w

$d3$

Substituting *d3* and *w* into Δz.

$$\Delta z = (d3 - R/2 + w)\sin(30 - \Delta\theta)$$

$$d3 = \left\{ \mathrm{R} - \left[\mathrm{R}\frac{\sin(30-\Delta\theta)}{\sin(30+\Delta\theta)} - \left(\frac{\sqrt{3}R\sin(\Delta\theta)\sin(30+\frac{\Delta\theta}{2})}{2\sin(180-\Delta\theta-30)\cos(\frac{\Delta\theta}{2})} + \frac{\mathrm{R}}{2} \right) \right] \right\} \times \frac{\sin(30+\Delta\theta)}{\sin(30-\Delta\theta)}$$

$$w = \frac{\sqrt{3}R\sin(\Delta\theta)\sin(150+\frac{\Delta\theta}{2})}{2\cos(\frac{\Delta\theta}{2})\sin(30-\Delta\theta)}$$

***d3* and *w* Substituted into**

$$\Delta z = (d3 - R/2 + w)\sin(30-\Delta\theta)$$

Calculation A (next page)

R=L

$(R : Circumradius)$

$$\Delta z = 2\sqrt{3}\,\mathrm{L}\sin(\Delta\theta)$$

Calculation A

$$d3=\left\{R-\left[R\frac{\sin(30-\Delta\theta)}{\sin(30+\Delta\theta)}-\left(\frac{\sqrt{3}R\sin(\Delta\theta)\sin(30+\frac{\Delta\theta}{2})}{2\sin(180-\Delta\theta-30)\cos(\frac{\Delta\theta}{2})}+\frac{R}{2}\right)\right]\right\}\times\frac{\sin(30+\Delta\theta)}{\sin(30-\Delta\theta)}=\frac{3R}{2}\frac{\sin(30+\Delta\theta)}{\sin(30-\Delta\theta)}-R+\frac{\sqrt{3}R\sin(\Delta\theta)\sin(30+\frac{\Delta\theta}{2})\sin(30+\Delta\theta)}{2\sin(150-\Delta\theta)\cos(\frac{\Delta\theta}{2})\sin(30-\Delta\theta)}$$

$$w=\frac{\sqrt{3}R\sin(\Delta\theta)\sin(150+\frac{\Delta\theta}{2})}{2\cos(\frac{\Delta\theta}{2})\sin(30-\Delta\theta)}$$

$$\Delta z=(d3-R/2+w)\sin(30-\Delta\theta)$$

Substituting *d3* and w into Δz.

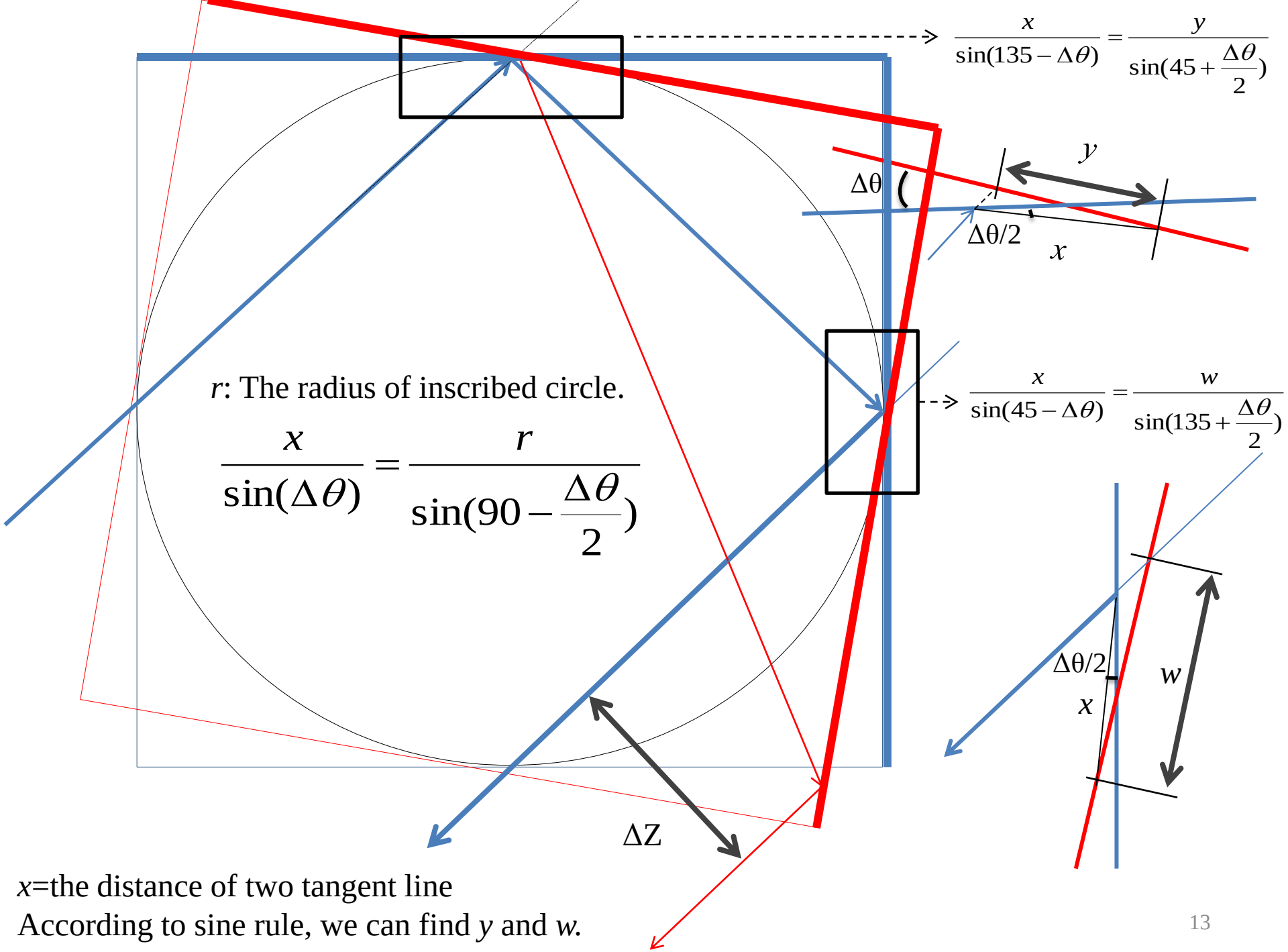
$$\Delta z=(\frac{3R}{2}\frac{\sin(30+\Delta\theta)}{\sin(30-\Delta\theta)}-R+\frac{\sqrt{3}R\sin(\Delta\theta)\sin(30+\frac{\Delta\theta}{2})\sin(30+\Delta\theta)}{2\sin(150-\Delta\theta)\cos(\frac{\Delta\theta}{2})\sin(30-\Delta\theta)}-R/2+\frac{\sqrt{3}R\sin(\Delta\theta)\sin(150+\frac{\Delta\theta}{2})}{2\cos(\frac{\Delta\theta}{2})\sin(30-\Delta\theta)})\sin(30-\Delta\theta)$$

$$\Delta z=\frac{3R}{2}\sin(30+\Delta\theta)-\frac{3R}{2}\sin(30-\Delta\theta)+\frac{\sqrt{3}R\sin(\Delta\theta)}{2\cos(\frac{\Delta\theta}{2})}\left[\sin(30+\frac{\Delta\theta}{2})+\sin(150+\frac{\Delta\theta}{2})\right];\ \sin(30+\frac{\Delta\theta}{2})+\sin(150+\frac{\Delta\theta}{2})=2\sin(30)\cos(\frac{\Delta\theta}{2})$$

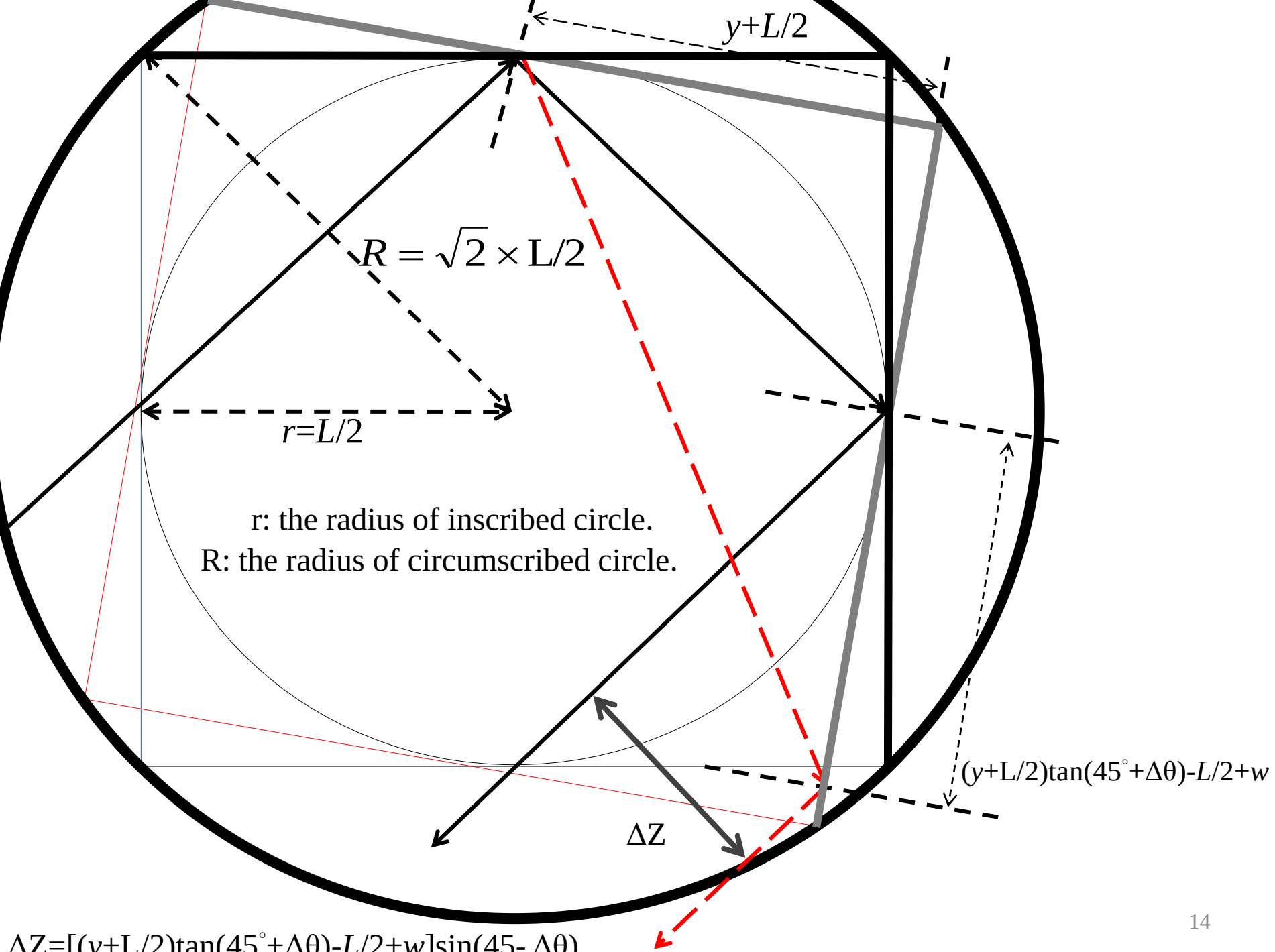
$$\Delta z=\frac{3R}{2}\times2\cos(30)\sin(\Delta\theta)+\frac{\sqrt{3}R\sin(\Delta\theta)}{2}=\frac{3\sqrt{3}R}{2}\times\sin(\Delta\theta)+\frac{\sqrt{3}R\sin(\Delta\theta)}{2}$$

$$\Delta z=2\sqrt{3}R\sin(\Delta\theta)$$

Square Mirror



x=the distance of two tangent line
According to sine rule, we can find *y* and *w*.



$\Delta Z=[(y+L/2)\tan(45^{\circ}+\Delta\theta)-L/2+w]\sin(45-\Delta\theta)$

$$x=\frac{r\sin(\Delta\theta)}{\sin(90-\frac{\Delta\theta}{2})},y=\frac{x\sin(45+\Delta\theta/2)}{\sin(135-\Delta\theta)}=\frac{r\sin(\Delta\theta)\sin(45+\Delta\theta/2)}{\sin(90-\frac{\Delta\theta}{2})\sin(135-\Delta\theta)},w=\frac{x\sin(135+\Delta\theta/2)}{\sin(45-\Delta\theta)}=\frac{r\sin(\Delta\theta)\sin(45-\Delta\theta/2)}{\sin(90-\frac{\Delta\theta}{2})\sin(45-\Delta\theta)}$$

Substituting *x, y and w into* Δz.

$$\Delta Z=[(y+L/2)\tan(45+\Delta\theta)-L/2+w]\sin(45-\Delta\theta);(L/2=r)$$

The individual multiplication of each term.

$$y\tan(45+\Delta\theta)\sin(45-\Delta\theta)=\frac{r\sin(\Delta\theta)\sin(45+\Delta\theta/2)}{\sin(90-\frac{\Delta\theta}{2})\sin(135-\Delta\theta)}\frac{\sin(45+\Delta\theta)}{\cos(45+\Delta\theta)}\sin(45-\Delta\theta)=\frac{r\sin(\Delta\theta)\sin(45+\Delta\theta/2)}{\cos(\frac{\Delta\theta}{2})}.....(1)$$

$$L/2\tan(45+\Delta\theta)\sin(45-\Delta\theta)=r\frac{\sin(45+\Delta\theta)}{\cos(45+\Delta\theta)}\sin(45-\Delta\theta).....(2)$$

$$-L/2\sin(45-\Delta\theta)=-r\sin(45-\Delta\theta).....(3)$$

$$w\sin(45-\Delta\theta)=\frac{r\sin(\Delta\theta)\sin(45-\Delta\theta/2)}{\sin(90-\frac{\Delta\theta}{2})\sin(45-\Delta\theta)}\sin(45-\Delta\theta)=\frac{r\sin(\Delta\theta)\sin(45-\Delta\theta/2)}{\cos(\frac{\Delta\theta}{2})}.....(4)$$

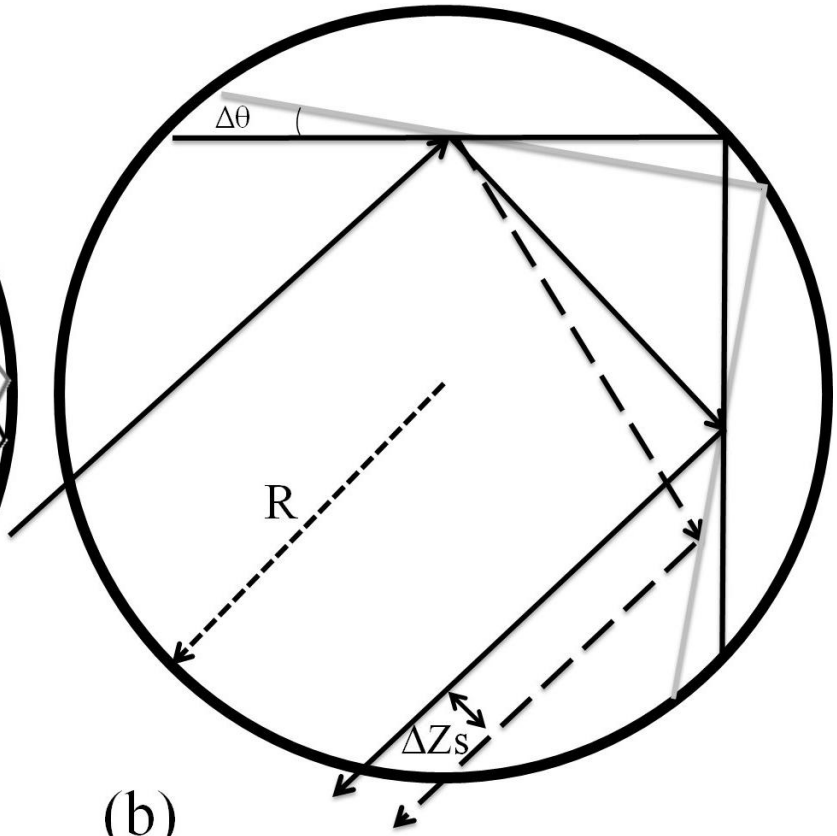
$$(2)+(3):r\sin(45+\Delta\theta)-r\sin(45-\Delta\theta)=r2\cos 45\sin(\Delta\theta).....(5)$$

$$(1)+(4):\frac{r\sin(\Delta\theta)}{\cos(\frac{\Delta\theta}{2})}(\sin(45+\Delta\theta/2)+\sin(45-\Delta\theta/2))=\frac{r\sin(\Delta\theta)}{\cos(\frac{\Delta\theta}{2})}2\sin 45\cos(\frac{\Delta\theta}{2})=r2\sin 45\sin(\Delta\theta).....(6)$$

$$(5)+(6):r2\cos 45\sin(\Delta\theta)+r2\sin 45\sin(\Delta\theta)=2r\sin(\Delta\theta)(\cos 45+\sin 45)=2\sqrt{2}r\sin(\Delta\theta)\ (r:\text{Inscribed circle radius})$$

$$\Delta Z=2\sqrt{2}r\sin(\Delta\theta)=2R\sin(\Delta\theta)\quad(\sqrt{2}r=R)\ (R:\text{Circumradius});$$

R
ΔZ
Δθ
(a)


Δθ
R
ΔZs
(b)