



Invited Lecture

An Exploration of Dynamics of the Moving Mechanism of the Growth Cone

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Abstract: A stochastic, nonlinear dynamic model is proposed to explain the growth cone at the tip of a cell process, such as a growing axon or dendrite of a neuron. The model explains the outward motion of the tip as an extension of the cytoskeleton, using the actin-myosin system as a molecular motor. The kinetic energy is supplied by heat from ATP hydrolysis in the form of random motion of water molecules embedding the actin-myosin. The mechanical structure is provided by the F-actin macromolecules forming a spiral filament. The myosin heads form a stochastic distribution of small spheres. They are attached by elastic springs to the spiral rods of the myosin filaments. Under thermal agitation the system sustains oscillation, which is directed by the interaction between the myosin heads and the actin filament. As the energy of oscillation is dissipated, the actin filament is moved toward the center of the growth cone. The joint probability density of movement of the actin filament is obtained by solving a non-stationary version of the FPK equation. By incorporating a probability distribution of actin filaments provided by the geometry of the tip, the directed motion of the tip is explained.

Keywords: Actin-myosin system, the growth cone, random noise and dissipation, stochastic elastic collision, the joint probability density, the non-stationary FPK equation.

1. Introduction

It is important to understand the mechanism of motion of the growth cone at the tip of an axon, in order to explain how it is accurately directed to its targets in the formation of functional neural networks. The various mechanisms of movement of the growth cone have been discussed [1-6, 12], which incorporate the theory of molecular clutch and slide in the actin-myosin system [3, 7, 8]. However, the source of the kinetic energy for motion remains unexplained. In this report a novel theory is proposed, by which the required energy is provided by thermal energy from the hydrolysis of ATP.

The proposed model incorporates the well documented model of the actin-myosin system [9, 11]. The actin and myosin molecules undergo random bombardment by water molecules. Owing to the elastic properties of the helical connections of the myosin molecules, the system vibrates. The distribution of the direction of oscillatory motion is guided by the network of actin filaments under the enclosing membrane of the axon tip [1, 2, 3]. The thermal energy serves to expand the center of the growth cone. It follows that the actin-myosin mechanism is a dissipative, non-equilibrium system feeding on thermal energy, and that the joint probability density is a function of the distributions of displacement and velocity of the molecules. The direction taken by the growth cone can be obtained by solving the Fokker-Planck equation [19, 20]. This model explains the outward thrust of the axon tip and the constriction in the diameter of the shaft.

Section 2 summarizes the actin-myosin model. Section 3 describes the mechanism for harnessing thermal energy to vibrations and describes the mechanism to focus the motion into the thrust of the apex of the growth cone. Section 4 gives the conclusions and discusses the problems clarified by the model for future study.

2. Mechanical model of the actin-myosin system

In this paper, according to the theory of molecular machines of the living cell we present a new idea and a new nonlinear mechanical model that can be used to describe why the growth cone can induct the reach of neuronal salience in probabilistic sense. In other words, what forces and conditions can occur at the tag end of axon and the dendrite to perform movement.

In the molecular biological sense the model originates from the actin-myosin system as a class of molecular motors, and the energy of motion of the system is supplied from the random noise of water molecules generated through heat energy of ATP hydrolysis. The structure of the model is composed of the elastic colliding spherical shapes by a set possessing spiral structure that is of the actin filament, the myosin heads and the springs (neck) by a set possessing elastic property that is of heads of the ATP

enzyme and the α -helical rods of the myosin. The model presented in the paper is supported by x-ray diffraction patterns [9,11].

In an open and non-equilibrium system, due to the interaction and the random collision among the actin molecules and the myosin molecules under random thermal noise all the actin filaments possess moving tendency along the direction of the growth cone in sense of the probabilistic statistics. In this procedure the myosin systems can work for molecules of the actin through consuming the partial energy of vibration, and the result arouses the actin filament is moved and is expand to the range of center of growth cone. It follows that the actin-myosin system is just a kind of dissipative non-equilibrium system.

The behavior procedure induces the transformation of the intermolecular energy of vibration to the intermolecular energy of translational motion in arbitrary instantaneity due to the special location of network of the actin filament formed under exterior membrane of the axon [1,2,3].

In order to understand the mechanism of movement of the growth cone a mechanical model on the actin-myosin system under the random noise of water molecules generated through heat energy of ATP hydrolysis is given in the Figure 1. Note that this imaginative figure is plotted in accordance with the molecular structure of the actin-myosin system.

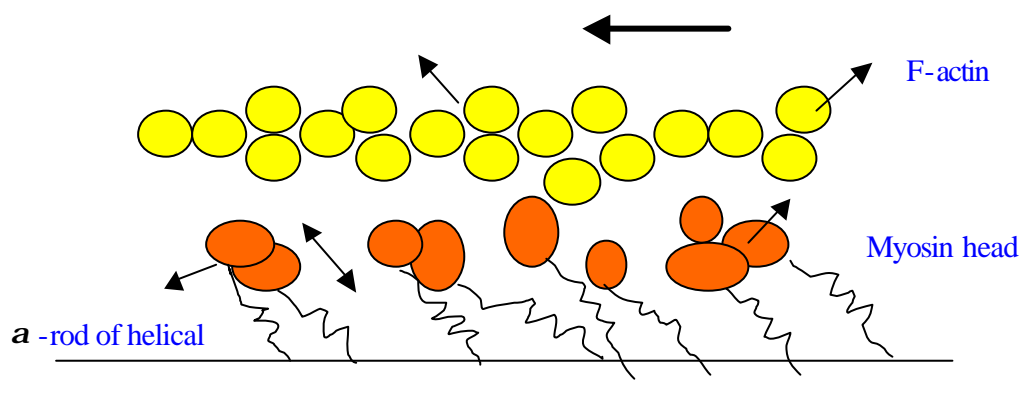


Figure 1. The imaging of the actin-myosin system

In Figure 1 we only describe the single side of the mechanical model of the molecular systems due to the actin-myosin system to be a symmetrical structure. Although the assuming of the stroke of rotating displacement of the myosin for the actin filament is here modeled to the stochastic elastic collided force that induce the movement of the actin filament, but the mechanical effect obtained in this model is in complete accordance with the stroke of rotation of the α -helical myosin presented in references [9-11]. As yet, no evidence can demonstrate a distribution of the myosin head in reference [9]. Hence, we can not give an extrapolation which point out the force of collision produced among the myosin heads and the actin filament to be counteract each other because of the stochastic distribution of the myosin heads, whereas x-ray crystallography demonstrated that the catalytic angle between the

myosin head and the actin filament is at about 45° angle given in Figure 2, and the catalytic angle is performed small fluctuate around at about 45° angle even if the catalytic angle can bring the stochastic diffusion under the random noise. The force of collision produced along with the direction at about 45° angle cause the movement of the actin filament. This point is that we establish the bases of the above mechanical model.

In the below there is a figure on *x*-ray crystallography with respect of catalytic angle between the myosin head and the actin filament given in references [9,11].

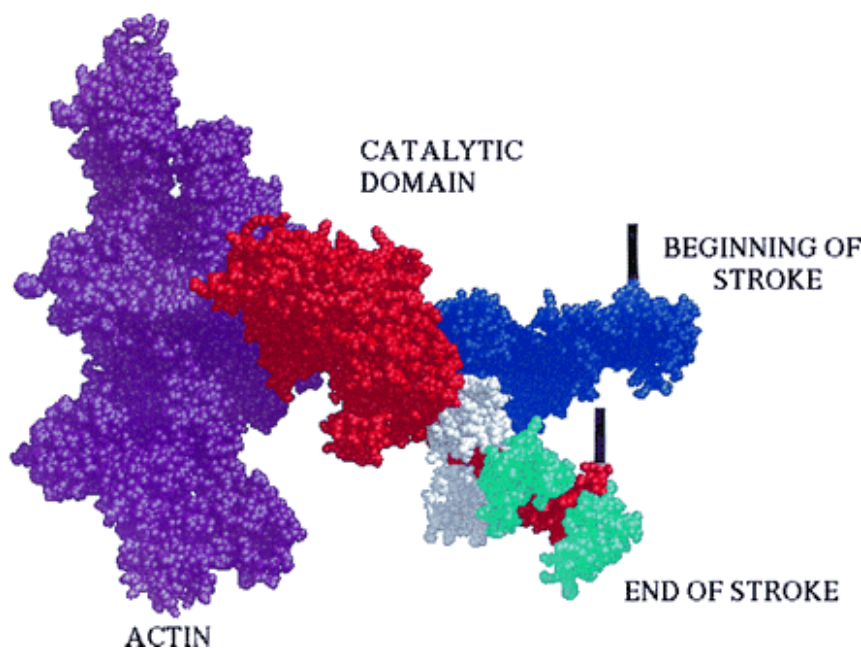


Figure 2. A model of the actomyosin complex illustrates how the spectroscopic results described by Baker et al [11]. Copyright (1998) National Academy of Sciences, U.S.A."

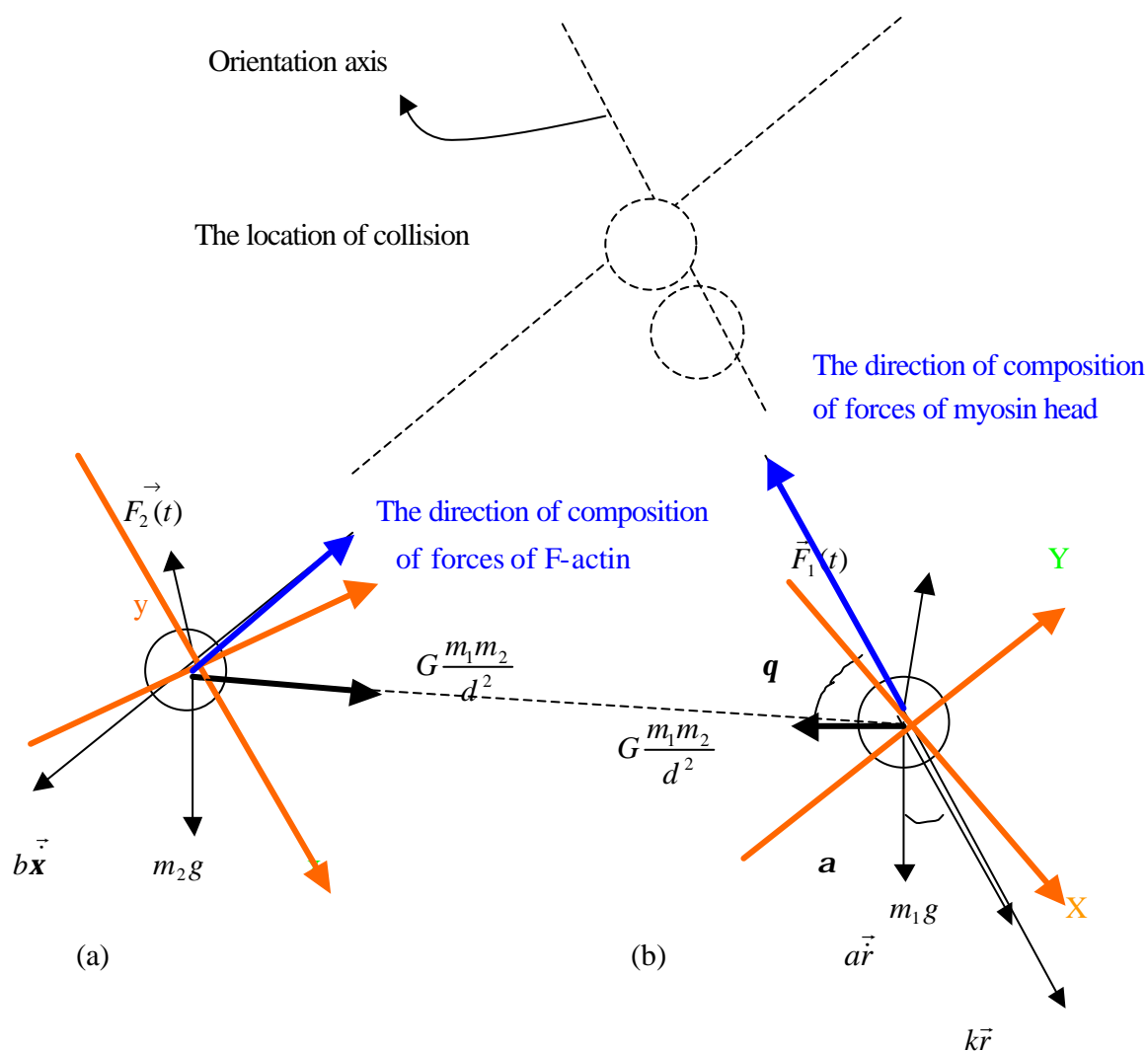
In this figure the actin filament is shown in purple on the left. The catalytic domain of the myosin head, shown in red, attaches rigidly to the actin filament, with its light chain domain extending down at about a 45° angle. The two light chains that help form this domain are shown in white and cyan, with the spectroscopic label of Baker *et al.* (11) attached to the latter. The light chain domain is also shown in a second orientation in blue, rotated upward by approximately 36° to simulate the beginning of the power stroke suggested by the results of Baker *et al.* (11). The power stroke would consist of a rotation of this domain from the upper position to the lower. This rotation would pull the tether that connects the myosin head to the thick filament, down by about 5 nm. The position of the tether is depicted schematically by the thick black vertical lines [9]."

Here is cited again the interpretation in reference [9] on the actin-myosin system. The interaction of the F-actin and myosin heads is commonly observed in three states: (1) In rigor-i.e., *rigor mortis*-the state obtained in the absence of ATP, all myosin heads are bound rigidly to actin in a configuration thought to resemble that found at the end of the power stroke; (2) In relaxation, myosin heads are

largely detached from actin and may be bound in a helical array around the thick filament or may be disordered and not bound to either filament; (3) In muscle, the myosin heads are undergoing a cyclic interaction with actin producing force. Comparing Figures 1 and 2 the mechanical model established in Figure 1 can be deemed to be completely correct.

3. The mechanism of vibration and the thrust of the apex of the growth cone

For the sake of convenience the actin-myosin systems are taken into account as two balls possessing each the center of mass modeled in the Figure 3. In this figure, taking into account the orientation axis of in collision as a fiducial axis, along the orientation of this fiducial axis performing the parallel transformation, the axis of coordinate in respect with the myosin and the F-actin can be obtained under the state of location of equilibrium.



(a) The analysis forced on the F-actin m_2 (before collision)
 (b) The analysis forced on the myosin head m_1 (before collision)

Figure 3. The analysis forced in the actin-myosin system.

Particularly, the motion equation on the center of mass of the actin filament is concerned with a step movement of the actin filament under the action of collision of the myosin head. In other words, the center of mass of the actin filament is continuously performed in way of the movement of step by step under the joint action of the stochastic and the composition forces. In short, according to the biological sense the actin filament and myosin head can be always collided although they are simplified to each centers of the mass. This interpretation is very important for understanding of the Figure 3 simplified by the Figure 1.

For the sake of simplicity the rotation occurred in the actin that is collided by the myosin head is not considered in the above mechanical model. In other words, on the analysis of forced in Figure 3, we only show the collision between the two masses in the system. According to Figure 3 we give the motion equations before collision and action time of collision for the two masses.

In here the motion equation of the center of mass of the all myosin heads before collision is given by

$$\left\{ \begin{array}{l} m_1 \ddot{X} + a\dot{X} + k \cdot X - G \frac{m_1 m_2}{d^2} \left(\frac{X}{r} \cos \mathbf{q} - \frac{Y}{r} \sin \mathbf{q} \right) - m_1 g \cos \mathbf{a} = \mathbf{I} F_1(t) \end{array} \right. \quad (1)$$

$$\left\{ \begin{array}{l} m_1 \ddot{Y} + a\dot{Y} + k \cdot Y - G \frac{m_1 \cdot m_2}{d^2} \cdot \left(\frac{Y}{r} \cos \mathbf{q} + \frac{X}{r} \sin \mathbf{q} \right) + m_1 \cdot g \sin \mathbf{a} = \sqrt{1 - \mathbf{I}^2} F_1(t) \end{array} \right. \quad (2)$$

where symbols a and k denote the viscous coefficient and the spring constant, respectively. And m_1, m_2 are the total mass of the myosin heads and the actin filament, respectively. G and g are the constant of gravitation and the gravitational acceleration, \mathbf{q} denotes the angle between the direction of motion of the myosin head and the gravitation, and \mathbf{a} is the angle between the gravity of the myosin head and X axis, and d denotes the active distant between centers of two masses, it is a function of time. In Figure 3 the vectors $r = \sqrt{X^2 + Y^2}$.

The equation of motion of the center of mass of myosin heads in the action time of collision is given by

$$\left\{ \begin{array}{l} m_1 \ddot{X} + b\dot{x} - G \frac{m_1 m_2}{d_0^2} - m_2 g \cos \mathbf{a} = \mathbf{b} F_2(t) \end{array} \right. \quad (3)$$

$$\left\{ \begin{array}{l} m_1 \ddot{Y} + a\dot{Y} + k \cdot Y + m_1 g \sin \mathbf{a} = \sqrt{1 - \mathbf{I}^2} F_1(t) \end{array} \right. \quad (4)$$

where $F_1(t)$ denotes the stochastic forces that is modeled by the Gaussian white noise, which is delta-correlated with zero mean value [6]:

$$\langle F_1(t) \rangle = 0 \quad (5)$$

$$\langle F_1(t) F_1(t + \mathbf{t}) \rangle = \mathbf{d}(\mathbf{t}) \quad (6)$$

Symbol \mathbf{I} denotes the coefficient of the stochastic force.

The motion equation of the center of mass of the actin filament before collision is given by

$$\left\{ \begin{array}{l} m_2 \ddot{x} + b\dot{x} + G \frac{m_1 m_2}{d^2} \left(\frac{X}{r} \cos \mathbf{q} - \frac{Y}{r} \sin \mathbf{q} \right) - m_2 g \cos \mathbf{a} = \mathbf{b} \cdot F_2(t) \end{array} \right. \quad (7)$$

$$\left\{ \begin{array}{l} m_2 \ddot{y} + b\dot{y} - G \frac{m_1 m_2}{d^2} \left(\frac{Y}{r} \cos \mathbf{q} + \frac{X}{r} \sin \mathbf{q} \right) + m_2 g \sin \mathbf{a} = \sqrt{1 - \mathbf{b}^2} \cdot F_2(t) \end{array} \right. \quad (8)$$

where $F_2(t)$ is the stochastic force which is modeled the Gaussian white noise, which is delta-correlated with zero mean value [6]:

$$\langle F_2(t) \rangle = 0 \tag{9}$$

$$\langle F_2(t)F_2(t+\mathbf{t}) \rangle = \mathbf{d}(\mathbf{t}) \tag{10}$$

Note that $F_1(t)$ and $F_2(t)$ are two mutual independent stochastic forces, so $\langle F_1(t)F_2(t') \rangle = 0$.

The equation of motion of the center of mass of the actin filament in the action time of collision is given by

$$\begin{cases} m_2\ddot{x} + a\dot{X} + kX - G\frac{m_1m_2}{d_0^2} - m_1g \cos \mathbf{a} = \mathbf{I} \cdot F_1(t) \end{cases} \tag{11}$$

$$\begin{cases} m_2\ddot{y} + b\dot{Y} + m_2g \sin \mathbf{a} = \sqrt{1-I^2} \cdot F_2(t) \end{cases} \tag{12}$$

where $d^2 = R^2 + (x-X)^2 + (y-Y)^2 + \frac{2R}{\sqrt{X^2+Y^2}}\{(x-X)X + (y-Y)Y\}\cos\mathbf{q} + (xY - yX)\sin\mathbf{q}$

$$\tag{13}$$

$$d_0^2 = (R + X - x)^2 + (y - Y)^2 \tag{14}$$

where symbol R denotes the distant from the balanceable location of the myosin head to the center of mass of the actin filament, and d_0 is the distant between centers of the two masses at the instantaneous of collision.

According to the above motion equations we obtain the following the Ito's stochastic differential equation on the actin-filament system

$$\vec{X} = \vec{B} + \vec{D} \cdot \vec{F} \tag{15}$$

where \vec{F} denotes a Winner process. In here the form of matrix of each vector in equation (15) in collision is given by

$$\vec{X} = \begin{bmatrix} x \\ y \\ X \\ Y \\ \dot{x} \\ \dot{y} \\ \dot{X} \\ \dot{Y} \end{bmatrix} \quad \vec{B} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{X} \\ \dot{Y} \\ -\frac{a}{m_2}\dot{X} - \frac{k}{m_2}X - G\frac{m_1}{d_0^2} + \frac{m_1}{m_2}g \cos \mathbf{a} \\ -\frac{b}{m_2}\dot{y} - g \sin \mathbf{a} \\ -\frac{b}{m_1}\dot{x} + G\frac{m_2}{d_0^2} + \frac{m_2}{m_1}g \cos \mathbf{a} \\ -\frac{a}{m_1}\dot{Y} - \frac{k}{m_1}Y - g \sin \mathbf{a} \end{bmatrix} \quad \vec{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{\mathbf{I}}{m_2} & 0 \\ 0 & \frac{\sqrt{1-b^2}}{m_2} \\ 0 & \frac{\mathbf{b}}{m_1} \\ \frac{\sqrt{1-I^2}}{m_1} & 0 \end{bmatrix} \tag{16}$$

$$\vec{F} = \begin{bmatrix} F_1(t) \\ F_2(t) \end{bmatrix}$$

Substituting equation (16) into equation (15) yields

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{X} \\ \dot{Y} \\ \ddot{x} \\ \ddot{y} \\ \ddot{X} \\ \ddot{Y} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{X} \\ \dot{Y} \\ -\frac{a}{m_2} \dot{X} - \frac{k}{m_2} X + G \frac{m_1}{d_0^2} + \frac{m_1}{m_2} g \cos \mathbf{a} \\ -\frac{b}{m_2} \dot{y} - g \sin \mathbf{a} \\ -\frac{b}{m_1} \dot{x} + G \frac{m_2}{d_0^2} + \frac{m_2}{m_1} g \cos \mathbf{a} \\ -\frac{a}{m_1} \dot{Y} - \frac{k}{m_1} Y - g \sin \mathbf{a} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{\mathbf{I}}{m_2} & 0 \\ 0 & \frac{\sqrt{1-\mathbf{b}^2}}{m_2} \\ 0 & \frac{\mathbf{b}}{m_1} \\ \frac{\sqrt{1-\mathbf{I}^2}}{m_1} & 0 \end{bmatrix} \begin{bmatrix} F_1(t) \\ F_2(t) \end{bmatrix} \quad (17)$$

Let the joint probability density of collision is

$$P = P(x, y, X, Y, \dot{x}, \dot{y}, \dot{X}, \dot{Y}, t) \quad (18)$$

Then the FPK equation corresponding to equation (17) is given by

$$\begin{aligned} \frac{\partial P}{\partial t} = & -\left\{ \dot{x} \frac{\partial P}{\partial x} + \dot{y} \frac{\partial P}{\partial y} + \dot{X} \frac{\partial P}{\partial X} + \dot{Y} \frac{\partial P}{\partial Y} + \frac{\partial}{\partial \dot{x}} \left[\left(-\frac{a}{m_2} \dot{X} - \frac{k}{m_2} X - G \frac{m_1}{d_0^2} \right. \right. \right. \\ & \left. \left. \left. + \frac{m_1}{m_2} g \cos \mathbf{a} \right) P \right] + \frac{\partial}{\partial \dot{y}} \left[\left(-\frac{b}{m_2} \dot{y} - g \sin \mathbf{a} \right) P \right] + \frac{\partial}{\partial \dot{X}} \left[\left(-\frac{b}{m_1} \dot{x} \right. \right. \right. \\ & \left. \left. \left. + G \frac{m_2}{d_0^2} + \frac{m_2}{m_1} g \cos \mathbf{a} \right) P \right] + \frac{\partial}{\partial \dot{Y}} \left[\left(-\frac{a}{m_1} \dot{Y} - \frac{k}{m_1} Y - g \sin \mathbf{a} \right) P \right] \right\} \\ & + \frac{1}{2} \left[\frac{\mathbf{I}^2}{m_2^2} \frac{\partial^2 P}{\partial \dot{x}^2} + \frac{1-\mathbf{b}^2}{m_2^2} \frac{\partial^2 P}{\partial \dot{y}^2} + \frac{\mathbf{b}^2}{m_1^2} \frac{\partial^2 P}{\partial \dot{X}^2} + \frac{1-\mathbf{I}^2}{m_1^2} \frac{\partial^2 P}{\partial \dot{Y}^2} \right] \\ & + \frac{\mathbf{I} \sqrt{1-\mathbf{I}^2}}{m_1 m_2} \frac{\partial^2 P}{\partial \dot{x} \partial \dot{Y}} + \frac{\mathbf{b} \sqrt{1-\mathbf{b}^2}}{m_1 m_2} \frac{\partial^2 P}{\partial \dot{y} \partial \dot{X}} \end{aligned} \quad (19)$$

We can not obtain the solution of closed form of the joint probability density for non-stationary FPK equation. However, when the parameters are determined, one can solve the FPK equation by means of a numerical method. This work will perform in the future.

The solution of the FPK equation (19) shows the probability density of change of moving state on the F-actin under the stochastic elastic collision. In Figure 4 we can fairly see the change of the moving direction of the F-actin in collision. The motion along the direction of composition of force is shown in red line on the F-actin. The angle between the red line and the horizontal line is just the catalytic angle between the F-actin and the myosin head in stroke shown Figure 4. In other words, the angle of collision is at about 45° angle and the catalytic angle is almost performed small fluctuating around at about 45° angle due to the influence of the stochastic noise. The moving changes of produced along this direction of composition of force causes the movement of the actin filament. This result shows that the mechanical model assumed in Figure 1 and Figure 3 as well as the mechanical analysis performed is

very effective.

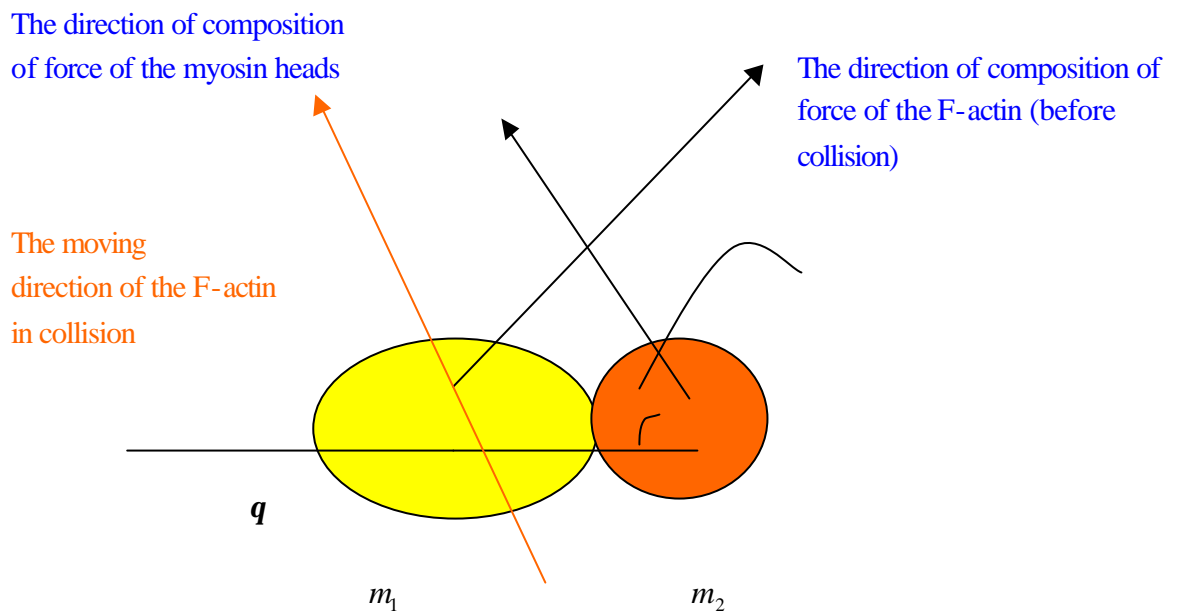


Figure 4. The state of collision on the actin-myosin system.

The form of matrix of each vector in equation (15) before collision is given by

$$\vec{X} = \begin{bmatrix} x \\ y \\ X \\ Y \\ \dot{x} \\ \dot{y} \\ \dot{X} \\ \dot{Y} \end{bmatrix} \quad \vec{B} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{X} \\ \dot{Y} \\ -\frac{b}{m_2} \dot{x} - G_1 \\ -\frac{b}{m_2} \dot{y} - G_2 - g \\ -\frac{a}{m_1} \dot{X} - \frac{k}{m_1} X + G_3 \\ -\frac{a}{m_1} \dot{Y} - \frac{k}{m_1} Y + G_4 - g \end{bmatrix} \quad \vec{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{b}{m_2} \\ 0 & \frac{\sqrt{1-b^2}}{m_2} \\ \frac{1}{m_1} & 0 \\ \frac{\sqrt{1-I^2}}{m_1} & 0 \end{bmatrix} \quad (20)$$

and $\vec{F} = \begin{bmatrix} F_1(t) \\ F_2(t) \end{bmatrix}$

where $G_1 = G \frac{m_1}{d^2} \left(\frac{X}{r} \cos \mathbf{q} - \frac{Y}{r} \sin \mathbf{q} \right)$
 $G_2 = G \frac{m_1}{d^2} \left(\frac{Y}{r} \cos \mathbf{q} + \frac{X}{r} \sin \mathbf{q} \right)$

$$G_3 = G \frac{m_2}{d^2} \left(\frac{X}{r} \cos \mathbf{q} - \frac{Y}{r} \sin \mathbf{q} \right)$$

$$G_4 = G \frac{m_2}{d^2} \left(\frac{Y}{r} \cos \mathbf{q} + \frac{X}{r} \sin \mathbf{q} \right)$$

Substituting equation (20) into equation (15) yields

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{X} \\ \dot{Y} \\ \ddot{x} \\ \ddot{y} \\ \ddot{X} \\ \ddot{Y} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{X} \\ \dot{Y} \\ -\frac{b}{m_2} \dot{x} - G_1 \\ -\frac{b}{m_2} \dot{y} - G_2 - g \\ -\frac{a}{m_1} \dot{X} - \frac{k}{m_1} X + G_3 \\ -\frac{a}{m_1} \dot{Y} - \frac{k}{m_1} Y + G_4 - g \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{\mathbf{b}}{m_2} \\ 0 & \frac{\sqrt{1-\mathbf{b}^2}}{m_2} \\ \frac{\mathbf{I}}{m_1} & 0 \\ \frac{\sqrt{1-\mathbf{I}^2}}{m_1} & 0 \end{bmatrix} \begin{bmatrix} F_1(t) \\ F_2(t) \end{bmatrix} \quad (21)$$

Let the joint probability density before collision is

$$P = P(x, y, X, Y, \dot{x}, \dot{y}, \dot{X}, \dot{Y}, t) \quad (22)$$

Then the FPK equation corresponding to equation (21) is given by

$$\begin{aligned} \frac{\partial P}{\partial t} = & -\left\{ \dot{x} \frac{\partial P}{\partial x} + \dot{y} \frac{\partial P}{\partial y} + \dot{X} \frac{\partial P}{\partial X} + \dot{Y} \frac{\partial P}{\partial Y} + \frac{\partial}{\partial x} \left[\left(-\frac{b}{m_2} \dot{x} - G_1 + g \cos \mathbf{a} \right) P \right] \right. \\ & + \frac{\partial}{\partial y} \left[\left(-\frac{b}{m_2} \dot{y} + G_2 - g \sin \mathbf{a} \right) P \right] + \frac{\partial}{\partial X} \left[\left(-\frac{a}{m_1} \dot{X} - \frac{k}{m_1} X + G_3 \right) P \right] + \frac{\partial}{\partial Y} \left[\left(-\frac{a}{m_1} \dot{Y} \right. \right. \\ & \left. \left. + G_4 - \frac{k}{m_1} Y - g - g \sin \mathbf{a} \right) P \right] \left. \right\} + \frac{1}{2} \left[\frac{\mathbf{b}^2}{m_2^2} \frac{\partial^2 P}{\partial \dot{x}^2} + \frac{1-\mathbf{b}^2}{m_2^2} \frac{\partial^2 P}{\partial y^2} + \frac{\mathbf{I}^2}{m_1^2} \frac{\partial^2 P}{\partial \dot{X}^2} \right. \\ & \left. + \frac{1-\mathbf{I}^2}{m_1^2} \frac{\partial^2 P}{\partial \dot{Y}^2} \right] + \frac{\mathbf{b}\sqrt{1-\mathbf{b}^2}}{m_2^2} \frac{\partial^2 P}{\partial \dot{x} \partial y} + \frac{\mathbf{I}\sqrt{1-\mathbf{I}^2}}{m_1^2} \frac{\partial^2 P}{\partial \dot{X} \partial \dot{Y}} \end{aligned} \quad (23)$$

Similarly, the solution of the closed form can be not obtained for equation (23), when the parameters are determined, we can solve the FPK equation (23) by means of a numerical method. This work will also perform in the future.

The solution of the FPK equation (23) shows the probability density of moving trend on the F-actin before collision or after collision. This solution is the condition of initialization of equation (19), in other words, the solution of equation (19) is the condition of initialization of equation (23), namely, solutions of equations (19) and (23) are each other's the condition of initialization.

There exist many F-actins at the lamellipodia of the growth cone, the interaction among the F-actin and the myosin in the lamellipodia and filopodia can produces a mechanical force that pushed the

F-actin into the range of center of the growth cone, according to the above analysis we know that this mechanical force is just the force of stochastic collision in the statistic sense. Under action of the force of stochastic collision the growth cone possesses a motivity of the expanse or the constriction.

4. Concluding remarks

In this paper a new nonlinear mechanical model has been proposed for the mechanism of movement of the actin-myosin system. This model can be used to interpretation the dynamic mechanism of the actin-myosin system. Due to the movement of the actin-myosin to be performed in surrounding of the random thermal noise we used the probability density function to describe behavioral procedure of the dynamic system to be more reasonable. The solutions of the joint probability density function of the FPK equation (19) can be used to describe the change of moving state on the F-actin under the stochastic elastic collision. The solutions of the joint probability density function of the FPK equation (20) can be used describe the moving tendency on the F-actin before collision or after collision. Although the numerical results on the FPK equations have been not performed, but the catalytic angle obtained under condition of collision agrees basically with the result observed by x-ray diffraction patterns. Therefore, the mechanical model presented in this paper is satisfactory. Due to the motion of the F-actin to determine the movement of the growth cone in large amount, and the lamellipodia of the growth cone exists the network formed by many F-actin, so the motion of the growth cone and the movement of the F-actin is mutual harmonious exception the behavioral procedure of the actin depolymerization and the actin polymerization. The mechanical mechanism of moving harmonious is the stochastic collided force and the random force from thermal noise. It follows that we can obtain the conclusion that the reach and constriction of the growth cone possess randomness.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (NSFC) (Number: 30270339).

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