VIBRATION ANALYSIS OF ROTOR BLADES OF A FARM WIND-POWER PLANT

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Abstract- In wind-power plants like in all engineering structures, vibrations are taken into consideration in design phases in order to avoid resonance. This condition occurs when the frequency of the exciting force coincides with one of the natural frequencies of the system, which causes dangerously large amplitudes. In the present study, natural frequencies of the Rotor blades of NACA (National Advisory Committee for Aeronautics) 4415 and NASA/Langley LS(1) 421MOD series of wind-power plants that deliver energy to be consumed by a farm family with four persons are calculated. Therefore, after designing the rotors, their natural frequencies are determined first by Rayleigh’s Method and next by finite element method. Further for both rotor blades, resonance analysis is carried out by the found excitation of external forces.

Key Words- Wind-power plant; Propeller blade; Vibration analysis; Resonance; Finite element method; Rayleigh’s Method.

1. INTRODUCTION

In engineering structures, a condition of resonance is encountered, when the frequency of the exciting force coincides with one of the natural frequencies of the system. This can result in dangerously large amplitudes, which leads the system to collapse. Therefore, in all systems exposed to vibrations and variable forces, the determination of natural frequencies is of great interest.

Wind energy, an environment-friendly renewable energy resource that is utilized in recent years more intensively, is transformed into the mechanical energy in wind-power plants through the rotor blades ([1], [2]). Nowadays, modern components of wind-power plants especially rotor blades and towers are constructed in elastic and slender characteristic that conducts these structures and components to inclination for vibration ([3], [4], [5]). Hence, vibrations in such engineering structures have to be investigated in design phases.

In this regard, El Chazly [6] carried out static and dynamic analysis of a tapered and twisted blade but with constant chord length and symmetric aerofoil-shaped cross-sections of NACA 0015 series. Maalawi and Negm [7] formulated and applied appropriate optimisation model for wind turbine design, which strongly was simplified by modelling symmetric aerofoil-shaped cross-sections as circle ones and by neglecting twist angle. Younsi et al. [8] investigated dynamical behaviours of a wind turbine blade, whose twist was neglected and its cross-sections were full, homogenous and made of
wood. Baumgart [9] developed a rod model for wind turbine blades for applications, in which a significant reduction in the number of co-ordinates was required. However, “the modelling includes uncertainties so that model shapes do not match well with experimentally found results.” Bechly and Clausen [10] chose a profile similar to a NACA 4412 series for designing a composite wind turbine blade using finite element analysis. The produced model, especially its mesh was simplified.

Nevertheless, due to maximizing energy to be obtained from the plants, the blades of NACA and NASA/LS series used by worldwide known manufacturers of the wind power plants possess the following features: Tapered (variable chord length), twisted and asymmetric aerofoil-shaped cross-sections. Thus, these characteristics require highly developed software programs for modelling and additionally for vibration analyses of blades, which were used in this study from the same reason. Moreover in this research, any assumptions that provide simplifications regarding the forms of the blades were avoided as far as possible.

In “wind farms” nowadays, wind turbine plants with high nominal powers operate. Hence a considerable amount of engineering expertise went into design of these plants. However, less attention was given to wind turbines, which produce low power in kilowatts. These plants usually deliver electrical energy to single use. For this purpose in this study, a wind-power plant that will supply energy to be consumed by a farm family with four persons was projected, after two optimal wind rotors were designed. Then, vibration analyses of the rotor blades of NACA 4415 and LS 0417 series were executed, because these rotor blades are mostly subjected to vibrations out of the components of wind-power plants.

Afterward, the each fundamental natural frequency of the both rotor blades was calculated by Rayleigh’s Method. Further, the natural frequencies and their directions were determined by the finite element method (FEM). Finally, the frequency of the exciting forces affecting the rotor blades was determined to evaluate possible resonance.

2. MATERIAL

2.1 Design of the rotor blades

In the present study, it was assumed that a person in Turkey consumes annual electrical energy of 1400 kWh. Thus, the annual electrical energy that is exhausted by a farm family with four persons was calculated to be 5600 kWh. When the annual electrical energy needed by this family will be obtained only from the wind-power plant, the radius of the rotor required is found as follows:

The power obtained from kinetic energy in air stream is determined by the Eq. (1) ([4]).

\[ P_w = C_p \frac{\rho}{2} \pi R^2 V_w^3 \]  

Here:
- \( C_p = 0.40 \) Power coefficient
- \( \rho = 1.23 \text{ kg/m}^3 \) Density of the air
- \( R \) Radius of the rotor blade [m]
- \( V_w \) Wind velocity [m/s]
In this study, the wind velocity was taken as a constant of $V_w = 6 \text{ m/s}$ that is the annual average wind velocity in wind-rich areas of Turkey ([4], [11], [12], [13]). When the wind-power affects in interval $\Delta t$, the wind energy is found as:

$$E = P_w \cdot \Delta t.$$  \hspace{1cm} (2)

The Eqs. (1) and (2) deliver the annual electrical energy that is obtained from the wind-power plant within a year by neglecting site wind characteristics, capacity and availability factor. This energy value to be obtained matches the maximal use and/or loading of the wind power plants without considering any cost analyses:

$$E_{\text{year}} = C_p \frac{\rho}{2} \pi R^2 V_w^3 8760.$$  \hspace{1cm} (3)

By means of the Eq (3),

$$5,600,000 \text{[Wh]} = 0.40 \frac{1.23}{2} \pi R^2 6^3 8760,$$

the radius of the rotor blades (of the wind-power plant) is found as $R=1.96 \text{ m}$, and it was chosen as $R=2 \text{ m}$, whereas Weibull wind distribution was neglected primarily due to focusing on the modal analysis of the blades to be determined. The blade number was determined as $z = 3$, because wind-power plants with $z = 3$ deliver generally the highest level of electrical energy ([3], [4], [5]).

In the design phase, the angle of attack was chosen as $\alpha_D = 10^\circ$ as well recommended in ([4]); the optimal twist angle was calculated by the Eq. (4):

$$\alpha_{\text{twist}} = \frac{2}{3} \arctan \left( \frac{R}{\lambda_{\text{tip}} r} \right) - \alpha_D.$$  \hspace{1cm} (4)

In Eq. (4), $r$ denotes the distance between the rotor hub and the chosen cross-section. Consequently, the profiles of the blade cross-sections were drawn for each $r$. Finally, the parameters in the rotor design of the wind-power plant were determined as follows ([4]):

i) Number of blades: $z = 3$,

ii) Radius of rotor (Blade length): $R=2 \text{ m}$,

iii) Tip-speed ratio: $\lambda_{\text{tip}} = 7$

iv) Profile of the blade: NACA 4415 and LS 0417

$$t_{\text{opt}}(r) = \frac{1.16\pi}{3 \cdot 1.3} r \sin^2 \left( \frac{1}{3} \arctan \frac{10}{7r} \right).$$  \hspace{1cm} (5)

The Eq. (5) delivers the optimal chord length of the rotor blades for each cross-section $r$ ([4]). Further, in order to design the rotor blades, blade length (radius of the rotor) was divided into 11 cross-sections of the same length. The first cross-section begins at $r = 0.05 R$ and the eleventh completes at $r = R$.

As an example, the optimal chord length of the profile at $r = 0.05 \cdot 2 = 0.1 \text{ m}$ was calculated from the Eq. (5) as follows, and this calculation was repeated for each cross-section so that Table 1 was filled out:
$$t_{opt}(0.1) = \frac{1}{3} \frac{16\pi}{1.3} \cdot 0.1 \cdot \sin^2 \left( \frac{1}{3} \arctan \frac{10}{7} \cdot 0.1 \right) = 0.2966 \text{ [m]}. $$

But the chord length at $r = 0.1R$ in Table 1 was chosen as $t = 0.2233\text{ m}$ to avoid constructive problems while assembling the blades at the rotor hub.

### 2.2 Modelling of the rotor blades

In the present study, the standard profiles of NACA 4415 and LS 0417 series were chosen for both rotor blades, and the coordinates of these profiles were given in Table 2.

**Table 1.** Optimal chord lengths calculated from the standard profiles of NACA 4415 and LS 0417 series

<table>
<thead>
<tr>
<th></th>
<th>NACA 4415</th>
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</thead>
<tbody>
<tr>
<td>$r$ [m]</td>
<td>0.1</td>
<td>0.29</td>
<td>0.48</td>
<td>0.67</td>
<td>0.86</td>
<td>1.05</td>
<td>1.24</td>
<td>1.43</td>
<td>1.62</td>
</tr>
<tr>
<td>$t_{opt}$ [m]</td>
<td>0.2233</td>
<td>0.2662</td>
<td>0.2124</td>
<td>0.1679</td>
<td>0.1367</td>
<td>0.1147</td>
<td>0.0985</td>
<td>0.0862</td>
<td>0.0765</td>
</tr>
<tr>
<td></td>
<td>LS 0417</td>
<td></td>
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<td>0.1</td>
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<td>0.48</td>
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<td>0.86</td>
<td>1.05</td>
<td>1.24</td>
<td>1.43</td>
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</tr>
<tr>
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<td>0.1679</td>
<td>0.1367</td>
<td>0.1147</td>
<td>0.0985</td>
<td>0.0862</td>
<td>0.0765</td>
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</tbody>
</table>

**Table 2.** Coordinate ratios of cross-section profiles of NACA 4415 and LS 0417 series

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<tr>
<th></th>
<th>NACA 4415</th>
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<tbody>
<tr>
<td>$x/t$</td>
<td>0</td>
<td>0.25</td>
<td>0.5</td>
<td>0.75</td>
<td>1</td>
<td></td>
<td></td>
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<tr>
<td>$y/t$</td>
<td>0</td>
<td>0.115</td>
<td>0.1</td>
<td>0.07</td>
<td>0</td>
<td></td>
<td></td>
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<tr>
<td>$y_b/t$</td>
<td>0</td>
<td>0.035</td>
<td>0.02</td>
<td>0.015</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>LS 0417</td>
<td></td>
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</tr>
<tr>
<td>$x/t$</td>
<td>0</td>
<td>0.25</td>
<td>0.5</td>
<td>0.75</td>
<td>1</td>
<td></td>
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<tr>
<td>$y/t$</td>
<td>0</td>
<td>0.115</td>
<td>0.09</td>
<td>0.065</td>
<td>0</td>
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<tr>
<td>$y_b/t$</td>
<td>0</td>
<td>0.065</td>
<td>0.055</td>
<td>0.03</td>
<td>0</td>
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</tbody>
</table>

**Table 3.** Coordinates of outer cross-section profiles of NACA 4415 and LS 0417 series for $r = 0.48\text{ m}$

<table>
<thead>
<tr>
<th></th>
<th>NACA 4415</th>
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</thead>
<tbody>
<tr>
<td>$t_{opt}=212.4\text{ mm}$</td>
<td>Outer</td>
<td>x</td>
<td>-53.1</td>
<td>0</td>
<td>53.1</td>
<td>106.2</td>
<td>159.3</td>
<td></td>
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<tr>
<td></td>
<td>$y_t$</td>
<td>0</td>
<td>24.426</td>
<td>21.24</td>
<td>14.868</td>
<td>0</td>
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<tr>
<td></td>
<td>$y_h$</td>
<td>0</td>
<td>-7.434</td>
<td>-4.248</td>
<td>-3.186</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>LS 0417</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_{opt}=212.4\text{ mm}$</td>
<td>Outer</td>
<td>x</td>
<td>-42.5</td>
<td>0</td>
<td>42.5</td>
<td>85.0</td>
<td>127.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$y_t$</td>
<td>0</td>
<td>19.55</td>
<td>15.3</td>
<td>11.05</td>
<td>0</td>
<td></td>
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<tr>
<td></td>
<td>$y_h$</td>
<td>0</td>
<td>-11.05</td>
<td>-9.35</td>
<td>-5.1</td>
<td>0</td>
<td></td>
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</table>

**Table 4.** Coordinates of inner cross-section profiles of NACA 4415 and LS 0417 series for $r = 0.48\text{ m}$

<table>
<thead>
<tr>
<th></th>
<th>NACA 4415</th>
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</thead>
<tbody>
<tr>
<td>$t_{opt}=176.6\text{ mm}$</td>
<td>Inner</td>
<td>$\alpha_{twist}=12^0$</td>
<td>x</td>
<td>-44.039</td>
<td>0</td>
<td>44.04</td>
<td>88.08</td>
<td>132</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$y_t$</td>
<td>0</td>
<td>20.258</td>
<td>17.616</td>
<td>12.331</td>
<td>0</td>
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<tr>
<td></td>
<td>$y_h$</td>
<td>0</td>
<td>-6.166</td>
<td>-3.523</td>
<td>-2.643</td>
<td>0</td>
<td></td>
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<tr>
<td></td>
<td>LS 0417</td>
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</tr>
<tr>
<td>$t_{opt}=176.6\text{ mm}$</td>
<td>Inner</td>
<td>$\alpha_{twist}=12^0$</td>
<td>x</td>
<td>-42.5</td>
<td>0</td>
<td>42.5</td>
<td>85.0</td>
<td>127.5</td>
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<tr>
<td></td>
<td>$y_t$</td>
<td>0</td>
<td>19.55</td>
<td>15.3</td>
<td>11.05</td>
<td>0</td>
<td></td>
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<tr>
<td></td>
<td>$y_h$</td>
<td>0</td>
<td>-11.05</td>
<td>-9.35</td>
<td>-5.1</td>
<td>0</td>
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</table>
The blades were modelled by filling out the solid volumes between the eleven cross-sections ([13]). Therefore first, the coordinates of the cross-sections were calculated. As an example, only the calculations for the cross-section \( r = 0.48 \text{ m} \) of the blades were given in Table 3 and 4 and all cross-sections of both were shown in Fig. 1a and Fig. 1b.

Each modelled profile cross-section describes a closed curve that consists of 8 points. As seen in Fig. 2a and Fig. 2b, to model the volume of the each blade, two guidelines that run at the beginning point of the curves were drawn. Further by means of these guidelines and the eleven cross-sections, the whole volume of the each blade was modelled as given in Fig. 3a and Fig. 3b.

**Figure 1a.** Perspective view of the eleven cross-sections of NACA 4415; **Figure 1b.** Perspective view of the cross-sections of LS 0417 series

**Figure 2a.** Jointing the blade cross-sections with guidelines for NACA 4415 series; **Figure 2b.** Jointing the blade cross-sections with guidelines for LS 0417 series
3. METHOD

3.1 Vibration analysis by Rayleigh’s method

Since the blades twisting along the blade axes (r) are slight, it was assumed that the change of initial moment of the blades in direction of the blade axes should be relatively insignificant. In addition, the wind power plants are machines with very low angular speeds (20-120 rpm). At very high wind speeds they must be stopped. Therefore the characteristics of the both blades were investigated by means of the Rayleigh’s method in the case of stopping period of the blades and in the case of its rotating period with very low angular speeds. According to Rayleigh’s Method, the natural frequency of the each blade was calculated with the Eq. (6), which is certainly the upper limit of it.

\[
\omega^2 = \frac{k}{m} = \frac{\int_0^L E(x) \left(\frac{d^2 q}{dx^2}\right)^2 dx}{\int_0^L \mu(x) dx}
\]  

In the Eq. (6), \(E\) and \(I(x)\) denote the modulus of elasticity in \(N/m^2\) and the moment of inertia in \(m^4\), respectively. In the present study, weldable aluminium alloy \(AlMg_5\) was used as material for both rotor blades. Hence, the modulus of elasticity of this material was determined as \(E = 71 \cdot 10^9\) \([N/m^2]\).

In determining cross-deflection of the blade by means of the value of \(q\) in Eq. (6), the expression \(q = (x/R)^2\) is differentiated twice according to \(x\), which is a variable for the radius of rotor or the blade length. Subsequently, the following constant is obtained:

\[
q^{\prime2} = \left(\frac{2}{R^2}\right)^2 = \text{const}
\]
Finally, the following term, in which $\rho$ is the density of the alloy $AlMg_5$ given as $\rho = 2700$ kg/m$^3$, denotes;

$$\mu(x) = \rho A(x).$$

When arranging the Eq. (6), the equation of the natural frequency is obtained as follows:

$$\omega^2 = \frac{\text{const} \cdot E}{\rho} \int_{x_0}^{x_1} I(x) dx$$

The integral terms in Eq. (6) were calculated by “MATLAB” program and the results of NACA 4415 series given below were obtained.

$$\int_{10}^{200} I(x) dx = 1.762 \cdot 10^{-7} \ [m^5] ; \quad \int_{10}^{200} A(x) \left( \frac{x}{R} \right)^4 dx = 9.155 \cdot 10^{-5} \ [m^3]$$

By means of these found values, the first natural frequencies of both rotor blades of NACA 4415 and LS 0417 series were determined as follows:

$$\omega_{\text{NACA}} = 124.63 \ [\text{rad/s}] ; \quad f_{\text{NACA}} = 19.836 \ [\text{Hz}]$$

$$\omega_{\text{LS}} = 111.62 \ [\text{rad/s}] ; \quad f_{\text{LS}} = 17.765 \ [\text{Hz}]$$

### 3.2 Vibration analysis by finite element package ANSYS

Vibration caused by the excitation of external forces is called forced vibration. The vibration is continued through periodical excitations. The amplitude of the vibration depends on system parameters and exciter characteristics. The Eq. (8) gives the general differential equation of a forced vibration:

$$[M][\ddot{u}] + [D][\dot{u}] + [K][u] = \{F(t)\}$$

In the Eq. (8), $M$, $D$, and $K$ denote the matrices of coefficients in FEM as follows ([14], [15], [16], [17]):

$$M = \sum_m M^{(m)} = \sum_m \int \rho^{(m)} H^{(m)\top} H^{(m)} dV^{(m)} ; \quad M: \text{Mass matrix of vibrating system},$$

$$D = \sum_m D^{(m)} = \sum_m \int \kappa^{(m)} H^{(m)\top} H^{(m)} dV^{(m)} ; \quad D: \text{Matrix of viscous damping coefficients},$$
\[ K = \sum_{m} K^{(m)} = \sum_{m} \int B^{(m)T} C^{(m)} B^{(m)} dV^{(m)}; \]

K: Matrix of elastic spring coefficients,
\( \rho^{(m)} \): Density of the mass of the \( m \)th element
\( \kappa^{(m)} \): Viscous damping coefficients
\( H^{(m)} \): Matrix of extension
\( B^{(m)} \): Matrix of deformation
\( C^{(m)} \): Matrix of elasticity of the \( m \)th element

\[ F(t) = R_B + R_S + R_C; \quad \text{External forces} \]

\[ R_B = \sum_{m} R_B^{(m)} = \sum_{m} \int H^{(m)T} f B^{(m)} dV^{(m)}; \quad \text{R}_B: \text{Forces of volume} \]

\[ R_S = \sum_{m} R_S^{(m)} = \sum_{m} \int H^{S(m)T} f S^{(m)} dS^{(m)}; \quad \text{R}_S: \text{Forces of surface} \]

\[ R_C = F_0; \quad \text{Single forces} \]
u: Displacement.

Free vibration begins with any excitation that delivers energy to the system, and then, this excitation disappears. When neglecting the friction and damping, the vibration is theoretically continued to infinity through a steady transformation of the potential energy into the kinetic energy and vice versa due to the displacement and elastic deformation. Indeed while transforming the mechanical energy, free vibrations decrease with time and they damp down finally due to losses caused by frictions.

Decrease of amplitudes depends on parameters of the system. The excitation determines the boundary conditions of the motion; hence, it affects amplitudes of the vibrations. Consequently, the main characteristic of this vibration is a function of physical properties of the system. In absence of viscous damping, the equation of free vibrations yields as follows:

\[ [M][\ddot{u}] + [K][u] = \{0\} \quad (9) \]

When arranging the Eq. (9) according to harmonic motion, the Eq. (10) is obtained:

\[ ([K] - \omega_i^2[M])[u_i] = \{0\} \quad (10) \]

In Eq. (10), \( \omega_i^2 \) denotes eigenvalues, and \( i \) shows degree of freedom, and the terms \( \{u_i\} \) mean eigenvectors. The square root of the eigenvalue delivers the natural frequencies \( \omega_i \). The eigenvectors of \( \{u_i\} \) express the modes. “Mode extraction” in ANSYS as a subprogram calculates eigenvalues and eigenvectors ([17]).

The lowest potential and deformation energy causes the first mode that explains the natural frequency. The second and third modes need more energy; consequently, they possess more deformation energy. Generally, many natural frequencies have to be calculated in engineering structures that are exposed to harmonic forces. This obtained information is used in dynamic analyses.

The modes, like in natural frequencies, depend on the weight and rigidity of structures and on distribution of the mass. Additionally, the aerodynamic elasticity of the rotor blades of wind-power plants has significant influence on vibrations and mode shapes of them. The mass moment of inertia can reflect the combined effect of all these
While increasing the inertia moment of mass, the natural frequency decreases due to the inverse ratio.

The aim of the modal analysis is to determine the natural frequencies of a system in order to avoid resonance so that the natural frequencies do not take place in the frequency range of excitations of the system. In this context, the FEM that is used for modal analysis of complicated engineering structures is very suitable and sometimes the only method.

In the commercial software ANSYS, different methods for calculating vibration modes are available. In the present study, the method of “Block Lanczos” was chosen for the following reasons, although it requires great data banks ([17]):

- Satisfactory results in analyses of complicated models including solid elements, shell elements and beams are obtained.
- It is suitable for modes, whose boundary conditions are not given.
- It is favourable for analyses of modes that have to be calculated in a defined interval.

In this study, the characteristics of the both blades also were investigated by means of the developed software based on the FEM in the case of stopping period of the blades and in the case of its rotating period with very low angular speeds, as explained in section 3.1. In order to execute modal analysis within the ANSYS program, first, the blades were modelled and meshed (Fig. 4a and Fig. 4b). Then, type of the analysis was chosen; the properties of the material and the boundary conditions were applied. Subsequently, calculations were executed. The stresses that occur due to vibrations were determined and visualised in Fig. 5. So, necessary vibration modes and directions of rotations were obtained as given in Table 5.

Figure 4a. Meshed model of the rotor blade of NACA 4415 series; Figure 4b. Meshed model of the rotor blade of LS 0417 series
Figure 5. Visualising of stress distribution of NACA 4415 series

4. RESULTS OF ANALYSIS AND CONCLUSION

Aerodynamic and elastic forces occurring cause aero-elastic vibrations, and these vibrations arise around all three axes. First, rotor blades generally vibrate across to the plane, in which the rotor rotates (1st mode, Table 5). The rotor blades clap into the wind direction and vice versa (clap vibration) ([4]). Further, rotor blades vibrate in the plane, in which the rotor rotates (2nd mode, Table 5). A typical vibration form of a rotor with three blades is such that two blades simultaneously in rotation direction and one blade in other direction vibrate (sway vibration) ([4]). These both types of vibrations in 3rd, 4th, 5th modes, and these vibrations and additionally torsional ones around all three axes in higher modes occur simultaneously (Table 5).

The kinetic energy in the wind stream is transformed into the mechanical energy in the rotor shaft through the rotor blades. In this study, it was assumed that the generator, which is driven by the rotor shaft, is a synchronous generator. Hence, the rotational speed of the rotor shaft remains as constant at all events. As determined in section “design of rotor blades”, the following parameters were used in the Eq. (10):

\[ \Omega = 21 \text{ rad/s} \quad \text{or} \quad n = 200.6 \text{ rpm}. \]

The 1st and 2nd modes of the natural frequencies of the both rotor blades arise outside of the frequency range of the exiting force such as \( n_{1 \text{ NACA}} = 1188.4 \) rpm; \( n_{2 \text{ NACA}} = 2711.9 \) rpm and \( n_{1 \text{ LS}} = 1059.0 \) rpm; \( n_{2 \text{ LS}} = 2374.7 \) rpm so that any danger of resonance does not exist for the both rotor as given in Fig. 6.
Figure 6. Resonance zones of the rotor blade of NACA 4415 and LS 0417 series

Table 5. Values of modal vibrations and rotational directions of NACA 4415 and LS 0417 series

<table>
<thead>
<tr>
<th>Mode</th>
<th>NACA 4415</th>
<th></th>
<th>LS 0417</th>
<th></th>
<th>Type of vibration</th>
<th>Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>f [Hz]</td>
<td>ω [rad/s]</td>
<td>f [Hz]</td>
<td>ω [rad/s]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>19.806</td>
<td>124.45</td>
<td>17.649</td>
<td>110.89</td>
<td>Bending</td>
<td>Around x axis</td>
</tr>
<tr>
<td>2</td>
<td>45.197</td>
<td>283.99</td>
<td>39.578</td>
<td>248.68</td>
<td>Bending</td>
<td>Around y axis</td>
</tr>
<tr>
<td>3</td>
<td>80.061</td>
<td>503.04</td>
<td>69.774</td>
<td>438.40</td>
<td>Bending</td>
<td>Around x and y axis</td>
</tr>
<tr>
<td>4</td>
<td>142.93</td>
<td>898.06</td>
<td>126.55</td>
<td>795.14</td>
<td>Bending</td>
<td>Around x and y axis</td>
</tr>
<tr>
<td>5</td>
<td>221.13</td>
<td>1389.4</td>
<td>181.68</td>
<td>1141.53</td>
<td>Bending</td>
<td>Around x and y axis</td>
</tr>
<tr>
<td>6</td>
<td>299.74</td>
<td>1883.4</td>
<td>269.74</td>
<td>1694.83</td>
<td>Torsional bending</td>
<td>Around x and y axis</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Torsion: Around z axis</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>398.93</td>
<td>2506.6</td>
<td>360.42</td>
<td>2264.59</td>
<td>Torsional bending</td>
<td>Around x and y axis</td>
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<td></td>
<td></td>
<td>Torsion: Around z axis</td>
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</tr>
<tr>
<td>8</td>
<td>520.33</td>
<td>3269.4</td>
<td>462.7</td>
<td>2907.23</td>
<td>Torsional bending</td>
<td>Around x and y axis</td>
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<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>Torsion: Around z axis</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>539.82</td>
<td>33918</td>
<td>599.18</td>
<td>3764.76</td>
<td>Torsional bending</td>
<td>Around x and y axis</td>
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<td></td>
<td></td>
<td>Torsion: Around z axis</td>
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<td>10</td>
<td>644.53</td>
<td>4049.7</td>
<td>692.34</td>
<td>4350.1</td>
<td>Torsional bending</td>
<td>Around x and y axis</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td>Torsion: Around z axis</td>
<td></td>
</tr>
</tbody>
</table>

In this study, two rotor blades for a wind-power plant that produces electrical energy for a farm family with four persons were designed. The vibration analyses of the blades from NACA 4415 and LS 0417 series were executed first by Rayleigh’s Method and next by the FEM. By the both methods, the 1st modes of free vibrations of the blades were calculated as $\omega_{1\text{ NACA}} \approx 124.5 \text{ rad/s}$ and $\omega_{1\text{ LS}} \approx 111.0 \text{ rad/s}$. The higher modes of the natural frequencies and the rotational directions for both blades were
determined by the FEM. The frequency of the forced vibration was found to be \( f_F = 3.343 \text{ Hz} \) that is very distant from possible resonance zones. Thus, it cannot be expected under these operating conditions that the blades and/or rotors do not get into resonance.

5. REFERENCES