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# Entropic localization in non-unitary Newtonian gravity.

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**Abstract:** The localizing properties and the entropy production of the Newtonian limit of a nonunitary version of fourth order gravity are analyzed. It is argued that pure highly unlocalized states of the center of mass motion of macroscopic bodies rapidly evolve into unlocalized ensembles of highly localized states. The localization time and the final entropy are estimated.

**Keywords:** Decoherence, Localization and gravity.

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Dealing with the quantum limits of the second law, a strongly related, somehow even unavoidable, issue is that of its quantum foundations: "...in order to gain a better understanding of the degrees of freedom responsible for black hole entropy, it will be necessary to achieve a deeper understanding of the notion of entropy itself. Even in flat space-time, there is far from universal agreement as to the meaning of entropy – particularly in quantum theory – and as to the nature of the second law of thermodynamics"[1]. A way out of the subjective and vaguely defined procedure of coarse graining is the assumption that even the evolution of a closed system is affected by a fundamental nonunitarity, as suggested by black hole formation and evaporation[2]. On the other hand the possibility that gravity may lead to nonunitary evolution was invoked also with reference to the measurement problem and the transition to classicality [3].

The rationale for such an assumption should finally be found in a future full theory of quantum gravity. In the meantime in some recent papers[4, 5] a specific non-Markovian nonunitary model for Newtonian gravity, without any free parameter, was derived as the nonrelativistic limit of a nonunitary version of fourth order gravity.

While for the non-Markovian nonunitary models considered by Unruh and Wald the basic idea is to have the given system interacting with a "hidden system" with "no energy of its own and therefore... not... available as either a net source or a sink of energy" [6], in the present model energy conservation is granted by the "hidden system" being a copy of the physical system, coupled to it only by gravity, and constrained to be in its same state and then to have its same energy. The unitary dynamics and the states referred to the doubled operator algebra are what we call respectively meta-dynamics and meta-states, while, by tracing out the hidden degrees of freedom, we get the non-unitary dynamics of the physical states. Pure physical states correspond then to meta-states without entanglement between physical and hidden degrees of freedom.

In particular it has been shown that, while reproducing the classical aspects of the Newtonian interaction, the model gives rise to a threshold, which for ordinary condensed matter densities corresponds to $\sim 10^{11}$ proton masses, above which self-localized center of mass wave functions exist. Moreover an initial localized pure state undergoes an entropic spreading, namely it evolves (very slowly in time ) into an unlocalized ensemble of localized states. That is consistent with the expectation that (self-)gravity may produce a growing entropy in a genuinely isolated system, as suggested by black hole physics.

We give here a concise definition of the model. Let $H[\psi^\dagger, \psi]$ be the non-relativistic Hamiltonian of a finite number of particle species, like electrons, nuclei, ions, atoms and/or molecules including also the halved gravitational interaction, where $\psi^\dagger, \psi$ denote the whole set $\psi_j^\dagger(x), \psi_j(x)$ of creation-annihilation operators, i.e. one couple per particle species and spin component. $H[\psi^\dagger, \psi]$ includes the usual electromagnetic interactions accounted for in atomic, molecular and condensed-matter physics. To incorporate that part of gravitational interactions responsible for non-unitarity, one has to introduce complementary creation-annihilation operators $\widetilde{\psi}_j^\dagger(x), \widetilde{\psi}_j(x)$ and the overall (meta-)Hamiltonian

$$H_G = H[\psi^\dagger, \psi] + H[\widetilde{\psi}^\dagger, \widetilde{\psi}] - \frac{G}{2} \sum_{j,k} m_j m_k \int dx dy \frac{\psi_j^\dagger(x)\psi_j(x)\widetilde{\psi}_k^\dagger(y)\widetilde{\psi}_k(y)}{|x-y|} \quad (1)$$

acting on the product $F_\psi \otimes F_{\widetilde{\psi}}$ of the Fock spaces of the $\psi$ and $\widetilde{\psi}$ operators, where $m_i$ is the mass of the $i$-th particle species and $G$ is the gravitational constant. The $\widetilde{\psi}$ operators obey the same statistics as the corresponding operators $\psi$, while $[\psi, \widetilde{\psi}]_- = [\psi, \widetilde{\psi}^\dagger]_- = 0$.

The meta-particle state space $S$ is the subspace of $F_\psi \otimes F_{\widetilde{\psi}}$ including the meta-states obtained from the vacuum $||0\rangle\rangle = |0\rangle_\psi \otimes |0\rangle_{\widetilde{\psi}}$ by applying operators built in terms of the products $\psi_j^\dagger(x)\widetilde{\psi}_j^\dagger(y)$ and symmetrical with respect to the interchange $\psi^\dagger \leftrightarrow \widetilde{\psi}^\dagger$, which, then, have the same number of $\psi$ (physical) and $\widetilde{\psi}$ (hidden) meta-particles of each species. As for the observable algebra, since constrained meta-states cannot distinguish between physical and hidden operators, it is identified with the physical operator algebra. In view of this, expectation values can be evaluated by preliminarily tracing out the $\widetilde{\psi}$ operators. In particular, for instance, the most general meta-state corresponding to one particle states is represented by

$$||f\rangle\rangle = \int dx \int dy f(x,y)\psi_j^\dagger(x)\widetilde{\psi}_j^\dagger(y)\,|0\rangle\,, \quad f(x,y) = f(y,x). \tag{2}$$

This is a consistent definition since $H_G$ generates a group of (unitary) endomorphisms of $S$.

Consider now a uniform matter ball of mass $M$ and radius $R$. Within the model the Schroedinger equation for the meta-state wave function $\Xi(X,Y,t)$ is given by

$$i\hbar\frac{\partial \Xi}{\partial t} = \left[-\frac{\hbar^2}{2M}(\nabla_X^2 + \nabla_Y^2) + V(|X-Y|)\right]\Xi \equiv H_G \Xi \tag{3}$$

where $X$ and $Y$ respectively denote the position of the center of mass of the physical body and of its hidden partner, while $V$ is the (halved) gravitational mutual potential energy of the two interpenetrating meta-bodies, whose explicit form can be found in Ref. [7].

In particular in Ref. [7] it was checked numerically that a slightly unlocalized pure state for such a matter ball just above the mass threshold evolves slowly into a mixed state with a small nonvanishing von Neumann entropy. Since such a state has approximately vanishing coherences for space points farther than the width of a highly localized state, it was argued that it can be obtained by tracing out the hidden degrees of freedom from a linear combination of highly localized bound metastates. In the present paper we want to show that a highly unlocalized pure state may evolve into an entropic unlocalized ensemble of highly localized states in a short time, even though meta-energy conservation prevents the formation of highly localized bound metastates. In order to do that, consider an initial Gaussian wave function $\Psi(X) \propto \exp[-X^2/\Lambda_0^2]$, corresponding to an unentangled meta-wavefunction

$$\begin{aligned}\Xi\,(X,Y;t_0) &= \Psi(X)\Psi(Y) \propto \exp[-(X-Y)^2/(2\Lambda_0^2)]\exp[-(X+Y)^2/(2\Lambda_0^2)] \equiv \\ \Phi(X-Y,t &= 0)\Theta(X+Y,t=0),\end{aligned} \tag{4}$$

and assume that $\Lambda_0$ is large enough to make (the expectation of) kinetic meta-energy irrelevant with respect to potential meta-energy, which, assuming $\Lambda_0 \gg R$, can be identified with the Newton energy for point particles. On the other hand, by the quantum virial theorem, (assuming, as it is natural, that the metastate can be well approximated by a linear combination including bound metastates only) we have for the time average $\overline{\langle K \rangle}$ of the expectation $\langle K \rangle$ of the kinetic energy $K$ of the relative motion:

$$\overline{\langle K \rangle} \simeq - \langle H_G \rangle \simeq \frac{GM^2}{\Lambda_0}. \tag{5}$$

The corresponding wave function $\Phi(X - Y, t)$ at a generic instant of time $t$ is expected to give an expectation for $K$ approximately coinciding with its time average, the initial value being exceptional. This corresponds to a typical length for phase variations of $\Phi(X - Y, t)$ given by

$$\Lambda \sim \frac{\hbar}{\sqrt{2M \langle K \rangle}} \sim \hbar \sqrt{\frac{\Lambda_0}{GM^3}}, \tag{6}$$

by which, if we consider the physical state

$$\rho(X, X', t) = \int dY \, \Phi(X - Y, t) \Theta(X + Y, t) \Phi^*(X' - Y, t) \Theta^*(X' + Y, t), \tag{7}$$

we have that (while the factors $\Theta(X+Y, t)$ and $\Theta^*(X'+Y, t)$ are slowly varying in space, since, for $M$ and $\Lambda_0$ large enough they essentially coincide with their value at $t = 0$, apart from an exceedingly slow spreading) the factors describing the relative motion have very rapid phase variations on the typical space scale $\Lambda$, this giving rise to cancellations except for $|X - X'| \leq \Lambda$, when the product $\Phi(X - Y, t)\Phi^*(X' - Y, t)$ is approximately real and positive.

We can conclude that, after a typical localization time $\tau_l \sim \hbar\Lambda_0/(GM^2)$, necessary for the Newton interaction to affect the state, the physical state $\rho(X, X', t)$ has vanishing coherences for $|X - X'| \gg \Lambda$, by which it is natural to assume that it can be represented as an ensemble of localized states with a localization length $\sim \Lambda$. The number $N$ of these states can be approximately evaluated as

$$N \sim \frac{\Lambda_0^3}{\Lambda^3} = \frac{\Lambda_0^{3/2} G^{3/2} M^{9/2}}{\hbar^3}, \tag{8}$$

which, in the assumption that $N$ is very large, allows to estimate the entropy as that of an ensemble of equiprobable states, namely as $S \simeq K_B \ln N$.

To give a numerical example, consider the case $M = 10^{-9} g$, $R = 10^{-3} cm$, $\Lambda_0 = 10^{-1} cm$, which gives $\Lambda \sim 10^{-10.5} cm$, $N \sim 10^{28.5}$, $S \simeq K_B (\ln 10) 28.5$, $\tau_l \sim 10^{-3}$ sec. It is immediate to check that the approximate equalities in Eq. (5), depending on $\Lambda_0$ being large enough, are in such a case practically exact. Furthermore an explicit evaluation gives $\langle H_G \rangle = -3.31 \cdot 10^{-24} erg$, $\sqrt{\langle H_G^2 \rangle - \langle H_G \rangle^2} = 3.09 \cdot 10^{-24} erg$, which is consistent with our assumption that the meta-state has a very small projection on the subspace of scattering metastates. Finally the spreading time of the center of mass motion is $\sim 10^{16}$ sec.

Some comments are in order to the present result. First, with reference to the quantum foundations of the second law, it shows only that in principle the present model can give rise to a growth of the von Neumann entropy even for a closed system. Of course this is possible because we start from a pure physical state, which within the model is a highly unlikely state for the center of mass motion of a macroscopic body. It is the analogue, for the entropy of the inner degrees of freedom of an ordinary matter system, of a state very far from equilibrium, while the localization process plays the role of the evolution towards equilibrium. To be specific, in the numerical example the principal quantum numbers of the hydrogen-like bound metastates corresponding to the average energy are $\sim 10^{10}$. This means that so many stationary metastates enter in the expression of the time dependent metastate that we have a substantial ergodicity with extremely long recurrence times. However possible in principle, the explicit evaluation, within the model, of the entropy growth for a realistic matter system is a rather hard task, which led us to consider a highly idealized initial state just of the center of mass motion of a matter ball.

As to the measurement problem, the initial pure state can be seen as a delocalized pointer state, while the final mixed one is the result of a dynamical wave function reduction. (The inclusion of a measured microscopic system would not change the picture [5].)

The most attractive feature of the model is precisely this possibility to address simultaneously, by a nonunitary version of Newtonian gravity without any free parameter, classically equivalent to the standard one, two unsettled questions: the quantum foundations of the second law and the transition to classicality. Another feature of the nonrelativistic model is the absence of any obstruction to its special-relativistic extension, at variance with other localization models [8].

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