Mediated Character of Economic Interactions

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Abstract: Economic interactions are conducted between economic agents - individuals and collectives, through exchange of natural or artificial entities - goods, services and money, in a myriad of combinations. In this article we adopt a microscopic point of view, concentrating on the exchanged entities, and extracting their relevant attributes as seen from structurally simple economic processes. Following that, we approach the interpretation of the economic interactions emphasizing their mediated character. Mediators of the interaction are locally available environment units. They are locally recognized and appropriately interpreted in a given value set as goods and money. The overall intensity of economic interactions considered is related to mediators’ spatial and temporal characteristics. Extracted characteristics of mediators and economic processes are compacted in the set of formal rules. The approach is connected with similar approaches in economy and physics.

Keywords: Economic processes, economic interactions, mediators, agents.

Introduction

There have been many realizations of money throughout history [1]. Moreover, the development of appropriate forms of money is still unfinished. A large number of types of objects and artifacts that have served as money share several functions: (i) a transactional function, (ii) a value storage function,
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and (iii) the function of a common measure of value [1,2]. If one concentrates on the transactional function, money belongs to a class of means for realization of a particular mode of communication among agents, and an established scale for measuring agents' wills to communicate in that way. This interpretation brings about the idea to look at money and its functioning in a society from the point of view of communications. For example, if money is considered as a representation of trust [2], then it enables a pair of humans to communicate with a level of trust that would not exist otherwise.

The starting point regarding the approach that is adopted here, is to look at money, other means of exchange and exchanged goods as means for mediating a specific type of communication between agents, which is conventionally known as an economic process. The valid communication includes mutual understanding and relationship. The former means that the communication be comprehensible and true, and the latter that agents consider it as trustworthy and appropriate, respectively [3]. The mutual understanding further implies that communication’s relevant characteristics are manifest in the very communicative action, while the relationship indicates that the communication is conducted following the widely accepted rules. The exchanged goods are considered as mediators within the economic system. The approach to description of economic interactions, in which the attributes of goods exchanged are considered explicitly, is regarded as mediated economic interaction (MEI). If the restriction to economic systems is released, then one encounters the general human interaction, in which a large number of mediator types is involved. Mediators in such an approach are generally exchangeable structures. The corresponding approach to description of human interaction as a set of processes including all available exchangeable structures is referred to as mediated human interaction [4].

The interpretation of economic processes as considered here has similarities with the praxeological analysis of economic processes [5], as the individual characteristics are manifestly crucial. However, while in praxeology the emphasis is put on the analysis of an individual's behavior in the economic context, in MEI the emphasis is put on the very mediator attributes, i.e. the way they influence individual behavior. The specifics of individual behavior are included at the beginning, and at the end the individual behavior is analyzed as influenced by the mediators. Between these two points, as stated, mediator attributes are considered, and are interpreted in the rather formal categories existing in the physical description of mediated interactions [6].

Mediator is a term used already in different fields [6 - 9]. In all fields where it is encountered it is a structure connecting two other structures, like two physical entities [6], bacteria [7], two humans in a triad, other groups of humans, animals, or general agents [8], or a human and a machine [9]. Depending on the context, mediators are given different attributes. The notion of mediators used here follows the closely related notion of mediators in theoretical physics [6]. There, fundamental interactions among elementary particles are described as conducted through a set of mediators. For example, from the point of view of mediator exchange, electrons interact through exchange of virtual photons, the quantized excitations of vacuum. This similarity is to be taken with some caution, as formalism of physics is highly developed consequence of a natural sciences’ positivism. To the contrary, positivistic elements in economics are not accompanied with correspondingly developed formalism, thus only a part of existing physics formalism is expected to have readily extractable counterparts in economy. Realizations of mediators in different, yet similar, descriptions of complex
phomena are given [10]. The approaches to social, political, economic or other human systems, and other natural realizations of complex systems with a reference to formalism from physics and chemistry are numerous [11 - 18]. The microscopic approaches [11, 12] and approach of mediators are rather similar. While seemingly the macroscopic approaches are not related to it, their broad field of applications [13 - 18], and in particular global character [12, 13, 18] provide one with a basis for inference about microscopic origin of phenomena encountered, of which mediators are one example.

The concept underlying this article - the concept of mediated human interaction [4] in the economic context - is developed through a constructive procedure, in which a part of existing economical descriptions [5], and a part of existing physical descriptions [6] are used in formulating a more formalized interpretation of economic terms. Such an approach is needed in order to obtain a basis sufficiently precise for development of analogue of theoretical physics concept of mediated interaction. The conceptual character is clearly seen from the text, because the similarities and differences of underlying structures are analyzed. The specific applications of the concept are money and the description of structurally simple economic interactions. Money is interpreted as a mediator that is used only as an exchangeable in economic interactions. In many cases money is not present naturally, i.e. spontaneously, in agent environment, what is more pronounced in technologically more developed human systems. Agents are individuals or economically active organizations. Structurally simple economic interactions are those that involve few mediators. The structural simplicity does not mean that MEI is not applicable otherwise. This simplification is introduced in order to add to understandability of the formalism presented in the text.

The structure of the article is a series of combinations of a well-known economic facts and its interpretation using terms developed originally within theoretical physics. In particular, the very economic interaction is interpreted as a mediated interaction between two agents. Following that, some regularly encountered attributes of mediators in economic interactions, e.g. the mediator energy, mass and duration, are related to the physical characteristics of regularly encountered mediators in physics.

The outline of the paper is as follows. In the second section auxiliary terms used to clarify the notion of mediators are introduced and described. They are described here in order for the article to be self-contained, while more detailed description is given elsewhere [4]. In the third section a similar approach is undertaken for exchangeables in economic processes. In that section, a quantitative description of elementary economic processes is given, and illustrative results obtained. The central quantity is a measure of economic process intensity. In the fourth section, as an example, a qualitative comparison of barter and monetary processes is presented.

**Hierarchy of environment excitations**

An environment excitation is a part of the environment of an agent system, which is *in some* value set attributed different characteristics from the rest of environment. Using that notion, the environment is represented as a set of environment excitations, with the number of their types depending on the relevant value set. This notion includes both of the traditional attributes: "natural" and "artificial". Moreover, the ways of obtaining some structure do not preclude its naming as an environment excitation.
Only a small part of possible environment excitations is used significantly in a particular agent system. Because of that, a notion of environment excitation is not very useful in the analysis of the connections between the agents’ value set and the plurality of environment excitations used in regular agent dynamics. Some of the environment excitations are recognized in a local agent system. The recognition denotes that their existence and functioning is represented in the local value set. An elementary excitation which is attributed a specific function in a given value set, is called the elementary environment excitation (EEE) within that value set.

The very adjective *elementary* means that excitations are attributed established functions in a local agent system, and points to their recognition. For example, a wheel has been an EEE in many societies for several thousand years. It is furthermore an EEE that is traditionally considered artificial, in that only its raw materials and not the total construction occur spontaneously in human environment. There are many other examples of elementary environment excitations: food, furniture, weapons, buildings, words, news, newspapers, Internet, stories, proverbs, songs, … in fact all structures which we consider part of our environment. The adjective *elementary* is to be interpreted strictly in connection with the referent value set of an agent system. It includes the existence of a function attributed only to that EEE. However, the very presence of a function does not mean that the corresponding part of environment is the least part of environment which is used in performing that function in the agent system. Therefore, the notion of elementary in EEE is needed. Furthermore, the convention that value set which is relevant for some system is the functional value set for that system is adopted. Such a convention then, is valid for parts of the value set, in particular EEEs, and need not be further emphasized once the EEE’s definition is known. For example, a function of a wheel is to transform periodic motion into aperiodic. It is the least part of a system that performs such a transformation. Half of a wheel, spokes, possibly included rubber or wooden parts of a wheel separately are not considered as periodic-to-aperiodic motion transformers. Motors, shafts and other similar structures are capable of such a transformation, but are more complex than a wheel. Their additional attributes are responsible for other, more complex functions they are linked to. The word elementary as related to a value set is somewhat inconvenient if one adopts the theoretical physics point of view in which elementary attributes are valid throughout the known universe. That inconvenience is consequence of too strict reference to analogue notions in theoretical physics, which is to be taken with caution. In other words, every value set is a universe of its own, and along with its dynamics, the notions of EEEs change. Certainly, such changes are seen only relatively, in comparison with earlier, or in other ways different value sets.

The relation of EEE and elementary excitations needs more clarification. In particular, the point to be emphasized is that only one value set figures in definition of EEE while two value sets figure in definition of elementary excitations. In other words, within one agent system, to which we will refer as the primary for the moment, if there is a recognized part of environment it is an EEE. Let as assume that there is another agent system, referred to here as the secondary, the value set of which does not include that part of environment. For the secondary system the considered part of environment is environment excitation, but not EEE. As an illustration to that, wheels were not EEEs in old enough societies, although then wheels were still possible environment excitations, as we can conclude from our point of view.
The EEEs are characterized with a value in a local agent system. Value, denoted as $V$, is considered here to be a scalar quantity. The EEEs have set of attributes uniquely determining them, denoted as $\{\alpha\}$. Value is influenced with several factors: availability and cost of components, cost of agent labor, durability, possibility of different usage and demand for it. One type of EEE includes sets of attributes with different values, e.g. a wheel can be wooden, metallic, combined, or produced from different materials, or their more complex combinations. Between the listed factors influencing an EEE's value, durability, denoted as $D$, has somewhat different meaning, as it measures the EEE's degradation in time. It is regarded here as imaginary counterpart of real value. Hence, total value of an EEE is

$$V\alpha + iD\alpha$$ (1)

The very recognition of an elementary excitation is generally a long-lasting attribute of a given agent system. Therefore, from the point of view of a given agent system, (functional) elementariness is an attribute which is effectively invariant in time if compared to characteristic time units of regular agent dynamics. Elementariness contributes to bringing about of time invariance. In order to have time invariant formulation of economic processes the precise form of elementariness is to be constant. Here, time invariance means that rules underlying exchange processes do not change significantly in time intervals during which a large number of processes are conducted. Furthermore, agent system has some area in which there are agents who, on average, adopts it. Generally, there will be differences between recognition of different environment excitations in different regions of the agent system. However, we consider them to be rather small. That brings about space invariance of elementariness. Space and time invariance of elementariness enables one to consider agent system's value set as a collection of elementariness attributes. A space thereby spanned is called a reciprocal space. In reciprocal space there are functions and abstract representations, which have direct or indirect material counterparts in geographical space of the agent system. The reference to the agent dynamics characteristic time is important, as the other time scale attributed to the system is that of value set dynamics.

The representation of an EEE in reciprocal space is called a propagator. For its form we take

$$P\alpha(V) = \frac{1}{V^2 - V\alpha^2 - D\alpha^2},$$ (2)

where $V$ is auxiliary value variable, a mathematically real quantity. The meaning of propagators will be clear after processes involving mediators are described in some detail. For the moment, it is to be emphasized that the value of $P\alpha(V)$ diverges if auxiliary value $V$ approaches the total mediator value.

**Elementary Exchange Processes**

State of an agent in reciprocal space is represented as a collection of values of EEEs which are considered as belonging to the agent. Agent dynamics in the reciprocal space is described as time dependent collection of values. Time dependence is introduced through degradation of an EEE,
possibly through its consumption by an agent, or exchange of EEEs among agents. The functioning of EEEs as means for interaction between agents is one possible EEEs function. The EEEs which are regularly used in performing an interaction between the agents are called mediators.

Characteristics of EEEs are suitably collected in their graphical representation, Figure 1a. In it, additional structure is representation of relevant agent states, Figure 1b, i.e. representation of values of states (i - initial and f - final) which are significantly changed in interaction with EEEs. In Figure 1, the time axis makes possible putting the agent-EEE interaction into a sequence. Explicit attributes of a particular state, like set of EEE's attributes are assumed, and are not shown in order to simplify the representation. Agent states are represented using full lines, and EEE's states using dashed lines. Among all the EEEs only those changing agent states are considered, hence the elementary agent-EEE

\[ \text{Figure 1. a) Graphical representation of an EEE, b) relation of the EEE with agent states.} \]

process is represented in Figure 2. A connection of two lines representing different agent states with a line representing a mediator is represented as a circle, and is called a vertex. Vertices are functions of attributes of the joined states, which expresses the probability of occurrence of a particular combination of joined states. As an illustration of these elementary processes, if the agent is a baker, and the EEE is a loaf of bread, then interaction of these two will not change the agents knowledge about automobiles, the very automobile owned etc., thus these characteristics will not be mentioned in the set of agent states. If, in some other case, the agent is a mechanical engineering student, then knowledge about automobiles is one relevant agent state, while the automobiles owned or a number of loaves of bread are not. Assume the EEE to be a book about automobiles, Let us describe processes shown in Figure 2 in more detail. The process in Figure 2a describes a situation in which an EEE is emitted from some agent. That emission can be a formation of the EEE which did not exist before, or a release of the EEE which existed before when it was conventionally described as agent’s property or responsibility. As an example of the former type let us consider bread which a baker bakes. Bread is the EEE and the baker is the agent. Bread is formed in a process starting with heat, wheat and some other EEEs, summarily known as baking. An example of the later type of EEE emission is a student with a textbook. The student carrying the book is interpretable as an agent whose present state includes
one EEE. The student may leave the book in the room, return it to the library, or in some other ways come in the state without the book which then becomes an independent EEE. These two, and a myriad of similar processes, are represented as shown in Figure 2a. In Figure 2b the converse processes are represented; collection or destruction of an EEE. In order to simplify the description, the processes in Figure 2a, and Figure 2b are considered as emission and absorption of an EEE, respectively.

However, the economic processes are not included in the representation in Figure 2. From these elementary processes one step further is needed to obtain a representation of an elementary economic process here considered as the elementary transaction. This is so because in the economic processes changes of quantities of agents’ assets are considered, realized as the mutual exchange of assets. The asset could be a material one, or not. Information is an example of an asset traditionally considered non-material.

Considering the world-wide identity of elementary economic processes observed throughout the history periods, it is opportune to consider as an inherent property of agents the following fact: a change of an agent state induced with only emission of one EEE is considered highly unwanted when economic processes are considered. The usual economic description involves a utility function of an agent, which is a scalar function referring to the combination of attributes the agents tend to augment. In economic interactions with a single EEE emission the value of utility function is considered too large to be acceptable. The larger the deviation of a final state in a process from some referent state, the smaller its probability. The utility function represents the local value set. Graphically, Figure 3 depicts favoring of two-mediator process using the sketch of some utility function. In Figure 3a the value of utility function is shown as a function of resource change, caused by mediator emission (negative changes) or absorption (positive changes). Having in mind that what is emission for one agent is the absorption for the other, the graph of combined change of utility functions for both interacting agents shown on Figure 3b is obtained.

Figure 3. a) Individual agent utility function, and b) utility function of two economically interacting agents.

The utility functions are shown as functions of the agents’ resource changes.

Because of a qualitative character of this graph, the maximum of the combined change is centered at zero net resource change. The precise form of the optimal combined utility function change is a separate, thoroughly examined problem; it is recognized as Nash equilibrium in game theory on the
one hand, and as the Luhmann’s contingency problem on the other hand. As that is out of the scope of this article, we will not analyze furthermore the precise form of the combined utility function change, i.e. the influence of both vertices. Further in the text we use a utility function as a functional representation of a vertex. Therefore, every vertex combining agent states $i$ and $f$ with corresponding resources $r_i$ and $r_f$, respectively, is in the reciprocal space represented as

$$U(r_f, r_i, V) = \frac{1 + \tanh(r_f - r_i)}{2} \delta(r_f - r_i \pm V),$$  

(3)

where it was explicitly stated that resource change occurs because of mediator emission or absorption in case of which the upper and lower signs are to be used, respectively. The delta distribution takes into account that locally, during these processes, value is conserved. Because of it, (3) may be written as $U(r_f, r_i, V) = (1 \pm \tanh(V)) \delta(r_f - r_i \pm V)/2$.

The function taken in (3) rises as the final resources are larger, maintain the calculation simplicity, and has relatively small resolution in the region of large resource transfer. Its precise form valid in a particular situation generally may differ from (3).

The final state in the process, in which it is separated by the interaction with an EEE from some initial state is generally highly unwanted, and is correspondingly effectively improbable. As that final state is highly unwanted, it is accompanied with the strong tendency towards its further change. The longer it lasts, the more probable its resolving through absorption of an EEE.

The change in value of utility function means occurrence of further interaction of the agent and a set of EEEs. Highly unwanted character of the agent state brings about tendency to shorten time interval spent in it, which means that second interaction is to occur as soon as possible after the first one. After the second interaction, the agent is again in a state characterized as wanted, or at least more appropriate than state before the first interaction with an EEE. Therefore, in cases when EEEs are interpretable as assets, hence providing economic context, it is highly probable that there will be more than one interaction of an agent with EEEs. Cases in which the first interaction brings about highly wanted final state are possible, but do not belong to economic processes as they imply that an EEE exists freely. In economic processes we assume that all, or at least a large majority of assets belong to some agents, and are considered more or less scarce. Cases with more than pair-wise EEE exchange we do not consider as structurally simple processes. Regarding the examples connected with description of Figure 2, it is clear that a baker who gives bread away is highly improbable. If notions from physics are exploited, then processes as shown in Figure 2a are usually connected with dissipative processes, e.g. emission of photons during deceleration, which drastically change a particle state.

Finally, phenomenological result is that processes involving smaller number of mediators are on average shorter in time. These processes, therefore, does not span time interval which is comparable with the value set dynamics, hence all parameters in their description are considered constant. Such a simplification is of technical character, and does not limit the applicability of MEI.
Therefore, elementary transaction, the simplest economic process, is realized as an exchange of two EEEs, e.g. Figure 4. It represents a pair-wise connected emissions and absorptions of two EEEs. It resembles the theoretical physics description of interaction between, e.g. electrons via exchange of two virtual photons. In a human system e.g., a baker and a customer may be the two agents. One EEE is a loaf of bread and the other is money. Or the agents may be a student and a bookshop salesman, while the EEEs are a book and a money. Having in mind that there is time axis attributed to the graph, the moments of emissions and absorptions may be considered as occurring before or after one another. The maximal number of possible realizations of their ordering in simple situations is $4!(2!2!) = 6$. E.g. a baker gives a loaf of bread to a customer and then the customer gives money. Or, conversely, firstly money is exchanged and after it the bread.

![Figure 4](image)

**Figure 4.** One example of elementary transaction seen as agent-agent interaction realized through exchange of two EEEs.

The situations in which a baker and a customer give a loaf of bread and a money, respectively, before taking the other EEE are also possible. A simple situation here means that each agent has the corresponding EEE before its emission. In general economic systems that does not have to be a case, and then a maximal number of their ordering is $4! = 24$, but the 18 further processes belong to the more complex economic systems, here excluded. In other words, here it is considered that in a double EEEs exchange each EEE is firstly emitted and later received. The two EEEs exchanged during economic interaction are only part of all the EEEs exchanged. Other exchanged EEEs, not shown in Figure 2 and Figure 4, are EEEs through which verbal and nonverbal communications take place, thus economic representation of agent interaction is a projection of total agent interaction.

The stochastic character of the concept of mediated economic interactions is clearly seen if one recognizes that the exchange described in general is repeated perpetually among all agents in the system. That is not the only source of stochasticity, because one expects the stochastic elements in internal agent dynamics. However, its explicit taking into account occurs when macroscopic quantities are derived through suitable averaging of the set of elementary transactions.

Formal representation of elementary economic interaction processes, the example of which is shown on Figure 4, contains several appropriately connected elements: mediator propagators and vertices. Representations of processes on Figure 5a and 5b, denoted as $M_a$ and $M_b$, respectively, are

$$
M_a = \sum_{\alpha, \beta} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\alpha_1 d\alpha_2 \int_{-\infty}^{\infty} dV_1 \int_{-\infty}^{\infty} dV_2 U(\alpha_1, \alpha_1)U(\alpha_3, \alpha_1)U(\alpha_j, \alpha_2)U(\alpha_4, \alpha_j)P_\alpha(V_1)P_\beta(V_2) = (4)
$$
\[
\frac{1}{16} \sum_{\alpha, \beta} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dV_{1} \cdot \delta(a_{1} + a_{2} - a_{3} - a_{4})}{\cosh^{2}V_{1} \cdot \cosh^{2}(V_{1} - \Delta a) \cdot [V_{1}^{2} - V_{\beta}^{2} - D_{\beta}^{2}] \cdot [(\Delta a - V_{1})^{2} - V_{\beta}^{2} - D_{\beta}^{2}]}.
\]

(5)

\[
M_{\alpha} = \sum_{\alpha, \beta} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dV_{1} U(a_{1}, a_{1}) U(a_{3}, a_{2}) U(a_{4}, a_{2}) P_{\alpha}(V) P_{\beta}(V) =
\]

(6)

\[
\frac{1}{16} \sum_{\alpha, \beta} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dV_{1} \cdot \delta(a_{1} + a_{2} - a_{3} - a_{4})}{\cosh^{2}V_{1} \cdot \cosh^{2}(\Delta a - V_{1}) \cdot [V_{1}^{2} - V_{\beta}^{2} - D_{\beta}^{2}] \cdot [(\Delta a - V_{1})^{2} - V_{\beta}^{2} - D_{\beta}^{2}]}.
\]

(7)

Expressions (5) and (7) equally contribute. In (5) and (7) \(\Delta a = a_{4} - a_{2}\). In Figure 5 the meaning of indices encountered in (4 - 7) is explained. As mediator transfers a finite value, relations \(V_{1} = a_{1} - a_{1} = a_{4} - a_{4}\) for one mediator, and \(V_{2} = a_{3} - a_{1} = a_{2} - a_{2}\) for the other mediator, hold for process in Figure 5a, thus agents’ intermediate states are known. Mediator auxiliary values are assumed equal to differences in agent’s values connected in a vertex, i.e. they are also known. Similarly, in case of a process shown in Figure 5b relations \(V_{1} = a_{1} - a_{1} = a_{2} - a_{2}\) for one, and \(V_{2} = a_{3} - a_{1} = a_{4} - a_{4}\) for the other mediator hold. Integrals and sums in (4 - 7) are put in order to take into account that in general there can be more than one type of mediators with a given value, hence their duration may differ. The sign ‘ in summation over mediator indices points to the fact that signs of values \(V_{\alpha}\) and \(V_{\beta}\) are equal. A complete representation of a process is

\[
M = M_{\alpha} + M_{\beta} = 2M_{\alpha} = 2M_{\beta}.
\]

(8)

\[\text{Figure 5. Elementary economic processes occurring in (4 - 7).}\]

Generally, values of \(M\) are larger for more pronounced augmentation of combined utility function, and in cases when mediator values match values which are attributed to mediators by agents, as seen from their intermediate states, i.e. when \(V_{1} = |V_{\alpha} + iD_{a}|\) and \(V_{2} = |V_{\beta} + iD_{b}|.\) Furthermore, values of \(M\) are larger if mediators are more durable, i.e. for lower \(D_{a, \beta}.\) Following that, one can interpret \(M\) as the indicator of economic activity – the larger its value, the more probable economic interaction. Strictly, this interpretation is valid for an elementary process between two agents. In order to obtain measure of a probability of process between two agents in specific states \(a_{1, 2}\) one needs to sum (4 - 7) over possible final agent states \(a_{3, 4}.\) Such a summation is formal representation of Luhmann’s contingency problem. Furthermore, a system indicator which measures intensity of agents’ economic interactions is
obtained after additional averaging over all possible initial agents’ states. Structure of a system is then explicitly taken into account.

**Elementary versions of barter and monetary transaction**

Further characteristics of an elementary transaction are seen clearly in case of a process with two agents and two EEEs. Let us emphasize here that the availability of an EEE, linking given initial with a particular final state, is connected with the probability of a change of a state. If the EEE needed is available, the probability of a process is non-zero. However, if the EEE needed is unavailable, the probability of a process is zero. On average, in a collection of agents, the average number of realizations of a particular process involves the quantity of available EEEs needed for that process. The very availability is a function of local characteristics. It is changed with the number of agents providing or needing some EEE. In case when there is small number of types of EEEs, it is simple to find a pair of agents such that their surpluses and lacks of EEEs are aligned, and to connect them, i.e. to perform an elementary transaction. In case when there is large number of types of EEEs, it is still formally simple to find such a pair of agents. However, practically it may become rather improbable for particular agents to realize such a pair, considering the probability of interaction, range of interaction, time available for interaction, screening of the two agents which tend to interact mutually by other agents, etc. The screening means that effective values of parameters of interaction differ from cases in which there are only two agents in environment. The situation changes significantly in case when there is a single EEE that is in a particular way equivalent to all other EEEs. The equivalence is such that there is a definite and widely accepted exchange ratio for expressing value of different types of EEEs. The very existence of that ratio is consequence of effective space and time invariance of set of values. In that case, for the realization of an exchange process it is needed to form a pair of agents with interests in exchanging a single general and the universally accepted EEEs. On average this is simpler to realize than the earlier case. Such a universally accepted EEE we denote as a global exchangeable (GE), and in further text EEE means additionally non-GE. A GE has several attributes. Primarily, it should be a sufficient substitute for a variety of EEEs, including slowly consumed ones. Hence, it should be durable. More formally, durability means that accompanied $D_α$ is negligible compared to values of EEEs' $D_α$. The GE should be available in large enough quantity for the very transaction including it be possible. The availability includes technical aspects, e.g. relatively simple transport. On the other hand, good transportability further implies relatively large range of interaction or, more precisely, relatively large space region around one agent in which the probability of interaction with another agent does not depend on the time connected with the GE transport. The ending recognition is that GEs have relatively low economic inertia, introduced here as the analogue of physical inertia, i.e. mass of an object. Economic inertia is a measure showing how large efforts are needed in order to change a state of some EEE. Between two EEEs, the one requiring larger amount of efforts for the equivalent transfer is attributed larger economic inertia. The origins of economic inertia are in the existing formal (e.g. laws about GE transfer between two regions) and informal rules (e.g. a possibility of robberies during transport tend to reduce somewhat the rate of GE transported. This is overcome by making larger the number of required actions of agents, of which one consequence is
enlargement of the quantity of energy required). Generally, in one system there may exist more than one type of GE. Universal acceptance means further that GE forms naturally a scale of value, i.e. it figures also as a measure of value. But, these several attributes are the functions of money stated in the introductory section of this article. Hence there is equivalence between the notion of money and the notion of GE.

Simple forms of exchange processes involving (i) two EEEs, (ii) one EEE and one GE are known as barter and monetary transaction. Such recognition contributes to realizing further characteristics of elementary transactions.

A) Barter

In elementary barter, each of two types of EEEs is needed by one of two interacting agents. Additionally, EEEs used in barter are of variable, generally finite, duration. Because of that, they are needed when prolongation of internal agent's dynamics requires so, and are henceforth soon transformed appropriately, e.g. food is consumed, building material included in houses built. This also reduces barter duration, hence it is localized in time. Its additional characteristic is relatively small probability of finding two agents with such a combination of initially owned resources, i.e. EEEs, that after mutual exchange they achieve more wanted final states. Hence, in the barter based economy, (i) economic processes are localized in time and space, (ii) overall activity depends crucially on the probability of agent-agent interaction. Interpretation using terms from theoretical physics includes statements that mediators involved are of relatively large economic inertia, hence are short-ranged. The changes in assets caused by emission or absorption of one EEE with a large economic inertia are considered relatively large (but that does not mean that accompanied utility function changes significantly, cf. Figure 3), hence are instantaneous.

Formal representation of that process is given with (8). Practical simplifications, which are consequences of the very barter characteristics, are that $V_\alpha$ and $D_\alpha$ are relatively large in comparison with achievable $V$, thus in a barter $M$ simplifies into

$$M = \frac{\delta(a_1 + a_2 - a_3 - a_4)}{8(V_\alpha^2 + D_\alpha^2)(V_\beta^2 + D_\beta^2)} \int_{-\infty}^{\infty} \frac{dx}{\cosh^2 x \cdot \cosh^2 (x - \Delta a)} = \frac{2\delta(a_1 + a_2 - a_3 - a_4)}{(V_\alpha^2 + D_\alpha^2)(V_\beta^2 + D_\beta^2)} f(\Delta a), \quad (9)$$

with

$$f(\Delta a) = \frac{e^{2\Delta a}[1 - e^{2\Delta a} + (1 + e^{2\Delta a})\Delta a]}{(1 - e^{2\Delta a})^2}. \quad (10)$$

The graph of $f(\Delta a)$ is shown in Figure 6. For $\Delta a > 1$ it is well approximated with linear function $\Delta a - 1$, while for $|\Delta a| \ll 1$ it is approximated with another linear function $5\Delta a/6$. For $\Delta a < 0$ it attains relatively small values, with minimum equal $-0.83$ for $\Delta a = -0.75$. The asymmetric shape of that function is a consequence of favoring the utility function augmentation because of economic interaction.
B) Transactions including money

In an elementary version of such a process one has different level of locality exchange process as compared to barter. The non-locality in space is pronounced because GE is durable and widely accepted. The non-locality in time is pronounced because of the confidence level established. The very existence of GE is a consequence of such a confidence level. In other words, the overall level of locality is determined not by the very mediator attributes, but by other characteristics. In particular the locality level is determined by individual agent dynamics, i.e. the changes in utility function during an exchange process. Hence, generally, monetary economy is intensified compared to the barter based economy partially because of the very existence of GE, i.e. universally accepted and durable realization of value.

Formal representation of monetary transaction simplifies after taking into account that for money, and GE in general, the $V^2 + D^2 \equiv \eta^2 = 0^7$.

\[
M = \frac{\delta(a_1 + a_2 - a_3 - a_4)}{8(V^2 + D^2)} \int_{-\infty}^{\infty} \frac{dx}{(x^2 - \eta^2) \cdot \cosh^2 x \cdot \cosh^2 (x - \Delta a)}.
\]  

Value of (11) is larger than (9) because only one mediator of large value figures in (11), with sub-integral function obtaining larger values than the one in (9). This means that the measure of economic activity M denotes enlargement of economic activity in case of introduction of GE, which is durable and of small value as an object.

Summary and Conclusions

The approach to elementary agent interaction with other agents and environment in terms of exchanged elementary environment excitations is described. It is applied to descriptions of elementary transactions as exchange of two elementary environment excitations, and to description and comparison of barter and monetary economy. It is argued that attributes of mediators used in economic context are one part of the influences on the duration and range of an elementary exchange process. Because of the perpetual repetition of exchange processes, these attributes are implicitly found in the averaged economic dynamics. The other part of the influences is the influences intrinsic to agents, which are parameterized, e.g. using individual agent utility function. The attributes involved are
interpreted using terms originating in the theoretical physics description of mediated interactions, which provides one with a number of possible parallels.

Formal description of elementary economic processes is given, starting with appropriate scheme. Non-trivial elements in the scheme: mediator lines and circles for vertices have mathematical counterparts given by (2) and (3), respectively. Integration is assumed for all variables of states other than final or initial: values of agents’ resources and auxiliary value of mediator. Summation is assumed for all mediator indices. Quantity thereby obtained, M, measures the intensity of economic interaction. It is mentioned how M is used as a starting point toward obtaining measures of economic activity valid for total system. Agent intermediate states are treated in a rather simple form.

The concept developed here opens in two different directions. Firstly, similarity with the concept of mediators as developed within theoretical physics here is only tackled. If one would like to set the limits of the analogies, then introduction of some new concepts, e.g. Hamiltonian or Lagrangian function, should be analyzed. Secondly, the functions encountered in the article, e.g. form of a vertex as described in (3), point to the necessity for application of the MEI onto realistic cases.

References and Notes

1. Davies, G. *A History of Money from Ancient Times to Present Days*; University of Wales Print, 1996.


