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Entropy – Some Cosmological Questions Answered by Model of Expansive Nondecelerative Universe

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Abstract. The paper summarizes the background of Expansive Nondecelerative Universe model and its potential to offer answers to some open cosmological questions related to entropy. Three problems are faced in more detail, namely that of Hawking‘s phenomenon of black holes evaporation, maximum entropy of the Universe during its evolution, and time evolution of specific entropy.
Keywords: Expansive Nondecelerative Universe; Black hole evaporation; Specific entropy; Maximum entropy.

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1. Background of Expansive Nondecelerative Universe model

Starting from the beginning of 80’s, the inflation model of the universe acquired dominant position in cosmology. The model has been able to eliminate certain cosmological problems, at the same time it has, however, open new questions, such as the Universe age, Hubble's constant or deceleration parameter values. It has not contributed to deepen our understanding of the gravitation and its relation to the other physical interactions. Moreover, in accordance with some analyses [1], the initial nonhomogenities should not be eliminated but they are rather enhanced within the inflation period.

Open questions have been a challenge for developing further models of the Universe, one of them being Expansive Nondecelerative Universe (ENU) model [2-5].

The cornerstones of ENU are as follows:

a) The Universe, throughout the whole expansive evolutionary phase, expands by a constant rate equals the speed of light c obeying thus relation

$$a = c\,t_c = \frac{2G\,m_U}{c^2} \tag{1}$$

where a is the Universe radius (gauge factor), t_c is the cosmological time, m_U is the Universe mass (their present ENU-based values are as follows: $a \cong 1.299\,x10^{26}$ m, $m_U \cong 8.673\,x10^{52}$ kg, $t_c \cong 1.373\,x10^{10}$ yr).

b) The curvature index k and Einstein cosmologic constant Λ are of zero value

$$k = 0 \tag{2}$$

$$\Lambda = 0 \tag{3}$$

c) The mean energy density of the Universe is identical just to its critical density.

d) Since a is increasing in time, m_U must increase as well, *i.e.* in the ENU, the creation of matter occurs. The total mass-energy of the Universe must, however, be exactly zero. It is achieved by a simultaneous gravitational field creation, the energy of which is negative. The fundamental mass-energy conservation law is thus observed.

e) Due to the matter creation, Schwarzschild metrics must be replaced in ENU by Vaidya metrics [6,7] in which the line element is formulated as

$$ds^2 = \left(\frac{d\Psi}{c\,dt}\right)^2 \frac{1}{f_{(m)}^2}\left(1-\frac{2\Psi}{r}\right)c^2\,dt^2 - \left(1-\frac{2\Psi}{r}\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta\,d\varphi^2\right) \tag{4}$$

and the scalar curvature R (which is, contrary to a more frequently used Schwarzschild metric of non-zero value in Vaidya approach also outside the body allowing thus to localize the gravitational energy density) in the form

$$R = \frac{6G}{c^3\,r^2}\cdot\frac{dm}{dt} = \frac{3\,r_g}{a\,r^2} \tag{5}$$

where m is the mass of a body, G (6.67259×10^{-11} kg^{-1} m^3 s^{-2}) is the gravitational constant, r is the distance from the body, r_g is the gravitational radius of the body, $f_{(m)}$ is an arbitrary function, and Ψ is defined [5] as

$$\Psi = \frac{G\,m}{c^2} \tag{6}$$

In order to $f_{(m)}$ be of nonzero value, it must hold

$$f_{(m)} = \Psi\left[\frac{d}{dr}\left(1-\frac{2\Psi}{r}\right)\right] = \frac{2\Psi^2}{r^2} \tag{7}$$

Based on (1), in the framework of the ENU model

$$\frac{d\Psi}{c\,dt} = \frac{\Psi}{a} \tag{8}$$

Dynamic character of the ENU is described by Friedmann equations. Introducing dimensionless conform time, equation (1) can be expressed as

$$c\,dt = a\,d\eta \tag{9}$$

from which

$$a = \frac{da}{d\eta} \tag{10}$$

Applying Vaidya metric and stemming from Robertson-Walker approach, Friedmann equations [8] can be then written in the form

$$\frac{d}{d\eta}\left(\frac{1}{a}.\frac{da}{d\eta}\right) = -\frac{4\pi G}{3c^4}a^2(\varepsilon + 3p) \tag{11}$$

$$\left(\frac{1}{a}.\frac{da}{d\eta}\right)^2 = \frac{8\pi G}{3c^4}a^2\varepsilon - k \tag{12}$$

where ε is the critical energy density (actual density within the ENU model) and p is the pressure. Based on (11) and (12) it follows

$$\varepsilon = \frac{3c^4}{8\pi G a^2} \tag{13}$$

$$p = -\frac{\varepsilon}{3} \tag{14}$$

Equations (13) and (14) represent the matter creation and the negative value of gravitational energy, respectively (for more details, see [2 - 5]).

A typical feature of the ENU model lies in its simplicity, in fact that no „additional parameters“ or strange „dark energy“ are needed, and in the usage of only one state equation in describing the Universe. Calculated gauge factor *a*, cosmological time t_c, and energy density ε match well the generally accepted values.

In spite of the fact that the conception of entropy and the second law of thermodynamics has been seemingly a textbook matter, it is clear that this conception is strongly coupled with several crucial scientific problems [9], those of cosmology and astrophysics including. Three of them will be addressed in the present contribution.

2. Quantum evaporation of black holes

One of the greatest intellectual achievements in the past century was formulation of hypothesis on the existence of black holes followed by its undirect experimental verification. Several problems relating to black holes have remained still open, one of them concerns ways of decreasing their mass and changing their entropy. Based on quantum mechanics and thermodynamics Hawking [10, 11] suggested a solution of the problem in the form of quantum evaporation. His theory has led, however, to possibility of the total evaporation of black holes, a phenomenon that has never been observed. In our previous contribution [5] we documented that a decrease in the mass of a black hole via its evaporation contradicts the second law of thermodynamics. In this section more details are given and, in addition, independent modes of an evidence on the improbability of a black hole mass decreasing, based on mutual consistency of the calculations treated the gravitational field energy quantum, output and density, E_g, P_g and ε_g are offered.

Gravitational output, *i.e.* the amount of the gravitational energy emitted by a body with the mass m in a time unit is defined as

$$P_g = \frac{d}{dt}\int \varepsilon_g \, dV \cong -\frac{mc^3}{a} = -\frac{mc^2}{t_c} \tag{15}$$

According to Hawking [10, 11], within the quantum evaporation a black hole (BH) with the diameter r_{BH} evaporates photons with the mean energy

$$E_{BH} = \frac{\hbar c}{r_{BH}} \tag{16}$$

The output of a black hole evaporation was expressed by Hawking as

$$P_{BH} = \frac{\hbar c^2}{r_{BH}^2} \tag{17}$$

A black hole with the mass m_{BH} would thus totally evaporate in time

$$t \approx \frac{G^2 m_{BH}^3}{\hbar c^4} \tag{18}$$

If the time t is substituted by the cosmological time t_c, then black holes completing their evaporation at present should have had the initial mass (primordial black holes)

$$m_{BH}^o \approx 10^{12} \text{ kg} \tag{19}$$

It was admitted by Hawking himself that in spite of a great efford, no such an evaporation was experimentally observed.

In the ENU model, the mutually related creation of matter and of gravitational energy simultaneously occurs. In case of black holes, the matter creation and evaporation are competitive counter-acting processes. Since a magnitude of the surface of a black hole horizon is proportional to entropy and the second law of thermodynamics should not be violated, during the matter creation and evaporation the surface of black hole event horizon must not decrease. In a limiting case, when amounts of the created gravitation and evaporated matter are just balanced, the surface of the black hole horizon (*i.e.* entropy) is constant. For such a case three postulates can be formulated:

i) the energy of gravitational field quanta is identical to the energy of photons emitted at evaporation,

ii) gravitational output equals to output of the evaporation,

iii) density of the energy of black hole radiation is equal to the density of gravitational energy.

Justification of the above postulates *i* – *iii* is verified below.

i) It follows from general expression for the energy of a gravitational field quantum [5]

$$\left|E_g\right| = \left(\frac{m\hbar^3 c^5}{a r^2}\right)^{1/4} \tag{20}$$

and introducing (16) to (20), it follows for a black hole

$$\left(\frac{m_{BH}\,\hbar^3 c^5}{a\,r_{BH}^2}\right)^{1/4} \cong \frac{\hbar c}{r_{BH}} \tag{21}$$

Using (1), Hawking relation (18) is directly obtained from (21) providing that time t represents the cosmological time t_c. This result manifests the compatibility of both approaches.

From the viewpoint of ENU, relation (19) represents a limiting (i.e. the lightest) black hole in a given cosmological time. Such a limiting black hole may exist in cases when requirements on the equilibrium of creation and evaporation, and those stemming from the second law of thermodynamics are met.

ii) At the same time it follows from relations (15) and (17) that (in absolute values)

$$\frac{m_{BH}.c^2}{t_c} \cong \frac{\hbar.c^2}{r_{BH}^2} \tag{22}$$

Based on (22), Hawking relation (18) can again be derived, its interpretation being identical to that offered in *i*).

iii) Using the Stefan-Boltzmann law, relation (23) expressing [5] the density of gravitational field energy ε_g created by a body with the mass m at the distance r, and taking (5) into account

$$\varepsilon_g = -\frac{Rc^4}{8\pi G} = -\frac{3mc^2}{4\pi a r^2} \tag{23}$$

It must hold in the mentioned limiting case that

$$\frac{3m_{BH}\, c^2}{4\pi\, a.r_{BH}^2} \cong \frac{4\sigma T^4}{c} \tag{24}$$

Relation (24) can be simplified using formulas

$$m_{BH} = \frac{r_{BH}\, c^2}{2G} \tag{25}$$

$$T \cong \frac{\hbar c}{r_{BH}\, k} \tag{26}$$

$$I = \sigma T^4 \cong \frac{c k^4 T^4}{(\hbar\, c)^3} \tag{27}$$

where I is the intensity of radiation, k is the Boltzmann constant. Based on (27), relation (28) is obtained

$$k^4 \cong \sigma \hbar^3 c^2 \tag{28}$$

Applying relations (25) - (28) to (24) again Hawking relation (18) is obtained in the meaning of limiting black hole at a given cosmological time. It should be noted that along with photons, other particles (i.e. neutrinos) can be considered as a product of evaporation. A conclusion on the evaporation possibility would be, however, identical.

This section can be concluded as follows:

- The present section offers an independent derivation of Hawking formula concerning the quantum evaporation of black holes.
- None of the arguments used contradicts the validity of the second law of thermodynamics.
- The calculations are of approximative nature when applying in the domain of strong fields.

3. Maximum entropy content of the Universe

In spite of the objections raised to the second law of thermodynamics in particuler cases, its validity for the Universe and the importance of entropy conception is generally recognized [12-15]. Along with its other meanings [9], entropy is also a measure of information needed to describe system properties.

The holographic model of the Universe, elaborated by Bohm [16], stems from the postulate stating that every point of the Universe (every elemental particle) is in mutual contact with the other points (particles) and holds the information about the whole Universe. The higher dimensionality of the Universe, the higher number of information is thus needed to its description. A postulate relating a maximum information $I_{\max}$ and a certain space dimensionality n of the Universe may be proposed as

$$\log_2 I_{\max} = 2^{(n-1)} \tag{29}$$

The ground of the postulate is based on relations between dimensionality and mass in n – dimensional space [17], entropy and information [9], and space and matter distribution.

As a starting point (rationalized later) let us suppose that

$$n = 10 \tag{30}$$

Then it follows from (29) and (30)

$$I_{\max} \cong 10^{154} \tag{31}$$

Stemming from relation between information and entropy [9], the maximum entropy of such a Universe is

$$S_{\max} \cong 10^{131} \tag{32}$$

The elementary particles did not exist at the very beginning of the Universe, the gravitational field quanta, however, did. This is why the entropy of the Universe can be expressed by means of a number of gravitational field quanta at a given cosmological time. If the mean energy of a gravitational field quantum is denoted as E_g, then

$$S = \frac{m_U c^2}{\left|E_g\right|} = \frac{t_c c^5}{2G\left|E_g\right|} \tag{33}$$

As shown in our previous paper [18], it holds in the ENU

$$i\hbar \frac{d\Psi_g}{dt} = E_g.\Psi_g \tag{34}$$

where Ψ_g is the wave function of the Universe defined as

$$\Psi_g = \exp\{-i(t_{Pc} t_c)^{-1/2} t\} \tag{35}$$

It follows from (34) and (35) that

$$\left|E_g\right| = \hbar (t_{Pc} t_c)^{-1/2} \tag{36}$$

where t_{Pc} is the Planck time (5.39056×10^{-44} s)

The entropy content at a time t is then, based on (33), (34) and (35), given by

$$S = \left(\frac{t}{t_{Pc}}\right)^{3/2} \tag{37}$$

i.e. at the time being

$$S \cong 10^{92} \tag{38}$$

A crucial conclusion stems from (32) and (37). It determines a cosmological time in which the maximum entropy should be reached

$$t_{c(\max)} \cong 10^{35} - 10^{36} \text{ yr} \tag{39}$$

A significance of this result lies in the fact that $t_{c(\max)}$ represents just the time for which a decay of baryonic matter is anticipated.

The ENU model is compatible with superstring theory having n = 10. Calculations based on relations present in this work lead to a result showing that if $n < 10$, the maximum cosmological time would be less than the present time.

New results might be expected when some features of the ENU model (such as the matter creation and gravitational energy localization) are incorporated into the superstring theory [19].

4. Specific entropy and its time evolution

Till the end of the radiation era, there had been a thermodynamic equilibrium of matter and radiation, and energy, temperature and gauge factor were related as follows

$$E_{CBR} \approx T_{CBR} \approx a^{-1/2} \tag{40}$$

The fact that the energy density in (13) is proportional to a^{-2} and not to a^{-3} can be rationalized by matter creation. Thus, ENU actually describes the Universe in which "eternal inflation" occurs. In classical inflationary models of universe, after completing its inflation stage the Universe should decelerate due to effects of gravitational forces. As a consequence, in the models of inflationary universe a new matter incessantly emerges from behind the event horizon and in this way the proportionality $\varepsilon \approx a^{-2}$ stated by (13) is explained.

Contrary, a nonzero value of the cosmological constant Λ or a newly elaborated quintessential model [20] gave rise to a presumption stating that the Universe expansion accelerates and that such an acceleration started at the beginning of the matter era. The hypothesis on the Universe acceleration leads, however, directly to a very important conclusion concerning impossibility of the relation (13) validity in the matter era and also to a conclusion on impossibility of critical energy density preserving.

It is generally accepted that the radiation era ended approximately in the time

$$t_r \cong 7 \times 10^5 \text{ yr} \tag{41}$$

when the temperature of radiation approached to

$$T_r \cong 5\times 10^3 \text{ K} \tag{42}$$

(the subscripts pt, r and m refer to the present-time, the end of radiation era, and matter era, respectively). The present-time temperature is

$$T_{pt} \cong 2.735 \text{ K} \tag{43}$$

Taking into account that the Universe expansion did not decelerate in the matter era, a presumption emerges stating that not only the event horizon but also the original part of the Universe extended in about four orders

$$\frac{t_{pt}}{t_r} = \frac{a_{pt}}{a_r} \cong 10^4 \tag{44}$$

Based on the fact that from the end of the radiation era the of cosmic background radiation (CBR) temperature has decreased by three orders but the gauge factor increased by four orders, a relation between the energy of cosmic background radiation E_{CBR} and its temperature T_{CBR} follows

$$T_{CBR} \approx E_{CBR} \approx a^{-3/4} \tag{45}$$

Introducing the value of a_{pt} into calculation of the present-time critical energy density of the Universe it follows that

$$\varepsilon_{crit(pt)} \cong 8.577\times 10^{-10} \text{ J/m}^3 \tag{46}$$

The energy density of radiation can be extracted from Stefan-Boltzmann law and for cosmic background radiation it is generally given as

$$\varepsilon_{CBR} = \frac{4\sigma T^4}{c} \tag{47}$$

Based on (43) and (47) the present-time energy density value of cosmic background radiation reaches

$$\varepsilon_{CBR(pt)} \cong 4.229\times 10^{-14} \text{ J/m}^3 \tag{48}$$

It follows from (45) and (47) that during the matter era

$$\varepsilon_{CBR(m)} \approx a^{-3} \tag{49}$$

and at the same time, stemming from (13), (46), (47), (48) and (49) it follows

$$\frac{a_{pt}}{a_r} = \frac{\varepsilon_{crit(pt)}}{\varepsilon_{CBR(pt)}} \tag{50}$$

Based on (50) the gauge factor value at the end of the radiation era can be calculated

$$a_r \cong 6.41\times 10^{21} \text{ m} \tag{51}$$

together with the cosmological time value at the same time

$$t_r \cong 6.6 \times 10^5 \text{ yr} \tag{52}$$

Treatment of relations (43), (45) and (51) leads to

$$\frac{T_{pt}}{T_r} = \left(\frac{a_r}{a_{pt}} \right)^{3/4} \tag{53}$$

Stemming from (53) the temperature at the end of the radiation era is directly calculable and it reaches the value of

$$T_r \cong 4650 \text{ K} \tag{54}$$

being in excellent agreement with the generally accepted value obtained using other independent modes of calculations.

Total average number of relict photons $n(h\nu)_m$ in a cubic meter during the matter era relates to the gauge factor according to (45) and (49) as follows

$$n(h\nu)_m \approx a^{-9/4} \tag{55}$$

and that of protons $n(p)_m$ (representing the matter particles) based on (13) as

$$n(p)_m \approx a^{-2} \tag{56}$$

Dependence of the specific entropy S, defined as a number of relict photons per one proton, on the gauge factor is in the matter era expressed as

$$S \approx a^{-1/4} \tag{57}$$

At the time being, the temperature of cosmic background radiation (2.735 K) leads to the following number of relict photons in a volume unit

$$n(h\nu)_{pt} = \frac{\varepsilon_{CBR(pt)}}{E_{CBR(pt)}} \cong 4 \times 10^8 \tag{58}$$

where $E_{CBR(pt)}$ is the mean energy of actual relict photons.

Given the present time energy density (13) and gauge factor values, a number of protons in a volume unit reaches

$$n(p)_{pt} \cong 5 \tag{59}$$

The present-time specific entropy calculated as a ratio of values provided in (58) and (59) is of the order

$$S_{pt} \cong 10^8 \tag{60}$$

At the end of the radiation era, the specific entropy value approached

$$S_r \cong 10^9 \tag{61}$$

Comparison of (60) and (61) verifies the correctness of (57), *i.e.* a slow decrease in the specific entropy with time.

Within a discussion on a time-dependence of specific entropy some contradictions emerge. If the specific entropy is constant, *i.e.* if relation (61) is valid at the present-time number of relict photons and gauge factor, the Universe density would have to have subcritical value. The assumption of permanent critical (nearly critical) density, however, excludes a constant value of specific entropy. The majority of current cosmological models take, however, critical mass-energy density a priori into account and tries to solve this discrepancy introducing some "exotic" nonbaryonic forms of matter.

Summarizing the conclusions offered in the present contribution it should be pointed out that in the majority of conventional models it is postulated that $E_{CBR} \approx T_{CBR} \approx a^{-1}$, $\varepsilon_{CBR} \approx a^{-4}, S = \text{const.}$

In the ENU, $E_{CBR} \approx T_{CBR} \approx a^{-3/4}$, $\varepsilon_{CBR} \approx a^{-3}, S \approx a^{-1/4}$. Experimental observations are in accord with the values derived by ENU model. Consequences related to black hole evaporation were supported also by independently formulated theoretical arguments [21]. Other models explain the above mentioned observed relations as a consequence of the matter emerging from behind event horizon due to the Universe expansion deceleration. The latest measurements, however, indicate that the Universe expansion might be accelerated and exclude its deceleration. In the ENU the expansion neither decelerates nor accelerates, it is constant and equal to the velocity of light.

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