

If the density is at equilibrium so that $\frac{d}{dt}\rho=0$, then

$$\begin{aligned}
 & -i(1-\alpha)(H\rho-\rho H)+\alpha \left[\begin{array}{cccc} \sum_n \rho_{nn} K_{1,n} & & & \\ & \sum_n \rho_{nn} K_{2,n} & & \\ & & \ddots & \\ & & & \sum_n \rho_{nn} K_{N,n} \end{array} \right] = 0 \\
 & \alpha \left[\begin{array}{cccc} \sum_n \rho_{nn} K_{1,n} & & & \\ & \sum_n \rho_{nn} K_{2,n} & & \\ & & \ddots & \\ & & & \sum_n \rho_{nn} K_{N,n} \end{array} \right] \\
 & = i(1-\alpha) \left[\begin{array}{cccc} \sum_n H_{1,n} \rho_{n,1} & \sum_n H_{1,n} \rho_{n,2} & \cdots & \sum_n H_{1,n} \rho_{n,N} \\ \sum_n H_{2,n} \rho_{n,1} & \sum_n H_{2,n} \rho_{n,2} & \cdots & \sum_n H_{2,n} \rho_{n,N} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_n H_{N,n} \rho_{n,1} & \sum_n H_{N,n} \rho_{n,2} & \cdots & \sum_n H_{N,n} \rho_{n,N} \end{array} \right] \\
 & -i(1-\alpha) \left[\begin{array}{cccc} \sum_n \rho_{1,n} H_{n,1} & \sum_n \rho_{1,n} H_{n,2} & \cdots & \sum_n \rho_{1,n} H_{n,N} \\ \sum_n \rho_{2,n} H_{n,1} & \sum_n \rho_{2,n} H_{n,2} & \cdots & \sum_n \rho_{2,n} H_{n,N} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_n \rho_{N,n} H_{n,1} & \sum_n \rho_{N,n} H_{n,2} & \cdots & \sum_n \rho_{N,n} H_{n,N} \end{array} \right]
 \end{aligned}$$

This is matrix equation. Every entry should equal.

$$\begin{aligned}
 \text{Diagonal: } \alpha \sum_n \rho_{nn} K_{j,n} &= i(1-\alpha) \left(\sum_n H_{j,n} \rho_{n,j} - \sum_n \rho_{j,n} H_{n,j} \right) \\
 &= i(1-\alpha) \left(H_{j,j} \rho_{j,j} + \sum_{n \neq j} H_{j,n} \rho_{n,j} - \sum_{n \neq j} \rho_{j,n} H_{n,j} - \rho_{j,j} H_{j,j} \right) \\
 &= i(1-\alpha) \left(\sum_{n \neq j} H_{j,n} \rho_{n,j} - \sum_{n \neq j} \rho_{j,n} H_{n,j} \right) \\
 &= i(1-\alpha) \left(\sum_{n \neq j} H_{j,n} \rho_{n,j} - \sum_{n \neq j} \bar{\rho}_{n,j} \bar{H}_{j,n} \right)
 \end{aligned}$$

$$\begin{aligned}
&= i(1-\alpha) \left(\sum_{n \neq j} H_{j,n} \rho_{n,j} - \sum_{n \neq j} \bar{H}_{j,n} \bar{\rho}_{n,j} \right) \\
&= i(1-\alpha) \sum_{n \neq j} (H_{j,n} \rho_{n,j} - \bar{H}_{j,n} \bar{\rho}_{n,j}) \\
&= i(1-\alpha) \sum_{n \neq j} 2i \operatorname{Im}(H_{j,n} \rho_{n,j}) \\
&= -2(1-\alpha) \sum_{n \neq j} \operatorname{Im}(H_{j,n} \rho_{n,j})
\end{aligned}$$

$$\begin{aligned}
\text{Nondiagonal: } 0 &= \sum_n H_{i,n} \rho_{n,j} - \sum_n \rho_{i,n} H_{n,j} \\
&= H_{i,i} \rho_{i,j} + H_{i,j} \rho_{j,j} + \sum_{n \neq i,j} H_{i,n} \rho_{n,j} - \sum_{n \neq i,j} \rho_{i,n} H_{n,j} - \rho_{i,i} H_{i,j} - \rho_{i,j} H_{j,j} \\
\Rightarrow H_{i,j} (\rho_{i,i} - \rho_{j,j}) &= (H_{i,i} - H_{j,j}) \rho_{i,j} + \sum_{n \neq i,j} H_{i,n} \rho_{n,j} - \sum_{n \neq i,j} \rho_{i,n} H_{n,j}
\end{aligned}$$

There are totally NxN equations and NxN unknowns. Due to the symmetry, we only need to consider 1+2+...+N-1=(N-1)N/2 nondiagonal unknowns.

Now let both K and H be real and tri-diagonal.

$$\begin{aligned}
\text{Diagonal: } \alpha \sum_n \rho_{nn} K_{1,n} &= -2(1-\alpha) \operatorname{Im}(H_{1,2} \rho_{2,1}) \\
\alpha \sum_n \rho_{nn} K_{j,n} &= -2(1-\alpha) \operatorname{Im}(H_{j,j-1} \rho_{j-1,j} + H_{j,j+1} \rho_{j+1,j}), j = 2, \dots, N-1 \\
\alpha \sum_n \rho_{nn} K_{N,n} &= -2(1-\alpha) \operatorname{Im}(H_{N,N-1} \rho_{N-1,N})
\end{aligned}$$

$$\text{Nondiagonal: } H_{i,j} (\rho_{i,i} - \rho_{j,j}) = (H_{i,i} - H_{j,j}) \rho_{i,j} + \sum_{n \neq i,j} H_{i,n} \rho_{n,j} - \sum_{n \neq i,j} \rho_{i,n} H_{n,j}$$

Due to nondiagonal: $i \neq j$

Due to tri-diagonal: $H_{i,j} \neq 0$ when $j = i-1$ or $j = i+1$. $H_{i,j} = 0$ when $j < i-1$ or $j > i+1$.

$$\text{When } j = i-1: H_{i,i-1} (\rho_{i,i} - \rho_{i-1,i-1}) = (H_{i,i} - H_{i-1,i-1}) \rho_{i,i-1} + H_{i,i+1} \rho_{i+1,i-1} - \rho_{i,i-2} H_{i-2,i-1}$$

$$\text{When } j = i+1: H_{i,i+1} (\rho_{i,i} - \rho_{i+1,i+1}) = (H_{i,i} - H_{i+1,i+1}) \rho_{i,i+1} + H_{i,i-1} \rho_{i-1,i+1} - \rho_{i,i+2} H_{i+2,i+1}$$

When $j < i-1$ or $j > i+1$:

$$0 = (H_{i,i} - H_{j,j}) \rho_{i,j} + H_{i,i-1} \rho_{i-1,j} + H_{i,i+1} \rho_{i+1,j} - \rho_{i,j-1} H_{j-1,j} - \rho_{i,j+1} H_{j+1,j}$$

Due to symmetry of rho and H, we only consider rho_ij and H_ij with $j \geq i$.

$$\begin{aligned}
\text{Diagonal: } \alpha \sum_n \rho_{nn} K_{1,n} &= -2(1-\alpha) \operatorname{Im}(H_{1,2} \bar{\rho}_{1,2}) \\
\alpha \sum_n \rho_{nn} K_{j,n} &= -2(1-\alpha) \operatorname{Im}(\bar{H}_{j-1,j} \rho_{j-1,j} + H_{j,j+1} \bar{\rho}_{j,j+1}), j = 2, \dots, N-1 \\
\alpha \sum_n \rho_{nn} K_{N,n} &= -2(1-\alpha) \operatorname{Im}(\bar{H}_{N-1,N} \rho_{N-1,N})
\end{aligned}$$

$$\text{When } j = i+1: H_{i,i+1} (\rho_{i,i} - \rho_{i+1,i+1}) = (H_{i,i} - H_{i+1,i+1}) \rho_{i,i+1} + \bar{H}_{i-1,i} \rho_{i-1,i+1} - \rho_{i,i+2} \bar{H}_{i+1,i+2}$$

$$\text{When } j > i+1: 0 = (H_{i,i} - H_{j,j}) \rho_{i,j} + \bar{H}_{i-1,i} \rho_{i-1,j} + H_{i,i+1} \rho_{i+1,j} - H_{j-1,j} \rho_{i,j-1} - \bar{H}_{j,j+1} \rho_{i,j+1}$$