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Concept Paper

Discussing Landscape Compositional Scenarios Generated with Maximization of Non-Expected Utility Decision Models Based on Weighted Entropies

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Abstract: The search for hypothetical optimal solutions of landscape composition is a major issue in landscape planning and it can be outlined in a two-dimensional decision space involving economic value and landscape diversity, the latter being considered as a potential safeguard to the provision of services and externalities not accounted in the economic value. In this paper, we use decision models with different utility valuations combined with weighted entropies respectively incorporating rarity factors associated to Gini-Simpson and Shannon measures. A small example of this framework is provided and discussed for landscape compositional scenarios in the region of Nisa, Portugal. The optimal solutions relative to the different cases considered are assessed in the two-dimensional decision space using a benchmark indicator. The results indicate that the likely best combination is achieved by the solution using Shannon weighted entropy and a square root utility function, corresponding to a risk-averse behavior associated to the precautionary principle linked to safeguarding landscape diversity, anchoring for ecosystem services provision and other externalities. Further developments are suggested, mainly those relative to the hypothesis that the decision models here outlined could be used to revisit the stability-complexity debate in the field of ecological studies.

Keywords: decision models; non-expected utility methods; weighted Shannon entropy; weighted Gini-Simpson index; economic values; landscape diversity; precautionary approach; landscape services; system manifold

1. Introduction

There are two distinct uses for the word *entropy* in landscape ecology, either related to the scientific field of thermodynamics or to information theory, and though there is a formal analogy these concepts are claimed to represent scientific disciplines [1].

In this paper we will focus on the information theory perspective, recalling that Shannon [2] derived the entropy of the set of probabilities $p_1, \cdots, p_n$ denoted $H = -K\sum_{i=1}^{n} p_i \log p_i$ (with $K > 0$) as a measure of uncertainty. Therefore, as Lindley [3] pointed out, Shannon introduced two fundamental ideas: that information is a statistical concept and that, on the basis of frequency distribution, there is an essentially unique function which measures the amount of information; function H is correlatively said to be a measure of the amount of uncertainty represented by a probability distribution [4] or the average randomness of a stochastic system [5]. Shannon based his deductive axiomatic procedure on previous work done by Hartley [6] aiming to compare capacities of several systems to transmit information, a concept there defined to be proportional to the number of selections, using a measure outlined as the logarithm of the number of possible sequences of symbols, following an analogy with the physical entropy in statistical mechanics first introduced by Boltzman. About a decade later, statistical entropy was introduced in ecological studies either as a measure of community stability by MacArthur [7] or as a diversity index of assemblages of species by Margalef [8], who used Shannon's formula as equivalent to Brillouin's expression [9] and interpreted it as the average number of bits per individual [10]. Pielou [11] reviewed the theme, stating that diversity in an ecological system is directly related to the amount of uncertainty regarding the identity of an individual selected at random from a community.

Shannon entropy was not the only source for diversity measures in ecology; the other main example was ascribed to Simpson's concentration measure [12], published in 1949, here denoted $C = \sum_{i=1}^{n} p_i^2$, also reformulated as $D = 1 - C$, which is usually referred to as Gini-Simpson index of diversity (e.g., [13,14]). In 1961, Rènyi [15] outlined a generalization of Shannon entropy as a 1-parameter functional family, and Hill [16], about a decade later, used the exponential form of Rènyi's generalized entropy to derive what he called diversity numbers, which were shown to be related with Shannon and Simpson diversity measures. In the 1980s, Rao introduced a generalization of Gini-Simpson index (see [13]), commonly mentioned as quadratic entropy and interpreted as the abundance-weighted mean distance between species [17], expressed as $Q = \sum_{i=1}^{s} \sum_{j=1}^{s} d_{ij} p_i p_j$. A review of diversity measures relative to community assemblages with focus on semantic content and distinction between simplex and complex issues was made by Ricotta [18].

In a process that can be classified as transference of concepts from community studies in ecology to broader space scales concerning landscapes or regions, thus replacing relative abundance of species in a community for land cover types (or habitats) areal proportions, we get into diversity metrics in landscape characterization and research (see [19–21] for a review). Landscape composition can be defined as the variety and abundance of different land cover types within the landscape [22] and different diversity indices measure distinct aspects of the partition of abundance between landscape elements [23]. In this paper, we will not deal with landscape spatial entropy measures related with heterogeneity and fragmentation of the habitats (see [1]).

There is a considerable body of evidence indicating, in the absence of legal constraints, change in human-dominated landscapes is substantially driven by economic values (e.g., [24]). Such land-use decisions are usually considered to follow utility maximization strategies associated with land conversion [25], although it can be argued that public preferences, for instance relative to forest management, are not influenced solely by the particular attributes of competing management objectives [26].

Non-expected utility models were developed in environmental and natural resources economics as a consequence for the realization that models must accommodate preferences that are non-linear in the probabilities [27], thus becoming an alternative to traditional expected utility methodology. For instance, recently, researchers concluded that a non-expected utility model was needed to evaluate environmental risks and preferences related to forest wildfires in Poland [28].

In this paper, we present two original decision models that combine expected utility and weighted entropies, which is a novel example of the non-expected utility framework, and discuss how the optimal points of the models using different utility valuations generate solutions to the composition of the landscape mosaic that will be assessed relative to the trade-off between economic value and landscape diversity.

2. Methodology

2.1. Background

A review of non-expected utility theory was made by Starmer [29], in which individuals made decisions based on finite lotteries as a function of a vector of fixed consequences $\boldsymbol{X} = (x_1, x_2, \cdots, x_n)$ mapped into real values named utilities, denoted $u(\boldsymbol{X}) = (u(x_1), \cdots, u(x_n))$, which are associated with the correspondent vector of unknown probabilities $\boldsymbol{p} = (p_1, \cdots, p_n)$. In this context, individuals are facing a decision problem of uncertainty and are assumed to maximize the functional $W(\boldsymbol{p}) = \sum_{i=1}^{n} u(x_i)\pi(p_i)$, where $\{\pi(p_i)\}_{i=1,\cdots,n}$ are decision weights. The general explanation for this concept is that when individuals are choosing between two lotteries they tend to perceive the probabilities subjectively, and transfer to decision weights with some systematic deviation between the decision weights and the objective probabilities [30]. The weighting function of the probabilities $\pi(\cdot) : [0, 1] \to [0, 1]$ is assumed to be continuous and non-decreasing, verifying the boundary conditions $\pi(0) = 0$ and $\pi(1) = 1$ (e.g., [31,32]).

A non-expected utility framework involving the sum of expected utility and Shannon entropy defined as a measure of risk was discussed as a decision-making model, later reframed into a normalized expected utility-entropy measure of risk [33]. Also, there are other utility-based motivations for different entropy measures, anchoring decision theoretic models (e.g., [34,35]).

Weighted Shannon entropy was conceived as a quantitative-qualitative measure of information, denoted $I = -c\sum_{i=1}^{n} u_i p_i \log p_i$ with $c > 0$, introduced and axiomatized by Belis and Guiasu [36] incorporating objective probabilities $\{p_i\}_{i=1,\cdots,n}$ and subjective utilities $\{u_i\}_{i=1,\cdots,n}$. About a decade later, Aggarwal and Picard [37], following a paper of Emptoz [38], generalized to the concept of entropy of degree β, defined as:

$$H_\beta = \sum_{i=1}^{n} u_i p_i \left(1 - p_i^{\beta-1}\right) / \left(1 - 2^{1-\beta}\right) \text{ with } \beta \neq 1. \tag{1}$$

Using l'Hôpital's rule, we get the result $\lim_{\beta\to 1} H_\beta = I$ (with $c = 1/\log 2$) thus allowing for an extension, using natural logarithms, to: $H_1 \times \log 2 = H_1' = -\sum_{i=1}^{n} u_i p_i \log p_i$. Also, a direct substitution shows that $H_2/2 = H_2' = \sum_{i=1}^{n} u_i p_i (1 - p_i)$, a formula named the weighted Gini-Simpson index (e.g., [39–41]) or weighted Simpson index [42]. Next, we shall consider that for $\beta = 1, 2$ the term H_β' stands for two different weighted entropies related through Equation (1), except for a multiplicative constant.

In addition, there are other links between H_1' and H_2' that we can highlight. Near $x = 1$ we have the first order Taylor's approximation $\log x \cong x - 1$, so it follows that $-\log p_i \cong 1 - p_i$ and the weighted Gini-Simpson index becomes the weighted average of the first order approximation of information values ($-\log p_i$) in Shannon entropy (when $p_i < 0.5$ the approximation may be considered quite poor).

2.2. Decision Models

To be considered relevant the problem must embody dimension $n \geq 3$ associated with a $n - 1$ simplex of compositional proportions. In any case, we will be dealing with a functional denoted $W_\beta = \sum_{i=1}^{n} u_i p_i + H_\beta'$ for $\beta = 1, 2$ where the term $\sum_{i=1}^{n} u_i p_i$ stands for expected utility $E[U]$. It can be shown that the functional W_β for each value of $\beta = 1, 2$ and for a fixed set of positive utilities $\{u_i\}_{i=1,\cdots,n}$ reduces to a real function which is smooth and concave, allowing determination of constrained maximum points with the method of Lagrange multipliers. In any case, there is only one optimal vector of proportions $\boldsymbol{p}^* = \left(p_1^*, \cdots, p_n^*\right)$ such that $\sum_{i=1}^{n} p_i^* = 1$ - a solution which is shown to be insensitive to a positive linear transformation of the utilities, or change in units, thus becoming irrelevant to assess the optimal proportions for an area.

2.2.1. The Case for $\beta = 1$

For $\beta = 1$ we get:

$$W_1 = \sum_{i=1}^{n} u_i p_i - \sum_{i=1}^{n} u_i p_i \log p_i = \sum_{i=1}^{n} u_i p_i (1 - \log p_i). \tag{2}$$

We can rewrite $W_1 = \sum_{i=1}^{n} u_i \pi_{1,i}$ with the decision weights defined with the expression $\pi_{1,i} = p_{1,i}(1 - \log p_{1,i})$ for $i = 1, \cdots, n$. It is clear that $p_{1,i} = 1$ is equivalent to $\pi_{1,i} = 1$, and (using l'Hôpital's rule) we have the result $\lim\limits_{p_{1,i} \to 0^+} \pi_{1,i} = 0$, allowing for the continuity statement defining the range $0 \leq \pi_{1,i} \leq 1$. The real function W_1 was previously presented and discussed (see [43]); in summary, W_1 is a differentiable concave function in the interior of the simplex, attaining minimum and maximum values in the domain. The minimum value is $\min W_1 = \min\limits_{i=1,\cdots,n} u_i$ and it is possible to locate the maximum point with a Lagrange multiplier method, the coordinates of the maximum point thus being evaluated by computing $p_{1,i}^* = \exp(-\alpha^*/u_i)$ for $i = 1, \cdots, n$, the optimal value of the Lagrange multiplier (α^*) defined implicitly by the equation $\alpha^* : \sum_{i=1}^{n} \exp(-\alpha/u_i) = 1$ which can be solved with numerical methods providing a unique solution.

2.2.2. The Case for $\beta = 2$

For $\beta = 2$ we have:

$$W_2 = \sum_{i=1}^{n} u_i p_i + \sum_{i=1}^{n} u_i p_i (1 - p_i) = \sum_{i=1}^{n} u_i p_i (2 - p_i) \tag{3}$$

In parallel with what was shown in the previous section relative to Equation (2), now we get $W_2 = \sum_{i=1}^{n} u_i \pi_{2,i}$ with the decision weights here defined as $\pi_{2,i} = p_{2,i}(2 - p_{2,i})$ for $i = 1, \cdots, n$. Also, we see that $p_{2,i} = 0$ implies $\pi_{2,i} = 0$ and $p_{2,i} = 1$ entails $\pi_{2,i} = 1$, so we get $0 \leq \pi_{2,i} \leq 1$. The real function W_2 displayed in Equation (3) was also previously studied (see [44]), and, analogously to what was stated relative to function W_1 shown in Equation (2), it is also a smooth and concave real function with minimum value evaluated like $\min W_2 = \min\limits_{i=1,\cdots,n} u_i$, while the maximum point is attainable with a Lagrange multiplier method checked for the feasibility of solutions verifying altogether the full set of inequalities $u_i > (n-1)/\sum_{i=1}^{n} 1/u_i$, for $i = 1, \cdots, n$. In general, the set of inequalities doesn't hold and we have to proceed with an algorithm obtaining the maximum point coordinates defined with the expression $p_{2,i}^* = 1 - (k-1)/\left(u_i \sum_{i=1}^{k} 1/u_i\right)$ and $2 \leq k \leq n$. When $k < n$ we have $n - k$ null coordinates in the optimal solution. Whether $k = 2$ we get the simple results $p_{2,i}^* = u_i/\left(u_i + u_j\right)$ and $p_{2,j}^* = u_j/\left(u_i + u_j\right)$.

2.3. Decision Space

The decision space is conceived with two dimensions: economic value *stricto sensu*, meaning economic assessment relative to forest products with market valuation, and landscape diversity. Economic value will be computed with the standard weighted average formula $V = \sum_{i=1}^{n} v_1 p_i^*$, where $\{v_i\}_{i=1,\cdots,n}$ is a set of economic values and $\{p_i^*\}_{i=1,\cdots,n}$ is the set of optimal proportions which will be computed for the cases W_1 and W_2 and different utility valuations.

Landscape diversity shall be assessed with the Hill number $N_1 = \exp\left(-\sum_{i=1}^{n} p_i^* \log p_i^*\right)$ defined in [15], which can be interpreted as the "number" of abundant habitats - making an analogy with the correspondent statement of Alatalo concerning species [45]. The option for this bi-dimensional decision space refers to the precautionary principle, as it is acknowledged that the resilience of a system can be lost because of optimal control strategies focused on a single variable [46], and it is believed that the trade-off between economic value and landscape diversity can be relevant to safeguard ecosystem services and other externalities. Also, a benchmark indicator represented by the formula $B = V \times N_1$ will be used to help the discussion of results. Lastly, we consider that the decision models we compare here are tools for optimizing landscape composition, not prescriptive methods.

3. Results

We used data from another source (see Table 1 in [47]), which reports the average economic values of different forest habitats in the region of Nisa, Portugal, expressed in euros per hectare (€/ha).

This simple example has dimensions $n = 4$, corresponding to four forest habitats or land cover types: holm oak; blue gum; cork oak and umbrella pine. Since our aim is merely to illustrate the methodology, the characterization of such economic values will be ignored except for highlighting that they just reflect forest products with market valuation and do not account for externalities and ecosystem services as other approaches do (e.g., [48]).

Table 1. Soil occupation economic values v_i $(/ha)$ of four forest habitat types in the region of Nisa, Portugal and different utility valuations according to exponent ω . Codes: Qr-*Quercus rotundifolia* (holm oak); Eg-*Eucalyptus globulus* (blue gum); Qs-*Quercus suber* (cork oak); Pp-*Pinus pinea* (umbrella pine).

Utilities u_i	ω	**Forest Habitat Type**			
		Qr	**Eg**	**Pp**	**Qs**
v_i (€/ha)	1	112	136	494	618
$\sqrt{v_i}$	0.5	10.583	11.662	22.226	24.860
${v_i}^2$	2	12,544.0	18,496.0	2.4404×10^5	3.8192×10^5

In Table 1 we present the economic values (v_i) as well as their utility valuation using the power function denoted $u_i = {v_i}^{\omega}$ (e.g., [49]) with $\omega = ,1\ ,2$. Those three cases are relative to different geometries of utility function, since utility theory states that risk-averse behavior is typically related to concave utility transformations, thus utility transformations (u_i) herein considered are: linear neutral with $\omega = 1$, square-root (concave) with $\omega = 0.5$ and quadratic (convex) with $\omega = 2$.

Applying formulas previously mentioned in Sections 2.2.1 and 2.2.2 to data in Table 1 we computed the results summarized in Table 2, relative to the optimal proportions in each case, respectively obtained with the maximization of W_1 and W_2 using the utilities specified by the exponent ω of the power value function.

Table 2. Optimal proportions relative to the maximization of expected utility and weighted entropies models W_1 and W_2 with data specified in Table 1.

Decision Model	ω	**Forest Habitat Type—Optimal Proportions**			
		Qr	**Eg**	**Pp**	**Qs**
W_1	1	$p^*_{1,1} = 0.02$	$p^*_{1,2} = 0.05$	$p^*_{1,3} = 0.43$	$p^*_{1,4} = 0.50$
W_2	1	$p^*_{2,1} = 0$	$p^*_{2,2} = 0$	$p^*_{2,3} = 0.44$	$p^*_{2,4} = 0.56$
W_1	0.5	$p^*_{1,1} = 0.11$	$p^*_{1,2} = 0.14$	$p^*_{1,3} = 0.35$	$p^*_{1,4} = 0.40$
W_2	0.5	$p^*_{2,1} = 0$	$p^*_{2,2} = 0$	$p^*_{2,3} = 0.47$	$p^*_{2,4} = 0.53$
W_1	2	$p^*_{1,1} \cong 0\ ^{\dagger}$	$p^*_{1,2} \cong 0\ ^{\ddagger}$	$p^*_{1,3} = 0.42$	$p^*_{1,4} = 0.58$
W_2	2	$p^*_{2,1} = 0$	$p^*_{2,2} = 0$	$p^*_{2,3} = 0.39$	$p^*_{2,4} = 0.61$

† 5.3635×10^{-8}; ‡ 1.1725×10^{-5}.

In Table 3 we assess the results presented in Table 2, by evaluating the economic value ($V = \sum_{i=1}^{n} v_1 p_i^*$ in €/ha) and landscape diversity associated with each solution, the last computed with the second Hill number N_1. Each case is benchmarked by the indicator value $B = V \times N_1$. The values of the measures relative to the indifference solution, or maximum landscape diversity, where each forest habitat type occupies 25% of the area and thus the correspondent value of the second Hill number is $N_1 = 4$ and the benchmark indicator evaluates as the sum of economic values at stake.

Table 3. Results shown in the bi-dimensional decision space with the dimensions defined as economic value (V) and landscape diversity (N_1); $B = V \times N_1$ is a benchmark indicator.

Decision Model		Measures		
		V	$\mathbf{N_1}$	***B***
W_1	$\omega = 1$	530.46	2.5536	1354.6
W_2		563.44	1.9856	1118.8
W_1	$\omega = 0.5$	451.46	3.4973	1578.9
W_2		559.72	1.9964	1117.4
W_1	$\omega = 2$	565.92	1.9745	1117.4
W_2		569.64	1.9518	1111.8
p_0		340.00	4.0000	1360.0

4. Discussion

In the example analysis we would attain the minimum value of the benchmark indicator with the whole area occupied by holm oak forest (Qr), scoring $B = 112$, as we have in that case $V = 112$ and $N_1 = 1$. On the other hand, the maximum landscape diversity would provide $B = 1360$ with the average economic value of $V = 340$ and $N_1 = 4$. A similar value for B is attained with the decision model W_1 and neutral utilities ($\omega = 1$), enhancing economic value to ca. $V = 530$, which is about 86% of the maximum of 618, and a diversity number of ca. $N_1 = 2.55$, meaning that the number of abundant habitats is over 63% of the maximum of 4. The highest value of the benchmark indicator is achieved with the decision model W_1 and square root utilities, ca. $B = 1579$, leading to an economic value of $V = 451$, about 73% of the maximum, and a diversity number of $N_1 = 3.5$, over 87% of the maximum value, which seems to be the most suitable compromise within the decision space, if we consider that landscape diversity is a general potential safeguard relative to ecosystem services and other externalities not accounted in the economic value. In fact, the number of patch types may indicate the level of resource diversity while the proportions may determine the dominance of critical resources [49], and it is believed that more variation in a landscape generally leads to greater genetic and species diversity and this, in turn, stabilizes populations and strengthens the different ecosystem elements in the landscape which provides for more varied ecosystem services, which may enhance the resilience of the local economy [50]. Even so, the framework here discussed has two major limitations: it presupposes that the habitats are spatially interchangeable in the whole area considered, which would not be the case in most actual situations where there will be ecological constraints associated with heterogeneity and fragmentation in patches; and also, it does not account for the distinction between the ecological value of the habitats concerning differences between abundant or rare, even endangered, species. The last issue could be bypassed with a parallel assessment using ecological values for conservation of the habitats, or a broader version of economic values incorporating externalities such as ecosystem or landscape services.

In summary, comparing the optimal proportions of the decision models W_1 and W_2 relative to the same set of utilities in a small dimension example ($n = 4$), the analysis shows that W_1 performs in a considerably more conservative way relative to differences in utilities enhancing landscape diversity, except for the quadratic risk-prone transformation where both models perform about the same. The link to this difference is rooted in the fact that each model incorporates a rarity valuation measure, which for W_1, defined in Equation (2), is the nonlinear factor $r_{1,i} = -\log p_{1,i}$ (e.g., [51]) and for W_2 is the linear function $r_{2,i} = 1 - p_{2,i}$ as we can check in Equation (3); when the habitat i vanishes we get respectively the results $\lim_{p_{1,i} \to 0^+} (-\log p_{1,i}) = +\infty$ and $\lim_{p_{2,i} \to 0^+} (1 - p_{2,i}) = 1$, the infinite rarity value helping counteracting extinction.

Highlighting areas for future research, we point out that the trends of the results presented here should be tested relative to an analogous higher dimensional problem, and also using other utility valuation functions such as the logarithmic or negative exponential and a valuation procedure that incorporates an ecological assessment of singularities.

Also, the framework we describe in this paper could be helpful in revisiting the diversity-stability debate relative to dynamic equilibrium points in ecological studies. In general, when dealing with equilibrium points in dynamic systems we consider minima under potential formulation and, in probability formulation, we look for the opposite: we consider maxima, while minima are excluded [52]. Thus, we could think of optimal solutions of the models here discussed as minimum points of the symmetric forms $-W_1$ and $-W_2$ considered to be potential functions governing dynamic systems. The utilities $\{u_i\}_{i=1,\cdots,n}$ then could better be replaced by some kind of time indexed characteristic positive values of the different habitats, say $\{w_i(t)\}_{i=1,\cdots,n}$, for example relative to a measure of their resilience. In fact, we can go back to 1937 and Volterra's works [53] and find a symmetric form of weighted Shannon entropy, though conceived with whole numbers, not proportions, which is to be minimized concerning an entity called "vital action" of the community, when searching for an equilibrium. About six decades later, we find weighted Simpson index as a potential function characterizing an inverse measure for antigenic diversity of virus populations [54].

Concerning the object landscape, a recent review based on agent-based modeling of landscape dynamics shows a cornucopia of methods with emphasis on comparative approaches, pursuing the continuation of innovative modeling for understanding landscape change, its causes and consequences for sustainability in the Anthropocene [55].

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