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The Shell Collapsar—A Possible Alternative to Black Holes

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Abstract: This article argues that a consistent description is possible for gravitationally collapsed bodies, in which collapse stops before the object reaches its gravitational radius, the density reaching a maximum close to the surface and then decreasing towards the centre. The way towards such a description was indicated in the classic Oppenheimer-Snyder (OS) 1939 analysis of a dust star. The title of that article implied support for a black-hole solution, but the present article shows that the final OS density distribution accords with gravastar and other shell models. The parallel Oppenheimer-Volkoff (OV) study of 1939 used the equation of state for a neutron gas, but could consider only stationary solutions of the field equations. Recently we found that the OV equation of state permits solutions with minimal rather than maximal central density, and here we find a similar topology for the OS dust collapsar; a uniform dust-ball which starts with large radius, and correspondingly small density, and collapses to a shell at the gravitational radius with density decreasing monotonically towards the centre. Though no longer considered central in black-hole theory, the OS dust model gave the first exact, time-dependent solution of the field equations. Regarded as a limiting case of OV, it indicates the possibility of neutron stars of unlimited mass with a similar shell topology. Progress in observational astronomy will distinguish this class of collapsars from black holes.

Keywords: gravitational collapse; Oppenheimer-Snyder; horizonless solutions; neutron stars

1. Introduction

This article shows that there is an approach to the field equations of General Relativity (GR) which avoids solutions with density singularities (black holes) and their paradoxical consequences [1,2], leading instead to horizonless solutions called collapsars. I am therefore proposing that the theme of *Black Hole Thermodynamics* should be placed within the wider one denoted by my article's title.

My argument is based on the classic article of Oppenheimer and Snyder [3] (OS), which, during the lifetime of its authors, was the only known exact solution of the field equations describing the evolution of a collapsing system under gravity. An analysis of the density of the OS collapsar shows a ball of uniform density at $t = -\infty$ evolving into a ball of gravitational radius $r_0 = 2GM/c^2$ at $t = +\infty$, with its mass strongly concentrated near the surface. The OS collapsar is a highly idealized model representing a cloud of 'dust' with zero pressure falling freely under its own gravity; it has a density discontinuity, and therefore a discontinuity in the curvature tensor, at its surface, and this becomes increasingly severe as the surface contracts. We should, of course, require that a real collapsar, with an equation of state (EoS) relating pressure to density satisfy curvature continuity at the surface. However, describing the time evolution of such an object remains a major challenge right up to the present day.

My colleagues' and my own studies of gravitational collapse fall into the categories of either the time-independent case, for which the pioneering article is that of Oppenheimer and Volkoff (OV) [4], or its partner, the time-dependent case of Oppenheimer and Snyder (OS) [3]. Our modification [5] to the conventional understanding of OV has its origin in footnote 10 of the OV article, where the authors, having acknowledged that there exists a family of solutions of the Hilbert-Einstein equations for which the pressure is zero at $r = 0$, failed to follow up that possibility. We discovered that such solutions have a shell-like density profile, with most of the material density concentrated just inside the surface; inside the shell there is a region consisting almost exclusively of negative gravitational energy.

The above description has some features in common with that of gravastars [6–8]. These objects, which, like the OS collapsar, are horizonless, have shells of undefined exotic material, in contrast with the neutron gas or liquid shells, which is what OV considered and which we consider in detail in Section 3. The stability of gravastars has been examined, in some cases with positive outcome, and it has also been suggested that the recently observed LIGO gravitational waves [9] from GW150914 could have been produced by the merger of two such objects [10].

There is, as yet, no calculation of the time evolution of a collapsar with a realistic equation of state like that of OV, so we have no rigorous means of deciding whether the shell model or the black hole model is correct for the final state. My study of the OS article [11–13], however, gives support to the shell model, because it shows that, as the radius of the OS collapsar approaches its asymptotic value at the gravitational radius r_0, the density at the surface becomes infinite. In Section 2.2 of the present article a theorem is established, namely that, at all times during the collapse, more than half the "dust" of the OS collapsar lies outside the radius $0.947r_0$. I also investigate the trajectory of a projected test particle as it travels from a surface point to the centre and then on to an antipodal point of the surface, thereby giving further illustration of the shell structure. However, the material of the OS collapsar, commonly referred to as "dust", is devoid of any self interaction; effectively it has dynamics but no thermodynamics. So, having used the OS collapsar to convince oneself that the shell version of OV is a plausible alternative to black holes, the programme one then follows can only be that of combining the normal thermodynamic variables of pressure, density and temperature with an OS-type gravitational field. Such a description will have much in common with that of normal stars; in particular there will be a smoothing of the infinite density at the surface of the OS collapsar in its final state. The shell structure of the OS collapsar indicates that we should take seriously the idea that gravity may become repulsive at high densities. In Section 3, applying the notion of repulsive gravity to neutron stars, we show that allowing the possibility of a positive pressure gradient at the centre can open the way to stationary shell collapsars with nuclear, that is nonexotic densities. This in turn means that such collapsars could exist for arbitrary mass, beyond the Tolman-Oppenheimer-Volkoff [14] limit but not composed of exotic hyperon or quark material.

2. The Dynamics of Oppenheimer-Snyder

The OS collapsar [3] with gravitational mass m has the vacuum Schwarzschild metric

$$ds^2 = \frac{r-2m}{r}dt^2 - \frac{r}{r-2m}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \quad , \tag{1}$$

in the exterior region $r > r_1(t) > 2m$, the surface separating it from the interior being defined by

$$t = -\frac{2}{3}\sqrt{\frac{r_1^3}{2m}} - \sqrt{2mr_1} + 2m\ln\frac{\sqrt{r_1}+\sqrt{2m}}{\sqrt{r_1}-\sqrt{2m}} \quad . \tag{2}$$

A particle projected radially inwards from $t = -\infty$ has the integrals of motion

$$\begin{aligned} \frac{r-2m}{r}\frac{dt}{ds} &= C \quad , \\ \left(\frac{dr}{ds}\right)^2 &= C^2 - 1 + \frac{2m}{r} \quad . \end{aligned} \tag{3}$$

If the particle is freefalling $C = 1$, and hence

$$\frac{dr}{dt} = -\frac{r - 2m}{r}\sqrt{\frac{2m}{r}} \quad . \tag{4}$$

Then Equation (2) is the integral of this, indicating, because t goes to infinity as r_1 approaches $2m$, that points at the surface of the OS collapsar are in free fall, and that a freefalling particle takes an infinite time to arrive at the surface, a result which is summed up by the statement OS made in their Abstract

> ".... an external observer sees the star asymptotically shrinking to its gravitational radius."

The title given to the OS article was "On continued gravitational contraction". My analysis of the OS metric will show that there is a single time coordinate t for both the exterior and interior regions, and that therefore the contraction is "continued" only in the sense that it continues to contract until $t = +\infty$; in spatial terms contraction stops at the gravitational radius, as was insisted upon by Einstein's article [15] published in the same year as OS.

In an early stage of black-hole theory the OS article's conclusion was seriously misquoted by Penrose [16] who stated

> "The general situation with regard to a spherically symmetrical body is well known [3]. For a sufficiently great mass, there is *no final equilibrium state* (my emphasis). When sufficient thermal energy has been radiated away, the body contracts and continues to contract until a physical singularity is encountered at $r = 0$. As measured by local comoving observers, the body passes within its Schwarzschild radius $r = 2m$."

OS had observed that a particle falling freely from any exterior point $r > r_1(t)$ would reach $r = 2m$ in a finite interval of proper time, which is because the gravitational red shift causes both falling particles and light rays to slow to zero speed as they approach the limiting value of r. But they also constructed a coordinate system for the interior region, which they matched to the exterior, as we shall see below. At no point does the surface go inside the "horizon". The relation of coordinate to proper time in the OS collapsar has also been demonstrated more recently [1,17]; the "trapped surfaces" of Penrose's theorem exist only at proper times which violate the *reality condition* (see Section 2.1), that is for values of t lying literally beyond infinity!

2.1. Horizonless Dust Metrics

Some eight years after the publication of Penrose's theorem the first time dependent dust metric was constructed [18] for which contraction continues all the way to a singularity at $r = 0$, and since that time the family has been widely extended [19]. The metrics of this class share the feature that their density decreases monotonically with r for $r > 0$, in contrast with the density of the OS metric which, as we shall see, *increases* monotonically from $r = 0$ to the surface at $r = r_1(t)$. It is clear that decreasing density allows the prerequisite of Penrose's theorem to be satisfied, leading to the formation of a horizon, while the increasing density of the OS metric leads to the formation of a shell at the surface, as we shall now demonstrate. The post-Penrose family of dust metrics, with a density maximum at $r = 0$, should be considered as *neo-newtonian*, whereas the OS and similar [11] metrics show the existence of solutions of the field equations with the feature of *repulsive gravity*, which we discuss for metrics with a realistic equation of state in the next section. An interesting property of the neo-newtonian family is that they have complicated initial states with nonzero ingoing radial speeds, in contrast with OS, for which the initial state is "a spatially constant matter density at rest" [19].

The OS interior metric replaces the coordinates (t, r) by (τ, R), with $0 \le R < 1$, in such a way that the surface point (t, r_1) maps into $(\tau, 1)$ and the metric tensor is continuous there

$$ds^2 = d\tau^2 - \frac{r^2}{R^2}dR^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \quad , \tag{5}$$

where

$$r = 2mR\left(1 - \frac{3\tau}{4m}\right)^{2/3} \quad . \tag{6}$$

A test particle in the interior falling freely from $\tau = -\infty$ has constant values for (R, θ, ϕ), that is R is a *comoving* ([20] Section 11.9) coordinate. The OS interior time coordinate t is then defined through the *cotime* y:

$$\frac{t}{2m} = -\frac{2}{3}y^{3/2} - 2\sqrt{y} + \ln\frac{\sqrt{y}+1}{\sqrt{y}-1} \quad , \tag{7}$$

with

$$y = \frac{r}{2mR} + \frac{R^2}{2} - \frac{1}{2} \quad , \tag{8}$$

So the cotime y is equal to $r_1/(2m)$ at the surface, and it must always be greater than 1. This is the reality condition. At the surface, $R = 1$, the limiting value of y is reached at $\tau = 0$, and at all interior points it is reached at earlier values of τ. Failure to see these limiting values of τ is what led to the Penrose error referred to above. Under the coordinate changes (6) and (7) this interior metric becomes

$$ds^2 = \frac{r^2(y-1)^2}{2mRy^3(r-2mR^3)}dt^2 - \frac{r}{r-2mR^3}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \quad . \tag{9}$$

At $R = 1$ this is the same as (1), thereby establishing continuity of the metric. The determinantal density factor for this metric is

$$\sqrt{-g} = \frac{r^{7/2}(y-1)\sin\theta}{(r-2mR^3)\sqrt{2mRy^3}} \quad . \tag{10}$$

2.2. Concentration of Matter into a Shell

The OS property regarding concentration of matter at $R = 1$ follows from consideration of the stress tensor there, that is the contracted curvature tensor divided by 8π. In the comoving coordinates the only nonzero component of the stress tensor is $T^{\tau\tau} = \rho$, the scalar density ρ being ([20], Section 11.9)

$$\rho = \frac{1}{16\pi}\left[2g^{RR}\frac{\partial^2 g_{RR}}{\partial\tau^2} + 4g^{\theta\theta}\frac{\partial^2 g_{\theta\theta}}{\partial\tau^2} - \left(g^{RR}\frac{\partial g_{RR}}{\partial\tau}\right)^2 - 2\left(g^{\theta\theta}\frac{\partial g_{\theta\theta}}{\partial\tau}\right)^2\right]. \tag{11}$$

Substituting (5) and (6), this leads, in the interior region $r < r_1(t)$, to

$$T^{\tau\tau} = \frac{3mR^3}{4\pi r^3} \quad , \tag{12}$$

and in the exterior, of course, $T^{\tau\tau} = 0$, which means the OS metric has a discontinuity in its curvature tensor at the surface. In the coordinates (t, r, θ, ϕ), the nonzero components of T are

$$T^{tt} = T^{\tau\tau}\left(\frac{\partial t}{\partial\tau}\right)^2, \quad T^{tr} = T^{\tau\tau}\frac{\partial t}{\partial\tau}\frac{\partial r}{\partial\tau}, \quad T^{rr} = T^{\tau\tau}\left(\frac{\partial r}{\partial\tau}\right)^2 \quad . \tag{13}$$

On substituting (6) and (7) T^{tt} becomes

$$T^{tt} = \frac{3m^2R^4y^3}{2\pi r^4(y-1)^2}, \tag{14}$$

that is, using the determinantal factor (10),

$$T^{tt}\sqrt{-g} \;=\; \frac{3m^2R^4y^{3/2}\sin\theta}{2\pi\sqrt{2mrR}(y-1)(r-2mR^3)} \quad . \tag{15}$$

The mass $\mu(r_2)$ contained in $r < r_2 \le 2my$ is then

$$\mu(r_2) = \int_{r<r_2} T^{tt}\sqrt{-g}drd\theta d\phi \quad . \tag{16}$$

The integration is to be carried out at constant y, which is readily achieved by putting

$$r = mR(2y + 1 - R^2) \quad , \tag{17}$$

and changing the variable of integration to R. Before doing that, however, we note that the denominator of $T^{tt}\sqrt{-g}$ contains a factor of $2y + 1 - 3R^2$, which diverges for $y \to 1$ and $R \to 1$, that is $r \to 2m$, and this is the concentration of mass density referred to in the Introduction. After the change of variable, this factor is cancelled by dr/dR and we obtain

$$\mu(r_2) = \int_0^{R_2(y)} \frac{3mR^2\sqrt{2y^3}}{(y-1)\sqrt{2y+1-R^2}}dR \quad , \tag{18}$$

where the limit $R_2(y)$ corresponds to $r = r_2$; in particular $R_2 = 1$ when $r_2 = 2my$. For given y we may calculate the *half-mass radius* r_{HM} defined by $\mu(r_{HM}) = \mu(2my)/2$. In the limit $y \to +\infty$, $r_{HM}/2my$ takes the value $2^{-1/3} = 0.7937\ldots$, which corresponds to the uniform density of the initial state. In the limit $y \to 1$ this quantity is $0.9470\ldots$, and the move of r_{HM} towards $2my$ with decreasing y reflects the concentration of mass near $r = 2my$.

2.3. Dynamics of Test Particles

We may now investigate the dynamical effect of the shell structure by considering the behaviour of a test particle falling along a geodesic in the interior region. Geodesics along a radius ($d\theta = d\phi = 0$) may be found by defining $x = r/(2mR)$ and expressing τ as a function of (x, R). The metric then becomes

$$\frac{ds^2}{4m^2} = xdx^2 - x^2dR^2 \quad , \tag{19}$$

and the cyclic variable R satisfies the equation

$$2m\frac{dR}{ds} = -\frac{\alpha}{x^2} \quad , \tag{20}$$

where the constant α is positive, and x takes the value $x_1 = r_1/(2m)$ at $R = 1$. Then the evolution of x is given, for a particle going towards $r = 0$, by

$$2m\frac{dx}{ds} = -\sqrt{\frac{1}{x} + \frac{\alpha^2}{x^3}} \quad . \tag{21}$$

If the particle was projected from $r = \infty$ with $C > 1$, as in (3), the constant α is related to C by

$$\left(\frac{2m\alpha}{r_1} + \sqrt{\frac{2m}{r_1} + \frac{8m^3\alpha^2}{r_1^3}}\right)^2 = C^2 - 1 + \frac{2m}{r_1} \tag{22}$$

The freefall case $C = 1$ corresponds to $\alpha = 0$, for which Equations (20) and (21) give, for all interior points,

$$R = const, \quad s - s_0 = \frac{2}{3}\left(x_0^{3/2} - x^{3/2}\right) \quad . \tag{23}$$

The constancy of R in this case establishes that "dust" particles inside the OS collapsar, like those at the surface, are falling freely in the gravitational field of the other particles, and confirms that R is a comoving coordinate.

For the general case $\alpha > 0$, we may eliminate s between (20) and (21) to obtain

$$R(x) = 1 - \int_{x}^{x_1} \frac{dx'}{\sqrt{x' + x'^3/\alpha^2}} \quad . \tag{24}$$

In particular the particle reaches $R = 0$ with $x = x_0$ given by

$$\int_{x_0}^{x_1} \frac{dx}{\sqrt{x + x^3/\alpha^2}} = 1 \quad . \tag{25}$$

Furthermore, after passing through the origin both dR/ds and dr/ds change sign leaving the sign of dx/ds unchanged, so that the particle exits at $R = 1$ with $x = x_2$ satisfying

$$\int_{x_2}^{x_1} \frac{dx}{\sqrt{x + x^3/\alpha^2}} = 2 \quad . \tag{26}$$

However, these values of x_0 and x_2 depend on the initial values x_1 and α allowing the time coordinate $t(x, R)$ to be real, that is $y > 1$, which implies, from (8), that $x_0 > 3/2$ and $x_2 > 1$. In particular the minimum value $x_{1m}(\alpha)$ allowing the particle to exit has $x_2 = 1$ and satisfies

$$\int_{1}^{x_{1m}(\alpha)} \frac{dx}{\sqrt{x + x^3/\alpha^2}} = 2 \quad . \tag{27}$$

We find, for example, that $x_{1m}(2) = 9.539$, $x_{1m}(4) = 4.878$, and $x_{1m}(\infty) = 4$; the latter value was obtained in [12] and represents the case where the initial value of dR/ds is infinite, that is our particle has become a light ray.

2.4. Consequences of the Reality Condition

I remark that, although my article [11] established that the OS choice (8) of the function $y(x, R)$ is not unique, the reality condition (27) is independent of that choice, and arises solely from the continuity of the metric tensor at $R = 1$. The wider family of solutions revealed in my article must also satisfy the reality condition, and in addition the condition that $y = 1$ when $R = 1$; this means the modification studied in my article [11] gives very similar dynamics to those of the original OS metric described above.

If the particle fails to satisfy the reality condition for x_2, that is if $x_1 < x_{1m}$, that means the trajectory reaches $y = 1$ before $R = 1$, and the final value of r will then be less than the gravitational radius $2m$, that is it will lie inside the "event horizon". We encountered an extreme case of this in [12], where x_1 itself was taken so close to the horizon that the particle did not penetrate beyond the dense concentration of matter at the collapsar's surface. The geodesics we obtained above enable us now to investigate what happens when the surface shell is approached from the interior. Defining the cartesian coordinate $Z = R\cos\phi$, where $\theta = \pi/2$, and $\phi = 0$ as far as the centre and $\phi = \pi$ after that, we consider the trajectory for $-1 < Z < 1$ with the initial values $Z = 1$ and $dZ/ds = -\alpha/x_{1m}(\alpha)^2$; such a particle just reaches $Z = -1$ after an infinite t-interval. A typical trajectory has been plotted in Figure 1 with the values $\alpha = 2, r_1/(2m) = 9.538$, and in this case we see that the 4-velocity component dr/ds changes sign at $R = 0.918$, which means the particle actually turns back towards the centre just before reaching the very high concentration of matter at the surface. We therefore have here further confirmation that the high-density shell produces a repulsive gravity field.

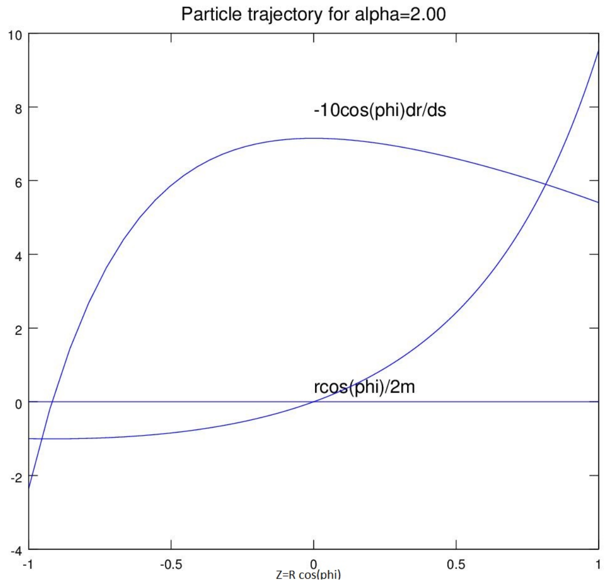

Figure 1. The trajectory of a particle projected inwards from the surface of an Oppenheimer-Snyder (OS) collapsar, with initial values $r_1/(2m) = 9.538, \alpha = 2$. The particle passes through $r = 0$ and then turns back towards the centre at $R = 0.918$, which corresponds to a value of r just inside the "horizon".

2.5. *Kerr Metrics for Spinning Collapsars*

In view of the importance now rightly accorded to the study of spinning collapsars, following the discovery of the Kerr [21] metric more than 50 years ago, my extensive discussion of the OS metric may seem hopelessly out of date. However, it seems to have been almost universally overlooked what Kerr stated in his final paragraph

> "If we expand the metric in Equation (5) as a power series in m and a .... and compare it with the third-order Einstein-Infeld- Hoffman approximation for a spinning particle, we find that m is the Schwarzschild mass and ma the angular momentum ... It has no higher order multipole moments in this approximation. Since there is no invariant definition of the moments in the exact theory, one cannot say what they are... It would be desirable to calculate an interior solution to get more insight into this."

The results obtained in the present article indicate that the time has come to resume the programme recommended by Kerr 50 odd years ago. Only then will we be able to extend the investigation of the interior geodesics from the spherosymmetric to the axisymmetric case. Such a programme now assumes new relevance in the description of the merged collapsar resulting after the inspiralling of a binary collapsar [9,10]; the merged object has a very large angular momentum even for the case where the inspiralling ones have no intrinsic spin.

3. The Thermodynamic Implications

The previous section considered the highly idealized OS dust model, whose properties are independent of the collapsar's gravitational mass m. With the addition of an appropriate equation of state (EoS) it would describe the formation of a neutron star; the EoS may be taken as that of a neutron gas as in OV, though one could, with minimal additional complication, use the Fermi liquid EoS of Cameron [22]. In contrast with the dust collapsar of OS, the final density profile will depend on the mass of the object.

Such an extension is easy to describe, but the magnitude of its undertaking is truly awesome. I reproduce here the description given by Tolman [23], who pioneered not only the dust model of a collapsar, but also the application of thermodynamics to cosmology [24], the subject of this issue of *Entropy* :

> ".... the fluid in the models was taken as dust exerting negligible pressure. Hence no allowance was made for effects such as thermal flow from one portion of matter to another, which in the actual universe might provide a non-gravitational kind of action which would tend to iron out inhomogeneities."

This was written over 80 years ago, and the author had in mind the "inhomogeneities" of the universe as a whole. What I have in mind is the application of the time-dependent OS model to the time-independent OV, and requires, among other things, finding the effect of nonzero pressure on the "inhomogeneity" presented by the OS surface $r_1(t)$, in particular the infinite density at that surface in the limit $t \to +\infty$. It seems modest by comparison with Tolman's programme, but it is a project which remains to be started.

Before starting this ambitious project, it is nevertheless possible to use what we have learned in the previous section from the OS model, namely that at high densities gravity may be repulsive. We may then appreciate that the Tolman-Oppenheimer-Volkoff (TOV) limit on the mass of a neutron star has been accepted through an incomplete understanding of what the field at its centre may be. The requirement that the maximum density must be at the centre is a relic left over from Newtonian gravity, so models which meet such a requirement are *neo-newtonian*.

It is tempting at this point to indulge a little in historical speculation. OV was published in February 1939, and included footnote 10 which conceded the possibility that the pressure could be zero at the centre. To arrive at the point where I suggest we now are, all that OS needed after their article appeared on September 1st, 1939 was to apply the knowledge gained therein, namely

> ".... an external observer sees the star asymptotically shrinking to its gravitational radius."

This knowledge, extended by a few steps in the previous section, got us to repulsive gravity. But September 1st, 1939 was the day Hitler invaded Poland, so maybe that is why it has taken an extra 77 years!

The TOV limit originated in the OV field equations [4]

$$\begin{aligned} \frac{du}{dr} &= 4\pi r^2 \rho \quad , \\ \frac{dp}{dr} &= -\frac{(p+\rho)(u+4\pi r^3 p)}{r(r-2u)} \quad . \end{aligned} \tag{28}$$

These may be numerically integrated. With $u(0) = 0$, and any initial value of ρ at $r = 0$, one finds [4,20], for a neutron-star EoS of the form $p = p(\rho)$, a value $r = r_1$ at which $p(r_1) = 0$. The interior metric for $r < r_1$ is smoothly matched to an exterior Schwarzschild metric for $r > r_1$ corresponding to a gravitational mass m, where m is equal to the integral of ρ over the interior. The TOV mass, above which no solution of this type exists, is when the initial value $p(0)$ tends to infinity.

This conclusion is based on the assumption, essentially Newtonian, that the gravitational field must be attractive, and that consequently p has its maximum at $r = 0$. However, if, instead of integrating outwards from $r = 0$, we fix m and integrate inwards from an arbitrary value of r_1 greater than $2m$ we find [5] that we have a family of solutions of (28), all of which have p increasing to a maximum and falling to $p = 0$ at a point r_2 between r_1 and zero.

A feature common to this family of solutions is that the variable u takes a negative value at r_2 which is determined by our choice of m and r_1. In Newtonian theory, which Chandrasekhar had previously used for white dwarfs (see Weinberg's discussion in [20]), $u(r_2)$ is the mass contained inside a sphere of radius r, so the negative value of $u(r_2)$ may be a surprise. Our proposed interpretation [5] is that the quantity $m - u(r_2) > m$ is the total or proper mass, part of which is cancelled by a substantial negative gravitational energy, contained in $r < r_1$; the fact that this total mass is greater than the gravitational mass was acknowledged in Weinberg's presentation, where, however, the two parts of the energy do not have the same values as we are proposing.

A further novel feature of the new family is that it apparently gives zero pressure and density, though not zero gravitational field, for $r < r_2$. However, this is because the OV equation of state, although it gives a neat cutoff in p at both r_1 and r_2, is inappropriate for small p; it should be replaced by the EoS for an electron gas as the density falls and a proportion of neutrons decay into protons and electrons, and this in turn should ultimately be replaced by an atmospheric EoS appropriate for a population of atoms, probably largely Fe^{56}. It is remarkable that Oppenheimer and Volkoff [4] in footnote 10 of their article seem already to have been aware of the possibility that $u(0) < 0$ and that the EoS at $r = 0$ may be of a different character from the EoS in the high-density region. It was recognized long ago [22] that there is an outer atmosphere in $r > r_1$, so now we should reckon with a complementary inner one in $r < r_2$. It seems likely that a more complete treatment of this inner atmosphere will give a positive pressure at the centre together with a positive pressure gradient, hland hence a positive density as was indicated in the analysis of the dust model of the previous section.

So far it has not been possible to make an estimate of how the surface radius r_1 may be related to the gravitational mass m. The values obtained experimentally indicate [14] that, for neutron stars in the solar-mass range, r_1 falls in the range $6m$–$10m$.

More work on the inner atmosphere is required before an estimate of this parameter may be made. Note that, in addition to locating where to put the exterior surface r_1, both r_1 and the new interior surface r_2 are really time dependent, and both variables should be treated along the lines of the Tolman programme [23] quoted at the beginning of this section. This requires a more ambitious programme than has been undertaken [14] for static solutions.

4. Discussion

Black-hole collapsars are not the only possible solutions of the field equations for a massive compact object; horizonless solutions of the gravastar family, which do not generate the same paradoxes of causality and information loss as black holes [1,2], provide an alternative description. The *superTOV neutron star* (STNS) described in the previous section may be considered a subvariety of gravastars. The family as a whole is characterized by a high-density shell of stellar material enclosing an interior region of small or zero material density, but containing a repulsive gravitational field. We have discussed the gravitational energy distribution of a STNS in previous articles [5,25,26]. No study has so far been attempted of the stability of such objects, but that of the more exotic gravastars has been investigated [10], in some cases with positive results.

The observation of the gravitational waves coming from the merging of two collapsars [9] now points a way to distinguish between black holes and gravastars. There is little difference between the inspiral part of the signal coming from either of these alternatives, but the 'merger-ringdown' parts are expected to be very different [10,27]; indeed, in the gravastar case, the merger is from a mass distribution concentrated in two contiguous, more or less spherical shells to a single axisymmetric shell, so it may be more appropriate to use the term 'ringup'. As more of such merger events are observed, it will be possible to examine the downchirp part of the signal, especially its frequency spectrum [27], with a view to resolving the nature of these collapsars.

Black hole thermodynamics has its origin in the study of the energy and entropy changes which accompany such mergers. It is now clear that, once we broaden the class of collapsars to include horizonless objects, we are introducing a substantial new family of thermodynamic variables. The suggestion coming from my study of STNS in the previous section is that some of these variables originate in the *formation* of collapsars, that is before the inspiralling and merger processes began. As far as the STNS case is concerned, it seems likely that these latter variables have their origin in the composition of the inner atmosphere mentioned at the end of the previous section.

The object at the centre of a galaxy is popularly classified as a *supermassive black hole* [28]. Following the argument about the boundary condition at $r = 0$, however, it may alternatively be treated as a member of the gravastar family, and in particular as a collapsar of our extended OV family. The only difference is that its radius and surface area are of a vastly higher order of magnitude than in STNSs, so that the maximum density of the surface shell is reduced below the threshold for beta decay [20] from neutrons to protons plus electrons. This means we should replace the neutron-star EoS by the white-dwarf EoS of Chandrasekhar [29]. The surface of this object must lie close to its accretion disc; for example our own galactic centre has a gravitational radius of 1.2×10^7km, and observations by the Event Horizon Telescope [30] (EHT) show that its accretion disc lies close to that radius. There is a variety of possible mechanisms for energy interaction with the disc, so effectively the galactic centre interacts directly with both its internal and external atmospheres, and ultimately the galaxy at large. Hence, both the galactic centre and its accretion disc have finite temperatures and may emit radiation across the electromagnetic spectrum. Although it seems likely that the accretion disc is the more intense source, there remains the possibility that the signal from the galactic centre, which has a different image [13] from the disc, may nevertheless also be visible. Thus with the EHT another way may be open for distinguishing between these two forms of collapsar.

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