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Article

On the Exact Solution of Wave Equations on Cantor Sets

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Abstract: The transfer of heat due to the emission of electromagnetic waves is called thermal radiations. In local fractional calculus, there are numerous contributions of scientists, like Mandelbrot, who described fractal geometry and its wide range of applications in many scientific fields. Christianto and Rahul gave the derivation of Proca equations on Cantor sets. Hao *et al.* investigated the Helmholtz and diffusion equations in Cantorian and Cantor-Type Cylindrical Coordinates. Carpinteri and Sapora studied diffusion problems in fractal media in Cantor sets. Zhang *et al.* studied local fractional wave equations under fixed entropy. In this paper, we are concerned with the exact solutions of wave equations by the help of local fractional Laplace variation iteration method (LFLVIM). We develop an iterative scheme for the exact solutions of local fractional wave equations (LFWEs). The efficiency of the scheme is examined by two illustrative examples.

Keywords: local fractional calculus; local fractional Laplace variation iteration method; local fractional wave equations

1. Introduction

Fractional calculus (FC) has attracted the attention of many scientists in different scientific fields due to its numerous applications in our day life problems. In these applications, there are contributions of different parts of FC. Among the different parts, local FC has a wide range of applications in the fields of physics and engineering based on the fractals. The fractal curves [1] are everywhere continuous but nowhere differentiable and therefore, the classical calculus cannot be used to interpret the motions in Cantor time-space [2]. Calcagni [3] studied continuous geometries with some specific dimensions. Local FC [4–7] started to be considered as one of the useful ways to handle the fractals and other functions that are continuously but non-differentiable.

Mandelbrot [8] described fractal geometry as a workable geometric middle ground between the excessive geometric order of Euclid and the geometric chaos of general mathematics and extensively illustrated wide range of applications fractals in many scientific fields like in, physics, engineering, mathematics and geophysics. Zhang and Baleanu [9] studied local fractional wave equations under fixed entropy. Srivastava *et al.* [10] studied an initial value problem by the help of Sumudu Transform. Li *et al.* [11] studied local fractional Poisson and Laplace equations and provided its application in fractal domain. Christianto and Rahul [12] gave the derivation of Proca equations on Cantor sets. Hao *et al.* [13] investigated the Helmholtz and Diffusion equations in Cantorian and Cantor-Type Cylindrical Coordinates. Carpinteri and Sapora [14] studied diffusion problems in fractal media in Cantor sets. Yang *et al.* [15] and Su *et al.* [16] studied wave equations in Cantor sets.

Many techniques are utilized for handling the local fractional problems in both ordinary and partial derivatives. For instance, The Yang-Laplace Transform [4], the local fractional Laplace variation iteration method (LFLVIM) [7] and many others. These methods are widely used in different scientific fields [6–10]. This area of research is much popular in the community of scientists and we are continuously observing recent developments in it. These developments are useful in engineering and physics and we feel further attention of scientists for the exploration of its different aspects.

This paper is organized as follows: In the first section, we have pointed out the essential and related work to local fractional differential equations (LFDEs) and the techniques which have been produced for handling the LFDEs. In Section 2, we have presented the preliminary results, which we will utilize for the production of iterative scheme. In Section 3, we produce an iterative scheme based on the LFLVIM for the solution of wave equations. Section 4 demonstrates the efficiency of our scheme by the help of several examples.

2. Preliminaries

In this section we are presenting the basic and related definitions and relations from local fractional calculus [1–5].

A function $g(t)$ is said to be local fractional continuous function if $f(t)$ satisfies

$$|g(t)-g(t_0)| < a^{\gamma}, \tag{1}$$

where $\gamma \in (0,1]$, $|t-t_0| < b$, for $a, b > 0$ and $a, b \in R$.

The local fractional derivative of a function $g(t) \in C_{\gamma}(a,b)$ of order γ is defined as:

$$\frac{d^{\gamma}f(t)}{dt^{\gamma}} = \frac{\Delta^{\gamma}\big(f(t)-f(t_0)\big)}{(t-t_0)^{\gamma}}, \tag{2}$$

where

$$\Delta^{\gamma}\big(f(t)-f(t_0)\big) = \Gamma(1+\gamma)\big(f(t)-f(t_0)\big) \tag{3}$$

and

$$\frac{d^{\gamma}}{dt^{\gamma}}\left(\frac{t^{m\gamma}}{\Gamma(m\gamma+1)}\right) = \frac{t^{(m-1)\gamma}}{\Gamma(\,(m-1)\gamma+1)}, m \in N. \tag{4}$$

The local fractional integral of a function $g(t)$ on [a, b] is defined by

$$I^{(\gamma)}{}_{[a,b]}f(t) = \frac{1}{\Gamma(\gamma+1)}\int_a^b g(x)(dx)^{\gamma} = \frac{1}{\Gamma(\gamma+1)}\lim_{\Delta x\to 0}\sum_{i=0}^{i=M-1} g(x_i)(\Delta x_i)^{\gamma}, \tag{5}$$

where $[a,b]$ is divided into $M-1$ sub-intervals (t_i, t_{i+1}) and $\Delta t_i = t_{i+1}-t_i$, with $a = t_0$, $b = t_M$ and

$$I^{(\gamma)}{}_{0,t}\left(\frac{t^{m\gamma}}{\Gamma(m\gamma+1)}\right) = \frac{t^{(m+1)\gamma}}{\Gamma((1+m)\gamma+1)}. \tag{6}$$

The Mittage-Leffler function in fractal space is defined by

$$E_{\gamma}(t^{\gamma}) = \sum_{j=0}^{\infty}\frac{t^{j\gamma}}{\Gamma(j\gamma+1)}. \tag{7}$$

Yang-Laplace Transforms (YLT)

In this subsection, we are giving definitions and some basic results related to the Yang-Laplace transforms (YLT). For a function $g(t)$ satisfying the following inequality,

$$\frac{1}{\Gamma(\gamma+1)}\int_0^{\infty}|g(t)|(dt)^{\gamma} < m < \infty, \tag{8}$$

the YLT is defined by

$$Ł_{\gamma}\{f(t)\} = \frac{1}{\Gamma(\gamma+1)}\int_0^{\infty}E_{\gamma}(-s^{\gamma}t^{\gamma})\,g(t)(dt)^{\gamma}, \tag{9}$$

where $\gamma \in (0,1]$ and $s^{\gamma} = \beta^{\gamma}+i^{\gamma}\omega^{\gamma}$ for i^{γ}, the fractal imaginary unit and $\mathrm{Re}(s^{\gamma}) = \beta^{\gamma} > 0$. The YLT has the following properties:

$$Ł_{\gamma}\{ah(t)+bg(t)\} = aŁ_{\gamma}\{h(t)\} + bŁ_{\gamma}\{f(t)\}, \tag{10}$$

$$\begin{gathered} Ł_{\gamma}\{h(t,x)\}^{m\gamma} = s^{m\gamma}Ł_{\gamma}\{h(t,x)\} - s^{(m-1)\gamma}h(0,x) - s^{(m-2)\gamma}h(0,x)^{(\gamma)} \\ -s^{\gamma(m-3)}h(0,x)^{(2\gamma)} - \cdots - h(0,x)^{((m-1)\gamma),} \end{gathered} \tag{11}$$

$$Ł_\gamma\{t^{m\gamma}\} = \frac{\Gamma(m\gamma+1)}{s^{(m+1)\gamma}}, \tag{12}$$

$$\cos h_\gamma(t^\gamma) = \sum_{j=0}^{\infty} \frac{t^{2j\gamma}}{\Gamma(2j\gamma+1)}. \tag{13}$$

3. Iteration Scheme

In this section, we produce an iterative scheme for the solution of LFWEs based on LFLVIM. For this we consider the following LFDE

$$v_t^{m\gamma} - p(x)v_x^{n\gamma} = 0, \tag{14}$$

where m, n are orders of local fractional partial derivatives with respect to t *and* x respectively. Applying the local fractional variation iteration method for the correction local fractional operator for (14), we have

$$v_{m+1}(t,x) = v_m(t,x) + I^{(\gamma)}{}_{0,t}\left(\frac{\mathfrak{X}(x)^\gamma}{\Gamma(\gamma+1)}\right)\left(v_t^{m\gamma} - p(x)v_x^{n\gamma}(t,x)\right), \tag{15}$$

where $\frac{\mathfrak{X}(x)^\gamma}{\Gamma(\gamma+1)}$ is the Lagrange multiplier and (15) leads to

$$v_{m+1}(t,x) = v_m(t,x) + I^{(\gamma)}{}_{0,t}\left(\frac{\mathfrak{X}(t-x)^\gamma}{\Gamma(\gamma+1)}\right)\left(v_t^{m\gamma}(t,x) - p(x)v_x^{n\gamma}(t,x)\right). \tag{16}$$

Applying the operator YLT, of order γ that is $Ł_\gamma$ on (16), we have

$$Ł_\gamma\{v_{m+1}(t,x)\} = Ł_\gamma\{v_m(t,x)\} + Ł_\gamma\left\{\frac{\mathfrak{X}(x)^\gamma}{\Gamma(\gamma+1)}\right\}Ł_\gamma\{v_t^{m\gamma} - p(x)v_x^{n\gamma}(t,x)\}. \tag{17}$$

Taking the γ order local fractional variation of (17) with respect to t, and assuming that the term $p(x)v_x^{n\gamma}(t,x)$ be invariant, we have

$$\begin{aligned} &\delta^{(\gamma)}Ł_\gamma\{v_{m+1}(t,x)\} \\ &= \delta^\gamma Ł_\gamma\{v_m(t,x)\} + Ł_\gamma\left\{\frac{\mathfrak{X}(x)^\gamma}{\Gamma(\gamma+1)}\right\}\delta^{(\gamma)}\big[s^{m\gamma}Ł_\gamma\{v_m(t,x)\} - s^{(m-1)\gamma}v_m(0,x) \\ &- s^{(m-2)\gamma}v_m(0,x)^{(\gamma)} - s^{\gamma(m-3)}v_m(0)^{(2\gamma)} - \cdots - v_m(0)^{((m-1)\gamma)} = 0, \end{aligned} \tag{18}$$

From (18) we obtain the Lagrange-Multiplier as follows

$$Ł_\gamma\{\frac{\mathfrak{X}(x)^\gamma}{\Gamma(\gamma+1)}\} = \frac{-1}{s^{m\gamma}}, \tag{19}$$

and by the help of (18) and (19), we have the following relation

$$\begin{aligned} Ł_\gamma\{v_{m+1}(t,x)\} = Ł_\gamma\{v_m(t,x)\} - \frac{1}{s^{m\gamma}}\{&s^{m\gamma}Ł_\gamma\{v_m(t,x)\} - s^{(m-1)\gamma}v_m(0,x) \\ &- s^{(m-2)\gamma}v_m(0,x)^{(\gamma)} - s^{\gamma(m-3)}v_m(0,x)^{(2\gamma)} - \cdots - v_m(0,x)^{((m-1)\gamma)} \\ &- p(x)v_x^{n\gamma}(s,x)\}. \end{aligned} \tag{20}$$

Hence, we obtain the following iterative scheme

$$\begin{aligned} v_{m+1}(t,x) = Ł_\gamma{}^{-1}\left\{\frac{1}{s^{m\gamma}}\{-s^{(m-1)\gamma}v_m(0,x) - s^{(m-2)\gamma}v_m(0,x)^{(\gamma)} - s^{\gamma(m-3)}v_m(0,x)^{(2\gamma)} - \cdots \right. \\ \left. - v_m(0,x)^{((m-1)\gamma)}\}\right\}. \end{aligned} \tag{21}$$

Consequently, we have the solution of (14) as

$$v(t,x) = \lim_{m\to\infty} Ł_\gamma{}^{-1}(Ł_\gamma\{v_m(s,x)\}). \tag{22}$$

4. Interpretation of the Iterative Scheme

This section is reserved for the interpretation of the iterative scheme (22). The iterative scheme is applied on some examples of wave equations for their solutions.

Example 1. Consider the following wave equation on Cantor sets,

$$\frac{\partial^{2\gamma}}{\partial t^{2\gamma}}v(t,x) - C\frac{\partial^{2\gamma}}{\partial x^{2\gamma}}v(t,x) = 0, \tag{23}$$

and the initial-boundary conditions read as

$$\frac{\partial^{\gamma}v(0,x)}{\partial t^{\gamma}} = 0, v(0,x) = E_\gamma(x^\gamma). \tag{24}$$

By the use of (20), we have

$$\begin{aligned} Ł_\gamma\{v_{m+1}(t)\} = Ł_\gamma\{v_m(t,x)\} - \frac{1}{s^{2\gamma}}\{s^{2\gamma}Ł_\gamma\{v_m(t,x)\} - s^\gamma v_m(0,x) \\ - v_m(0,x)^{(\gamma)} - C\frac{\partial^{2\gamma}}{\partial x^{2\gamma}}v_m(s,x)\}. \end{aligned} \tag{25}$$

Using the initial conditions (24), we get

$$v_0(s,x) = Ł_\gamma\{v_0(0,x)\} = \frac{E_\gamma(x^\gamma)}{s^\gamma}, \tag{26}$$

From (24)–(26), we proceed to

$$\begin{aligned} Ł_\gamma\{v_1(t,x)\} &= Ł_\gamma\{v_0(t)\} - \frac{1}{s^{2\gamma}}\{s^{2\gamma}Ł_\gamma\{v_0(t,x)\} - s^\gamma v_0(0,x) - v_0(0,x)^{(\gamma)} \\ &- C\frac{\partial^{2\gamma}}{\partial x^{2\gamma}}v_0(s,x)\} = \frac{1}{s^{2\gamma}}\left\{s^\gamma E_\gamma(x^\gamma) + C\frac{E_\gamma(x^\gamma)}{s^\gamma}\right\} = \frac{E_\gamma(x^\gamma)}{s^\gamma} + C\frac{E_\gamma(x^\gamma)}{s^{3\gamma}} \\ &= v_1(s,x). \end{aligned} \tag{27}$$

For the second iteration, we utilize (24)–(27), as under

$$\begin{aligned} Ł_\gamma\{v_2(t,x)\} &= Ł_\gamma\{v_1(t,x)\} - \frac{1}{s^{2\gamma}}\{s^{2\gamma}Ł_\gamma\{v_1(t,x)\} - s^\gamma v_1(0,x) \\ &- v_1(0,x)^{(\gamma)} - C\frac{\partial^{2\gamma}}{\partial x^{2\gamma}}v_1(s,x)\} \\ &= \frac{1}{s^{2\gamma}}\left\{s^\gamma E_\gamma(x^\gamma) + C\left(\frac{E_\gamma(x^\gamma)}{s^\gamma} + C\frac{E_\gamma(x^\gamma)}{s^{3\gamma}}\right)\right\} \\ &= \frac{E_\gamma(x^\gamma)}{s^\gamma} + C\frac{E_\gamma(x^\gamma)}{s^{3\gamma}} + C^2\frac{E_\gamma(x^\gamma)}{s^{5\gamma}} = v_2(s,x) \end{aligned} \tag{28}$$

For the third iteration, using (24)–(26) and (28), we have

$$\begin{aligned}
Ł_\gamma\{v_3(t,x)\} &= Ł_\gamma\{v_2(t,x)\} - \frac{1}{s^{2\gamma}}\{s^{2\gamma}Ł_\gamma\{v_2(t,x)\} - s^\gamma v_2(0,x) - v_2(0,x)^{(\gamma)} \\
&\quad -C\frac{\partial^{2\gamma}}{\partial x^{2\gamma}}v_2(s,x)\} \\
&= \frac{1}{s^{2\gamma}}\left\{s^\gamma E_\gamma(x^\gamma) + C\left(\frac{E_\gamma(x^\gamma)}{s^\gamma} + C\frac{E_\gamma(x^\gamma)}{s^{3\gamma}} + C^2\frac{E_\gamma(x^\gamma)}{s^{5\gamma}}\right)\right\} \\
&= \frac{E_\gamma(x^\gamma)}{s^\gamma} + C\frac{E_\gamma(x^\gamma)}{s^{3\gamma}} + C^2\frac{E_\gamma(x^\gamma)}{s^{5\gamma}} + C^3\frac{E_\gamma(x^\gamma)}{s^{7\gamma}} = v_2(s,x).
\end{aligned} \tag{29}$$

Continuing this process up to the *n*th approximation, we deduce

$$Ł_\gamma\{ v_n(t,x)\} = \frac{E_\gamma(x^\gamma)}{s^\gamma} + C\frac{E_\gamma(x^\gamma)}{s^{3\gamma}} + C^2\frac{E_\gamma(x^\gamma)}{s^{5\gamma}} + C^3\frac{E_\gamma(x^\gamma)}{s^{7\gamma}} + \cdots + C^n\frac{E_\gamma(x^\gamma)}{s^{(2n+1)\gamma}}. \tag{30}$$

Applying ${Ł_\gamma}^{-1}$ on (30), we obtain

$$\begin{aligned}
v_n(t,x) &= E_\gamma(x^\gamma) + C\frac{t^{2\gamma}E_\gamma(x^\gamma)}{\Gamma(2\gamma+1)} + C^2\frac{t^{4\gamma}E_\gamma(x^\gamma)}{\Gamma(4\gamma+1)} + C^3\frac{t^{6\gamma}E_\gamma(x^\gamma)}{\Gamma(6\gamma+1)} + \cdots + C^n\frac{t^{2n\gamma}E_\gamma(x^\gamma)}{\Gamma(2n\gamma+1)} \\
&= E_\gamma(x^\gamma)\lim_{n\to\infty}\sum_{j=0}^{n} c^j\frac{t^{2j\gamma}}{\Gamma(2j\gamma+1)} = E_\gamma(x^\gamma)\cos h_\gamma(ct^\gamma).
\end{aligned} \tag{31}$$

Example 2. Consider the following wave equation on Cantor sets,

$$\frac{\partial^{2\gamma}v(t,x)}{\partial t^{2\gamma}} - \frac{x^\gamma}{\Gamma(\gamma+1)}\frac{\partial^{2\gamma}v(t,x)}{\partial x^{2\gamma}} = 0, \tag{32}$$

and the initial-boundary conditions read as

$$\frac{\partial^\gamma v(0,x)}{\partial t^\gamma} = 0, v(0,x) = \frac{x^{2\gamma}}{\Gamma(2\gamma+1)}. \tag{33}$$

From (33), we form

$$v_0(s,x) = Ł_\gamma\{v_0(0,x)\} = \frac{x^{2\gamma}}{s^\gamma\Gamma(2\gamma+1)}. \tag{34}$$

From (20) and (32)–(34), we get

$$\begin{aligned}
Ł_\gamma\{v_1(t,x) &= Ł_\gamma\{v_0(t,x) - \frac{1}{s^{2\gamma}}\left\{s^{2\gamma}Ł_\gamma\{v_0(t,x)\} - s^\gamma v_0(0,x)\right. \\
&\quad -v_0(0,x)^{(\gamma)} - \frac{x^\gamma}{\Gamma(\gamma+1)}\frac{\partial^{2\gamma}v(s,x)}{\partial x^{2\gamma}}\} \\
&= \frac{x^{2\gamma}}{s^\gamma\Gamma(2\gamma+1)} - \frac{x^\gamma}{s^{3\gamma}\Gamma(\gamma+1)} = v_1(s,x)
\end{aligned} \tag{35}$$

For the second iteration, using (33)–(35), we proceed to

$$\begin{aligned}
Ł_\gamma\{v_2(t,x) &= Ł_\gamma\{v_1(t,x) - \frac{1}{s^{2\gamma}}\left\{s^{2\gamma}Ł_\gamma\{v_1(t,x)\} - s^\gamma v_1(0,x)\right. \\
&\quad -v_1(0,x)^{(\gamma)} - \frac{x^\gamma}{\Gamma(\gamma+1)}\frac{\partial^{2\gamma}v(s,x)}{\partial x^{2\gamma}}\} \\
&= \frac{x^{2\gamma}}{s^\gamma\Gamma(2\gamma+1)} - \frac{x^\gamma}{s^{3\gamma}\Gamma(\gamma+1)} = v_1(s,x).
\end{aligned} \tag{36}$$

For the third iteration, (34)–(36) are utilized and we get

$$\begin{aligned}
Ł_\gamma\{v_3(t,x) &= Ł_\gamma\{v_2(t,x) - \frac{1}{s^{2\gamma}}\{s^{2\gamma}Ł_\gamma\{v_2(t,x)\} - s^\gamma v_2(0,x) - v_2(0,x)^{(\gamma)} \\
&\quad - \frac{x^\gamma}{\Gamma(\gamma+1)}\frac{\partial^{2\gamma}v_2(s,x)}{\partial x^{2\gamma}} \\
&= \frac{x^{2\gamma}}{s^\gamma\Gamma(2\gamma+1)} - \frac{x^\gamma}{s^{2\gamma}\Gamma(\gamma+1)}\frac{\partial^{2\gamma}}{\partial x^{2\gamma}}\left(\frac{x^{2\gamma}}{s^\gamma\Gamma(2\gamma+1)} - \frac{x^\gamma}{s^{3\gamma}\Gamma(\gamma+1)}\right) \\
&= \frac{x^{2\gamma}}{s^\gamma\Gamma(2\gamma+1)} - \frac{x^\gamma}{s^{3\gamma}\Gamma(\gamma+1)} = v_3(s,x).
\end{aligned} \tag{37}$$

Continuing this process up to nth approximation, we get

$$Ł_\gamma\{v_{n+1}(t,x)\} = \frac{x^{2\gamma}}{s^\gamma\Gamma(2\gamma+1)} - \frac{x^\gamma}{s^{3\gamma}\Gamma(\gamma+1)}. \tag{38}$$

Applying ${Ł_\gamma}^{-1}$on (38), we deduce the following result as the solution of (32), (33), by the help of proposed method LFLVIM and our scheme in (22):

$$v(t,x) = \frac{x^{2\gamma}}{\Gamma(2\gamma+1)} - \frac{t^{2\gamma}x^\gamma}{\Gamma(2\gamma+1)\Gamma(\gamma+1)}. \tag{39}$$

5. Conclusions

This paper describes an iteration scheme based on the LFLVIM for the solutions of LFWEs which is a powerful technique and the efficiency of the iterative scheme is examined by two illustrative examples. The solutions obtained are graphically presented by the figures, Figure 1, Figure 2 respectively for $\gamma = \frac{\ln 2}{\ln 3}$. The prescribed technique is a better approach for the approximation of LFWEs in particular and LFEs in general.

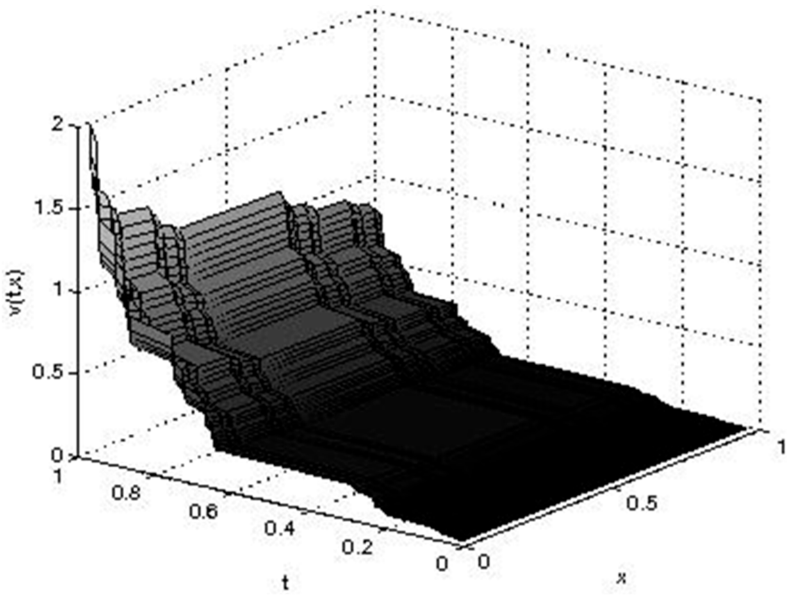

Figure 1. Exact solution of Equation (23) for $\gamma = \frac{\ln 2}{\ln 3}$.

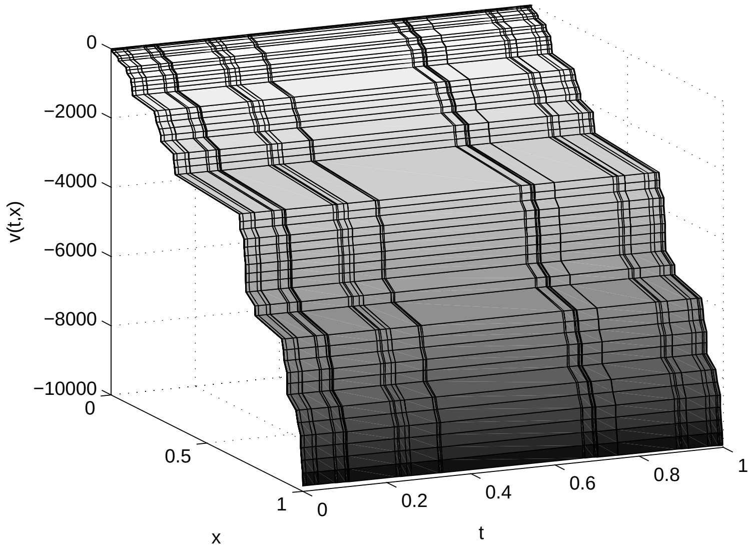

Figure 2. Exact solution of Equation (32) for $\gamma = \frac{\ln 2}{\ln 3}$.

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Author Contributions

All authors have contributed equally to the study and preparation of the article. All authors have read and approved the final version of the paper.

Conflicts of Interest

The authors declare no conflict of interest.

Nomenclature

$I^{(\gamma)}$	Local fractional integral
$\delta^{(\gamma)}$	Local fractional variation
E_γ	Mittage-Leffler function
$Ł_\gamma$	Yang-Laplace transform

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