



Figure S1. Distribution and mean of Φ^{Max} , #concepts and $\Sigma\phi^{Max}$ across all ECA for $N=5$ ECA of the $N+1$ evaluated states. The 88 rule equivalence classes are ordered by their $\langle\Phi^{Max}\rangle$ (see Fig. 4, main text). The labels indicate some of the example rules discussed in the main text. The value of all evaluated states for a particular rule are indicated by gray '+'. Gray lines connect the maximum and minimum values of different rules. $\langle\Phi^{Max}\rangle$, $\langle\#concepts\rangle$, and $\langle\Sigma\phi^{Max}\rangle$ averaged across $N+1$ system states with different numbers of '0' and '1' are shown in red. Note that for certain rules, all states with a particular number of '0's and '1's may be impossible (non-reachable) states. Since the IIT measures are not defined for impossible states, these states were not further taken into account. While some rules show no or only little variance in their IIT measures across states, others, particularly rules of Wolfram class II, such as rule 232, show larger variances. Nevertheless, the measured values tend to scatter evenly between the maximum and minimum values, which also grow steadily with the average. This indicates that the IIT measures averaged across $N+1$ states with varying numbers of '0's and '1' are reasonable approximations of the true mean values across all 2^N states. Moreover, since the cause-effect structure of any particular state depends on all counterfactuals (perturbing the system into all possible states), it is possible to infer general causal properties of a rule already by evaluating only a small set of states (see Fig. 3, main text, for examples). In sum, the relations between dynamical system properties and IIT measures reported in this study should hold even for the extrema and thus for any state sample out of the 2^N possible states.