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Article

Entropy Production of Stars

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Abstract: The entropy production (inside the volume bounded by a photosphere) of main-sequence stars, subgiants, giants, and supergiants is calculated based on *B–V* photometry data. A non-linear inverse relationship of thermodynamic fluxes and forces as well as an almost constant specific (per volume) entropy production of main-sequence stars (for 95% of stars, this quantity lies within 0.5 to 2.2 of the corresponding solar magnitude) is found. The obtained results are discussed from the perspective of known extreme principles related to entropy production.

Keywords: non-equilibrium thermodynamics; main-sequence stars; giants; supergiants; stability; metastability; minimum and maximum entropy production

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1. Introduction

Entropy production is a crucial characteristic of a nonequilibrium system that has attracted the interest of scientists for many decades. In part, this can be explained by the close relationship of this quantity with a number of variational principles of nonequilibrium thermodynamics. Minimum and maximum entropy production principles are among the most well-known [1–4]. Despite a seeming mutual negation, these variational principles do not contradict each other but are complementary and valid depending on the restricting conditions under which a system is considered [3,4]. The use of such variational principles proves to be important both for the foundation of nonequilibrium physics and for calculations: from heat-conduction problems to the solution of the Boltzmann equation [1–4]. It should be noted that the quantity of entropy production is convenient from the standpoint of theory and calculations but not always easy to determine in experiments: it is measured only indirectly and special conditions of a nonequilibrium system are preferable (for example, isolated or stationary). All these facts severely restrict the accuracy of determinations of this quantity and the range of systems for which it can be experimentally determined. In this regard, stars are extremely important. For such calculations, we believe they have the following advantages: (1) this nonequilibrium system remains in a nonequilibrium steady state for a long period of time because of the nuclear fusions in its core (see, for example, [5–9]); (2) entropy production can be easily calculated on the basis of experimental data of star luminosities and surface temperatures, and these data can be obtained from astronomical observations with a sufficiently high accuracy [8–12]; (3) there are reliable experimental data and developed theoretical models related to stellar evolution (see, for example, [11,12]); and (4) enormous number of stars and their photometric data represents an almost inexhaustible supply of statistical material which individual samples can be used to test various hypotheses regarding the behavior of entropy production, its extremeness included.

In spite of the above, as well as an obvious interest in the stars themselves as crucial objects in the Universe (constituting its predominant visible part and having a great impact on its evolution), experts in nonequilibrium thermodynamics have previously paid little attention to them. The authors are aware of only two series of theoretical papers. The first one [5–7] considers the issues of steadiness of the stars with different reaction types (proton–proton and carbon–nitrogen–oxygen) from the viewpoint of Prigogine’s criterion, whereas in the second one [9], the four stellar structure equations are reformulated as two alternate pairs of variational principles related to entropy production. In fact, these studies use entropy production and principles based thereon to reformulate, in terms of nonequilibrium thermodynamics, laws and restrictions that are already known for stars. However, these papers have an important corollary: the contributions of nuclear burning in the core, of conductive, radiative and convective transfer, and neutrinos to the total entropy production of a star are estimated. As the analysis shows [9], the main contribution to entropy production is connected with the heat-transfer processes occurring near a photosphere's surface (their input is inversely proportional to the effective temperature of a star, whereas, for instance, the contribution of nuclear burning turns out to be inversely proportional to the temperature in a star’s core (T_{core}), which is thousand times higher).

We have recently started a systematic experimental study of entropy production and its dependence on various stellar characteristics for main-sequence (MS) stars in open clusters [10]. The most interesting result of this investigation is the found relatively narrow range of specific (per volume) entropy productions (0.5 to 1.8 solar magnitudes for 90% of all the studied MS stars). These data indicate the peculiarity of specific entropy production for the considered nonequilibrium objects.

The objectives hereof are as follows: (1) additional check and substantiation, based on considerably larger statistics, of the peculiarity of specific entropy production previously found for the MS stars. This will be achieved by both considering the nearest star clusters (in order to study low-luminosity stars including red dwarfs) and calculating entropy productions of the stars belonging to globular clusters. (2) Calculation of entropy production for other types of stars (subgiants, giants, and super giants).

The obtained results can be used as a basis to analyze the validity of a number of variational principles related to entropy production with regard to stars. This issue is discussed in the concluding part hereof.

2. Photometry-Based Technique of Entropy-Production Calculation

This study uses photometric data of open and globular star clusters from the WEBDA and Exoplanets & Stellar Populations Group (ESPG) databases [13,14]. These databases contain the following data for every cluster star obtained from direct observations: apparent stellar magnitude V and color index $(B-V)$, as well as metallicity [Fe/H], color excess $E(B-V)$, and distance modulus $(m-M)_V$.

A detailed algorithm for calculating effective temperature T_{eff} and luminosity L with reference to the observed photometric data is given in the paper [10]. Based on the values of T_{eff} and L for each star, a simple formula for calculating the total entropy production Σ inside a star's photosphere can be obtained. Indeed, using traditional assumptions that a star's photosphere is, on the first approximation, an absolutely black body (see e.g., [8]), we can write the classical formula for an entropy flux leaving a star's photosphere surface in the form: $4L/(3T_{eff})$. Then let us use the second commonly-accepted assumption [5–9] that, for the time interval under study, a star is in a nonequilibrium *steady* state. Therefore, we can neglect entropy change in a star with time, and according to the balance equation for entropy [1–3], an entropy flux from a star's photosphere will be equal to the total entropy production Σ inside a star's photosphere. As a result:

$$\Sigma = 4L/(3T_{eff}) \tag{1}$$

It should be noted that only entropy production related to the conversion of thermonuclear and mechanical energy to heat energy in the course of irreversible processes occurring inside a star is analyzed herein. Gravitational entropy and its change (as well as other generalizations of Clausius's classical entropy that have recently appeared in a number of theoretical papers) are not considered in the present study. In addition to the total entropy production of a star, let us use the Stefan–Boltzmann formula to determine the specific (per unit volume) entropy production Σ_V as [10]:

$$\Sigma_V = \chi \cdot T_{eff}^5 / \sqrt{L} \tag{2}$$

where $\chi = 2 \cdot \sigma^{3/2} \cdot \pi^{1/2}$ and σ is the Stefan–Boltzmann constant. Whereas the total entropy production characterizes a star as a whole (independently of its mass), the specific entropy production is related to the rate of the heat released from the star's unit volume (this is an average local characteristic because, in reality, some local regions can have higher local entropy productions and some can have lower ones).

As is seen from the above, the quantities Σ and Σ_V can be obtained from direct photometric measurements. For this reason, only their values are calculated and analyzed below (mass-specific entropy production is not considered herein, because in order to determine a star's mass, some stellar model must be used and, hence, additional assumptions are required).

Based on the accuracy of the given photometric data and the calibration errors, the error of entropy production calculation can be found. The calculations show [10] that the entropy-production error is primarily connected with the accuracy of photometric data, so for the stars of the types considered herein with the temperature of up to 5000 K, the error for Σ and Σ_V does not exceed 19% and 15%, respectively; for the stars with the temperature of 5000 K to 10,000 K, the error does not exceed 17% and 32%, respectively; and for the stars with the temperature above 12,000 K, the error value is found to be higher than 23% and 40%, respectively.

To facilitate the calculation of the temperature, luminosity, and entropy production of stars (and statistic characteristics thereof) based on the above algorithm, we have developed and used a special software complex, Star Clusters (SC) [15].

3. Research Data

Stars are classified depending on their locations in the Hertzsprung–Russell (HR) diagram (Figure 1). The present study deals with main-sequence stars (including the so-called red dwarfs) as well as subgiants, giants, and supergiants. As is well known, stars of these classes have considerably different masses and types of nuclear fusions. We have used stars belonging to both open and globular clusters. It is very convenient and informative to analyze the samples of cluster stars because, according to the modern concepts, stars of a single cluster have approximately the same and relatively easily determinable ages and chemical compositions. Globular and open clusters have an important difference in age (9–14 billion years in the case of globular clusters, and 10 million to 8 billion years in the case of open ones) and in the number of stars (10^4–10^6 in globular clusters, and 10–10^3 in open ones).

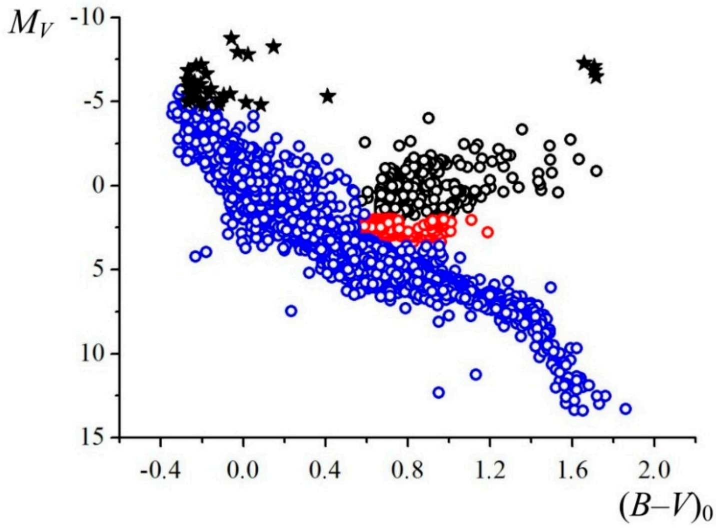

Figure 1. Hertzsprung–Russell diagram of 15 clusters considered herein. The diagram is plotted in the following coordinates: color index (*B–V*) *versus* absolute stellar magnitude M_V. Blue circles indicate main-sequence stars, red circles indicate subgiants, black circles indicate giants, and black stars indicate supergiants.

To select clusters from the WEBDA [13] and ESPG [14] databases for investigation, the following main criteria were applied: (1) availability of all the necessary photometric information (see Section 2); (2) availability of information on the probability that stars belong to a cluster; and (3) the HR diagrams of clusters must be well approximated by theoretical isochrones [16,17] using the corresponding values of age and metallicity.

As a result, the data of the stars belonging to 13 open clusters (NGC 884, NGC 869, IC 4725, NGC 2516, NGC 1039, NGC 3532, NGC 2099, NGC 2281, NGC 2506, NGC 2682, NGC 188, NGC 2632, Hyades) and two globular clusters (NGC 6121 and NGC 6656) were used. Stars were chosen for investigation if their membership probability was above 0.5 and the root mean square error in the B and V magnitude was 0.02 mag.

The total number of the stars was 10,017 main-sequence stars (including 64 red dwarfs), 1018 subgiants, 816 giants, and 36 supergiants. The age of the studied clusters expressed in the logarithmical form ($\log_{10}$ (t, year)) was: 7.10 (NGC 884), 7.28 (NGC 869), 7.83 (IC 4725), 8.08 (NGC 2516), 8.42 (NGC 1039), 8.45 (NGC 3532), 8.54 (NGC 2099), 8.70 (NGC 2281), 8.896 (Hyades), 8.9 (NGC 2632), 9.05 (NGC 2506), 9.41 (NGC 2682), 9.80 (NGC 188), 10.098 (NGC 6121) and 10.103 (NGC 6656).

4. Results and Analysis

For ease of analysis, the data below were normalized to solar magnitudes. Their values are as follows: $T_{eff\,\odot} = (57 \pm 2) \times 10^2$ K, $L_\odot = (3.8 \pm 0.6) \times 10^{26}$ W, $\Sigma_\odot = (9 \pm 1) \times 10^{22}$ W·K^{-1}, and $\Sigma_{V\,\odot} = (6 \pm 2) \times 10^{-5}$ W·K^{-1}·m^{-3}.

Calculation results of Σ and Σ_V for the stars are shown in Figures 2–7. Using these results, the following conclusions can be reached:

(1) In the case of the main-sequence stars, the distribution intervals of entropy productions show a considerable difference for the globular and open clusters (Figure 2). In the case of the globular clusters, this interval is especially narrow covering the domain of smaller values of Σ (and smaller luminosities, according to Equation (1)).

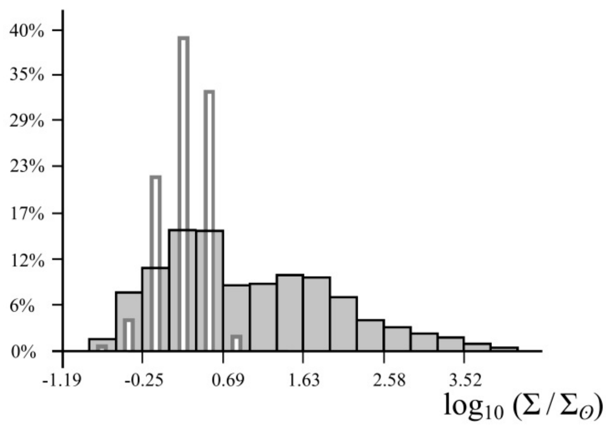

Figure 2. Bar chart showing the distribution of entropy productions Σ for the main-sequence stars. Wide bars correspond to the open-cluster data and the narrow ones correspond to the globular-cluster data. The number of the stars is 3074 and 6943 in the open and the globular clusters, respectively.

This is connected with the fact that globular clusters, as opposed to open ones, have older ages. Correspondingly, the stars with large masses (and, accordingly, large luminosities) belonging to globular clusters have already left the main sequence after evolving. The bar chart (Figure 2) is the only case when the globular and open clusters have different distributions. Other entropy-production distributions (for the subgiants, giants, and supergiants) of both the globular and the open clusters are the same; for this reason, the bar charts of Figure 3 summarize the samples of stars for the globular and open clusters. As is seen, when transiting from subgiants to supergiants, the entropy production grows, and every type of stars has its own interval which does not overlap the neighboring one.

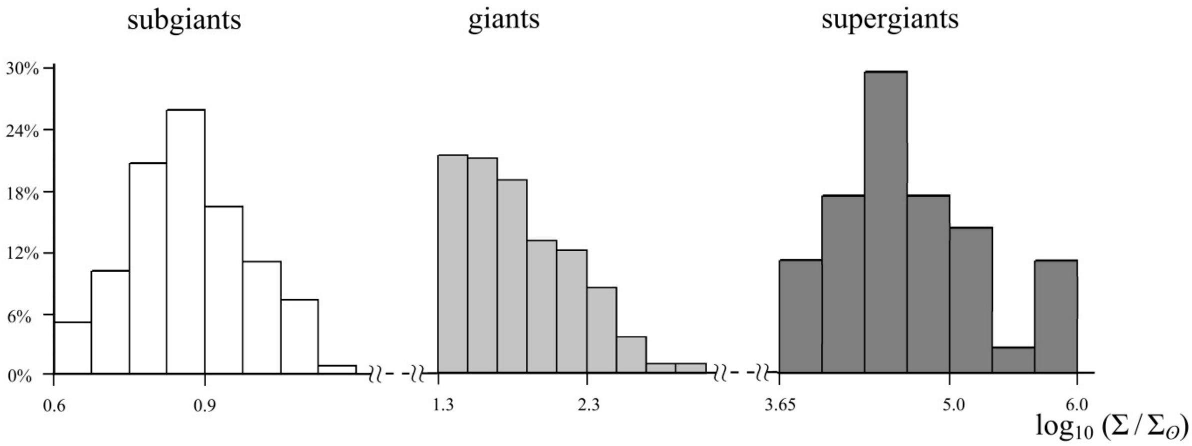

Figure 3. Bar chart showing the distribution of total entropy productions Σ for the subgiants, giants, and supergiants. The number of the stars: 1018 subgiants, 816 giants, and 36 supergiants.

(2) Bar charts of the distribution of specific entropy productions Σ_V of various types of stars are the same for the open and globular clusters and they are therefore combined in Figures 4 and 5. It is important that Σ_V, as compared with Σ, has a very narrow range of the possible values of specific entropy production for the stars belonging to the main-sequence (see Figures 2 and 4). Thus, 95% of Σ_V values fall within the range of 0.5 to 2.2 $\Sigma_{V\Theta}$, which is 200 times less than the corresponding interval for Σ.

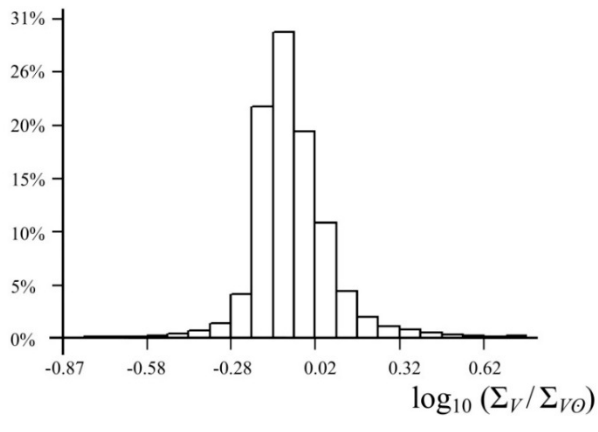

Figure 4. Bar chart showing the distribution of specific entropy productions Σ_V for the main-sequence stars. The total number of the MS stars is 10,017.

This conclusion repeats the one stated in our previous paper [10] but now is based on considerably larger statistics. As in the case of Σ, the distribution of Σ_V values for the subgiants, giants, and supergiants is within their typical and rather wide domains (Figure 5).

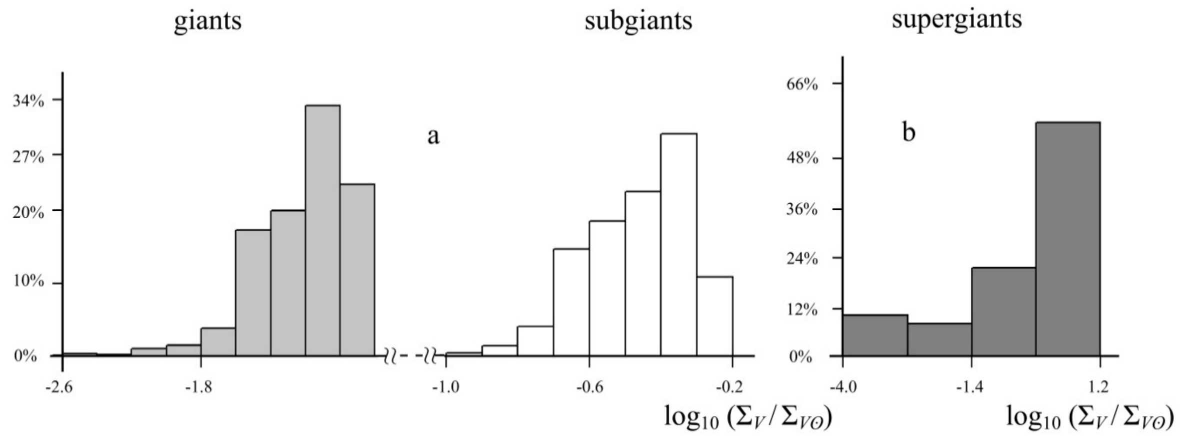

Figure 5. Bar chart showing the distribution of specific entropy productions Σ_V for the subgiants, giants, and supergiants. The number of the stars: 1018 subgiants, 816 giants, and 36 supergiants.

(3) The entropy production considerably depends both on the luminosity and the temperature (Figure 6). The dependence on the luminosity is very close to the power-law one. So, it has the form $\Sigma = 3\times10^{-1}L^{0.88}$ for the MS stars, $\Sigma = 4\times10^{-7}L^{1.10}$ for the subgiants, and $\Sigma = 4\times10^{-5}L^{1.03}$ for the giants (with R^2 above 0.99). It should be noted that in the case of the supergiants, one cannot make a statistically grounded conclusion on the type of dependence due to a small number of points.

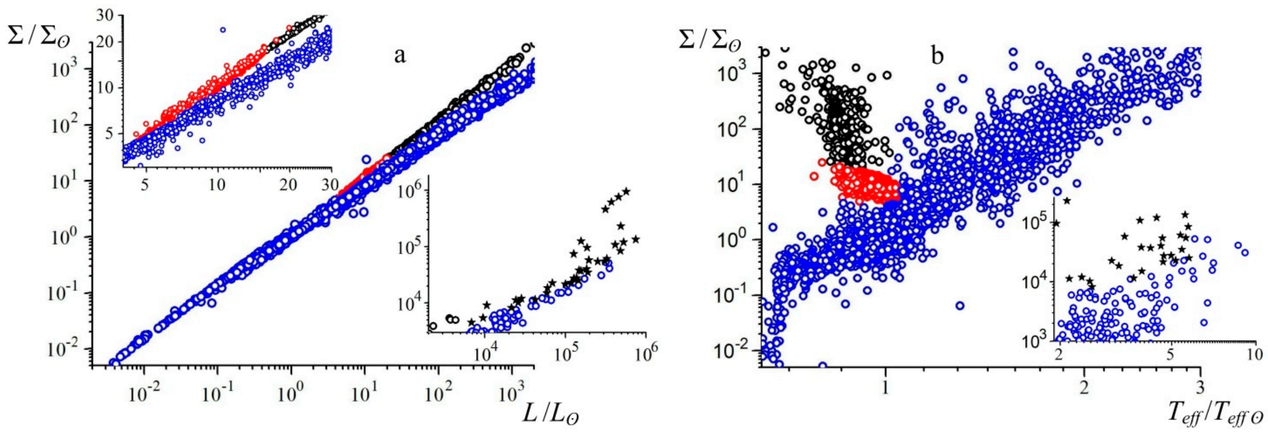

Figure 6. Dependence of the total entropy production Σ on the luminosity L (**a**) and the effective temperature T_{eff} (**b**). Blue circles indicate main-sequence stars, red circles indicate subgiants, black circles indicate giants, and black stars indicate supergiants.

Since a star's luminosity can be considered (if the terms of nonequilibrium thermodynamics are used) a thermodynamic flux J, then, taking into account the fact that $\Sigma = J \times X$ (where X is a thermodynamic force) [3,4], $X(J) = 3\times10^{-1}J^{-0.12}$ for the MS stars, $X(J) = 4\times10^{-7}J^{0.10}$ for the subgiants, and $X(J) = 4\times10^{-5}J^{0.03}$ for the giants. From the standpoint of nonequilibrium thermodynamics, the dependence $X(J) \propto J^{-0.12}$ found for the MS stars is rather unusual. Indeed, if the heat flux (luminosity) grows, then the thermodynamic force decreases. This fact is explained by the corollary of the well-known virial theorem for stationary self-gravitating systems according to which the increase of heat emission (electromagnetic radiation of a star) results in the increase of the star's heat energy (temperature) [11]. However, as is known [1–4], $X \propto (T_{core} - T_{eff}) / T_{core}T_{eff} \approx 1/T_{eff}$, and therefore the thermodynamic force decreases with the growth of temperature.

(4) For the main-sequence stars, the specific entropy production very little depends on the luminosity and the temperature (Figure 7). Indeed, if the luminosity changes from 10^{-3} to 10^{6} $L_{\odot}$ (and the temperature changes from 0.6 to 10 $T_{eff\odot}$), Σ_V remains almost the same and lies within the range of 0.5 to 2.2 $\Sigma_{V\odot}$.

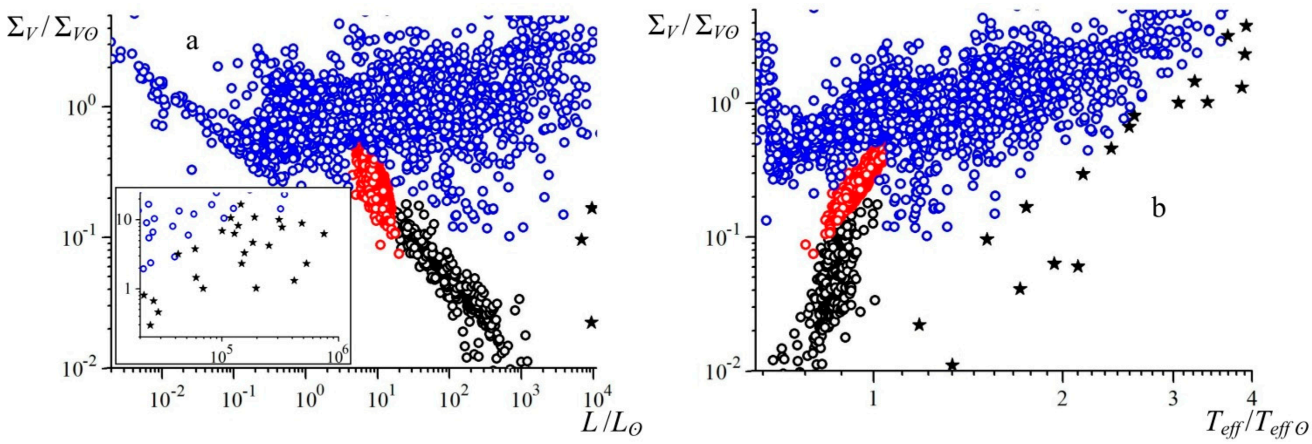

Figure 7. Dependence of the specific entropy production Σ_V on the luminosity (L) (**a**) and the effective temperature (T_{eff}) (**b**). Blue circles indicate main-sequence stars, red circles indicate subgiants, black circles indicate giants, and black stars indicate supergiants.

The power-law decrease of Σ_V in the domain of small values of L (from 10^{-3} to 10^{-1} $L_{\odot}$) and T_{eff} (from 0.6 to 0.7 $T_{eff\odot}$) found for the so-called red dwarfs is within the above range of Σ_V values for the MS stars and requires an additional study using larger statistics (the mentioned values rely on the data of the stars belonging to two open clusters only: NGC 2632 and Hyades).

The subgiants, giants, and supergiants have explicit power-law dependences of $\Sigma_V(L)$ and $\Sigma_V(T_{eff})$ (Figure 7). Σ_V decreases with the increase of L (Σ_V demonstrates the opposite behavior only for the supergiants), and Σ_V increases with the growth of T_{eff}.

The obtained result with respect to the invariability (within the accuracy of the given calculations) of the specific entropy production of the main-sequence stars is rather surprising. Indeed, if nuclear fusions were the only contribution to the entropy production, then such a behavior could be supposed for all main-sequence stars with different masses (luminosities) based on the similarity of initial and final products of hydrogen-to-helium conversion. However, due to a considerably higher temperature T_{core} (as compared with T_{eff}) in the zone of such a reaction, this contribution to the total entropy production of a star proves to be negligible, while Σ_V is mainly contributed by the heat-transfer processes occurring near a photosphere's boundary with the temperatures and sizes considerably differing for MS stars.

It follows from the obtained data that the stars with slightly different chemical compositions (belonging to different clusters) and considerably different masses (from 0.2 to 11.8 $M_{\odot}$) reach the main-sequence phase through a transformation (a very complex, according to the modern concepts, and still not fully understood process) with almost the same specific entropy productions close to the solar value. These stars remain in this MS phase for the most of their lives (in the case of the stars having a mass similar to the one of the Sun, about 10 billion years; and in the case of the stars having the mass of 10 $M_{\odot}$, about 50 million years) occupying a rather extensive region (Figure 1) in the HR diagram. It is natural to consider it as some steady state to which a system (some volume of galaxy from which a cluster was formed) transits and where stars co-exist for a relatively long time. Does this state correspond to the minimum or the maximum possible specific entropy production when stars evolve to the main sequence? Presently, it is difficult to answer this question using the experimental data (for a number of evolutionary tracks of cluster stars the specific entropy production may have some intermediate—neither maximum nor minimum—value). However, based on the obtained data, it is presently possible to point out a connection between the steady co-existence of stars in a formed and yet rather young cluster (where all the stars belong to the main sequence) and the equality of specific entropy productions of individual stars in this cluster.

It is interesting to note that the equality of specific entropy productions needed for the co-existence of nonequilibrium subsystems (existing in approximately the same external conditions) was previously supposed as a corollary of the maximum entropy production principle and confirmed during the study of nonequilibrium crystallization and a process of hydrodynamic instability [3,4,18]. The same papers state that the maximum specific entropy production ensures the stability of a subsystem's nonequilibrium state, whereas a smaller value of Σ_V for a number of subsystems leads to their metastability, instability, and hence, considerably smaller lifetimes as compared with a subsystem having the maximum entropy production. Let us consider the data (Figure 7) from this perspective. The clusters in the considered moment of time (we have analyzed the clusters aging 12.6 Myr to 12.67 Gyr) contain the stars belonging to a number of subsystems of which we have studied the main-sequence stars as well as the subgiants, giants, and supergiants. The three latter types of stars (subsystems) have, according to our calculations, smaller specific entropy productions and, therefore, have to be less stable, observable for a smaller time, and transform into other objects. Indeed, as is well known from astrophysics [11,12,16,17], these star types do have considerably smaller lifetimes as compared with less massive MS stars and transform relatively fast into white dwarfs, neutron stars, *etc.*

5. Conclusions

The paper calculates entropy production for the main types of stars in a number of open and globular clusters on the basis of photometric data. From the standpoint of nonequilibrium thermodynamics, we have obtained the following main results: (1) the nonlinear dependence of the entropy production on the thermodynamic flux (particularly for the MS stars, $\Sigma = 3\times10^{-1}L^{0.88}$) and (2) an almost constant specific (per volume) entropy production of the MS stars (thus, 95% of all the studied stars fall within the interval of 0.5 to 2.2 $\Sigma_{V\odot}$) which is higher than the specific entropy productions of the subgiants, giants, and supergiants. The latter result allows considering specific entropy production as a parameter characterizing a stable state (attractor) during the formation and evolution of stars.

The most interesting question arising from the present study is whether the specific entropy production is minimized or maximized (or maybe something different) during this evolution. The solution of this problem requires mathematical modeling based on the existing theories of stellar formation and evolution and represents an important task for future investigations.

Acknowledgments

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Author Contributions

Leonid M. Martyushev proposed the idea of research and the method of entropy-production calculation demonstrated in the paper. Sergey N. Zubarev calculated the data. Both authors analyzed the data, prepared the manuscript, and read and approved its final version.

Conflicts of Interests

The authors declare no conflict of interest.

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