

Editorial

Entropy Methods in Guided Self-Organisation

Mikhail Prokopenko^{1,2,3}, and Carlos Gershenson^{4,5}

¹ CSIRO Digital Productivity, PO Box 76, Epping, NSW 1710, Australia

² Department of Computing, Macquarie University, E6A Level 3, Eastern Rd, Macquarie Park, NSW 2113, Australia

³ School of Physics, University of Sydney, Physics Rd, Camperdown NSW 2050, Australia

⁴ Instituto de Investigaciones en Matemáticas Aplicadas y en Sistemas, Universidad Nacional Autónoma de México, A.P. 20-126, 01000 Mexico D.F., Mexico

⁵ Centro de Ciencias de la Complejidad, Universidad Nacional Autónoma de México, A.P. 20-126, 01000 Mexico D.F., Mexico

* Author to whom correspondence should be addressed; E-Mail: mikhail.prokopenko@csiro.au; Tel.: +61-2-9372-4716.

External Editor: Kevin H. Knuth

Received: 12 August 2014; in revised form: 22 September 2014 / Accepted: 29 September 2014 / Published: 9 October 2014

Abstract: Self-organisation occurs in natural phenomena when a spontaneous increase in order is produced by the interactions of elements of a complex system. Thermodynamically, this increase must be offset by production of entropy which, broadly speaking, can be understood as a decrease in order. Ideally, self-organisation can be used to guide the system towards a desired regime or state, while “exporting” the entropy to the system’s exterior. Thus, Guided Self-Organisation (GSO) attempts to harness the order-inducing potential of self-organisation for specific purposes. Not surprisingly, general methods developed to study entropy can also be applied to guided self-organisation. This special issue covers a broad diversity of GSO approaches which can be classified in three categories: information theory, intelligent agents, and collective behavior. The proposals make another step towards a unifying theory of GSO which promises to impact numerous research fields.

Keywords: entropy; guided self-organisation

1. Introduction

Examples of self-organising systems can be found practically everywhere: a heated fluid forms regular convection patterns of Bénard cells, neuronal ensembles self-organise into complex spike patterns, a swarm changes its shape in response to an approaching predator, ecosystems develop spatial structures in order to deal with diminishing resources, and so on. One may ask whether it is possible to guide the process of self-organisation towards some desirable patterns and outcomes? Over the last decade, it has become apparent that this question can be rigorously formalised across multiple domains, leading to the emergence of a new research field: guided self-organisation (GSO) [1–4].

Guided self-organisation attempts to reconcile two seemingly opposing forces: one is guiding a self-organising system into a better structured shape and/or functionality, while the other is diversifying the options in an entropic exploration within the available search space. At first glance, these two alternatives may even appear irreconcilable in principle, given an apparent contradiction between the concepts of guidance (implying control) and self-organisation (implying autonomy). However, the resolution of this paradox capitalises on the distinction between the concepts of “control” and “constraint”: rather than trying to precisely control a transition towards the desirable outcomes, one puts in place some constraints on the system dynamics to mediate behaviors and interactions [5].

Intuitively, the imposed constraints guide the dynamics by reducing the bandwidth of relevant channel(s), so that the system progresses to preserve its information by self-improving and self-structuring. Here, information is understood in Shannon’s sense [6], as a general reduction in uncertainty, making it applicable to a wide range of processes. A swarm reacts to a predator; bacteria search for sugar; a player selects a winning strategy in a dilemma game; an animal optimises an assortative mating choice: all of these decisions benefit, in the simplest case, from a reduction in uncertainty, and in a general case, from specific information dynamics [7–12]. This is especially relevant for systems that constantly generate uncertainty (random, chaotic or open). Therefore, it may be hypothesised that developing and placing constraints that shape more efficient information processing within a self-organising system would present an abstract way to guide it, without having to specify individual interactions and trajectories.

Some of the success on this path was underpinned by novelties in combining information-theoretic, graph-theoretic and computation-theoretic models, on the one side, with dynamical systems techniques and methods of statistical mechanics, on the other side. Fundamental connections between information-theoretic and thermodynamic (or statistical-mechanical) models reflect the rich common dynamics underlying guided self-organisation in open systems [13]. In general, interactions among the system’s components (neurons, particles, sensors, actuators, agents, *etc.*) induce statistical regularities, structuring information processing within the system. These regularities can be interpreted thermodynamically [14–16]. For example, a thermodynamic interpretation of transfer entropy shows that this quantity is proportional to the external entropy production by the system, attributed to a source of irreversibility [17], and is related to transient limits of computation [18].

The mathematical study of graphs has had several advances in the last fifteen years into what is now described as “network science” [19,20]. Networks are useful for modelling complex systems, where interactions are relevant [21], as nodes represent elements of a system, links represent interactions, while

meso- and macro- measures can be obtained, as well. This opens the possibility of studying phenomena at multiple scales under the same formalism [22], relating topological and information-theoretic properties [23].

Thus, it is not surprising that many studies of guided self-organisation turn their attention to the generic concepts of entropy and information, utilised in various thermodynamic, information-theoretic and graph-theoretic methods.

2. Special Issue

The 6th International Workshop on guided self-organisation was held in Barcelona on September 18, 2013, as a satellite Workshop at the 2013 European Conference on Complex Systems (ECCS'2013). Following the Workshop, a call for papers for a special issue on entropy methods in guided self-organisation was launched. Ten papers were selected after several rounds of comprehensive reviews.

The issue begins with three papers devoted to information-theoretical modelling of complexity and guided self-organisation.

The paper by Fuentes [24] proposes a new quantitative definition of emergence, using the measure of effective complexity introduced by Gell-Mann and Lloyd [25]. Attempts to capture the phenomenon of emergence information theoretically, while relating it to studies of complex systems, have been continuing vigorously over the last several decades [26–30]. Gell-Mann and Lloyd's original approach contrasted (1) the Shannon entropy, which measures the information required to describe the random aspects of the entity, given an ensemble in which it is embedded, with (2) the effective complexity, *i.e.*, the length of a compact description of the identified regularities of an entity, computed as the algorithmic information content, not of the entity, but of the ensemble in which it is embedded. These two measures can be combined within the total information. The proposal described by Fuentes suggests that a given property of an entity is emergent if, for a given set of the control parameters, “the information content of its regularities” increases abnormally. In other words, the property is emergent, at a particular value of some control parameter, if its effective complexity shows a jump discontinuity at this critical value. Since control parameters describe different ways to couple a system with its environment, as well as different characteristics of the system, the suggested definition of emergence may be argued to capture the relative, rather than absolute, nature of the phenomenon. This work contributes to the current vigorous debate on the elusive subject of emergence, employing information theory yet again in order to analyze the intricate relationship between emergence and complexity.

The study by Griffith *et al.* [31] is also well-grounded in information theory. In fact, it considers one of the most challenging information-theoretic topics in guided self-organisation and complex systems, in general—the formalisation of intersection information—in an attempt to quantify how much of “the same information” two or more random variables specify about a target random variable. This question is immediately related to measuring information modification or synergistic mutual information [32–35]. Several information-theoretic measures of synergy, developed over the last two decades, were reviewed and proposed in a previous work by Griffith and Koch [35], and the work presented in this issue makes another important step. Specifically, the introduced intersection information measure, based on the Gács–Körner common information (a stricter variant of Shannon mutual information) [36], is

the first to satisfy the important property of target monotonicity. This property is underlined by a partition of the set of all random variables into disjoint “information-equivalence” classes, produced by a well-defined ordering, and requires that intersection information about an informationally richer target variable is at least as high as the intersection information about an informationally poorer target variable. This exemplifies, and extends, an axiomatic framework for information processing which has been developed over the last few years, promising to provide a solid foundation for information-theoretic modelling of self-organising processes.

Ivancevic *et al.* [37] propose an action-amplitude model for controlled entropic self-organisation (CESO). The authors provide physical, global functional, local geometric and computational views on CESO. In organisational decision-making, it is assumed that optimal behavior is given by minimising perceptual error. The perceptual error of an agent is defined as the difference between the intent and the consequences of the action. Relating this concept with information theory and Prigogine’s extended second law of thermodynamics, three phases are described: Intent, where entropy increases, action, where entropy (and information) is conserved, and control, where entropy is reduced. Proposed formalisms for modeling and simulation are derived. Applications of this work can be made in collective decision-making using formal frameworks.

The next three papers are devoted to applying information-theoretic models and tools to the generation of complex self-organising behaviors in intelligent agents.

As pointed out by Salge *et al.* [38], “one aspect of intelligence is the ability to restructure your own environment, so that the world you live in becomes more beneficial to you”. This paper extends the increasingly maturing empowerment formalism to a methodology providing task-independent, intrinsic motivations driving an agent to manipulate and restructure a deterministic and discrete external world. Empowerment was first introduced in [39] and studied over the subsequent years as a mechanism generating self-organising behaviours in agents [40–43]. Formally, it is the channel capacity of the exterior part of an agent’s action-perception loop, measured via the maximum quantity of Shannon information that an agent could potentially inject into the environment and recover via its sensors [13]. The paper exemplifies how different agent embodiments and changing environmental conditions result in the agents producing discernibly different worlds, even though the agents are controlled by the same internal motivation. The study opens several important avenues for future research, reaching beyond deterministic and discrete worlds and including multi-agent cooperation in a joint re-structuring of their common environment, as well as self-modifying agent behaviors and morphological computation in general.

The paper by Ristic *et al.* [44] deals with an unknown structured environment and presents a framework for the autonomous search for a diffusive source. An environment is modeled as an unknown discretised map with randomly placed and shaped obstacles. The solution is formulated in the sequential Bayesian framework and implemented as a Rao–Blackwellised particle filter [45], augmented with an entropy-reduction motion control. Some robots have been built using chemotaxis, *i.e.*, following a chemical gradient. However, chemotaxis is not effective when turbulent flows are present. As an alternative, infotaxis has been proposed with successful applications [46]. The authors combine this approach with navigation in an unknown environment with encouraging results.

A novel internal control structure for a robot, considered as a general dynamic embodied system, is also investigated in the paper by Nurzaman *et al.* [47]. This study presents a GSO approach based on a coupling between the mechanical dynamics of the robot and its internal control structure, known as the attractor selection mechanism [48]. The resulting architecture attains a balance between deterministic and stochastic dynamics. The crucial assumption is that “the deterministic dynamics can be represented by a number of attractors, and the tendency to gracefully change the behaviors depends on internally generated stochastic perturbation and sensory input”. The approach advocated in the paper not only illustrates the guided self-organisation of a specific embodied system, but also highlights a methodological perspective on the research field: guidance and self-organisation within a dynamic system may be combined through a proper coupling of the behavioral primitives with (selectable) attractors, setting suitable levels of noise and appropriately expressing current goals via the sensory feedback function. This research perspective is well aligned with the view on GSO developed at the intersection of the theory of dynamical systems and machine learning [49–56]: in order to guide a dynamical system, one may restrict its flow to a certain region in phase space, allowing for an otherwise unrestricted development within this bounded area of phase space [57].

The next four papers apply information-theoretic, game-theoretic and graph-theoretic tools to studies of multi-agent collective behavior.

In the study carried out by Guckelsberger and Polani [58], the concept of empowerment plays a central role once more, in deriving skills of multiple agents that compete for a scarce resource. The work is motivated by the insight that “self-organization and survival are inextricably bound to an agent’s ability to control and anticipate its environment”. It has been noted in the past that an information exchange modulates the empowerment mechanism in a way that triggers complex collective behaviour [41]. The results reported in this issue show that initial assumptions about an agent’s peers and anticipation of their behavior have a strong effect on the agent’s individual behaviour, producing, via maximisation of empowerment, diverse survival strategies. Furthermore, different degrees of scarcity are shown to affect survival strategies. Importantly, multiple homogeneous agents driven by empowerment maximisation, are capable of surviving in a flat hierarchy without direct communication, but with some level of anticipation. This again highlights the role of the empowerment maximisation as a universal drive for GSO in collective agent systems. Another salient outcome of the study is the observation that agents “develop the most efficient behaviour locally if they assume their peers to act in a way that would be indeed the most efficient at a global level”. This aspect is immediately related to game-theoretic modeling of rational agent behavior, studied in the next paper.

Harré and Bossomaier [59] consider, in a game-theoretic setting centered on the quantal response equilibrium, the issue of how changes in the players’ underlying incentives can move the outcome from an optimal economy to a sub-optimal economy. This problem is complicated by the ensuing dynamics that may make it impossible for the players to collectively navigate a way to a better strategy without passing through a socially undesirable “tipping point” (a discontinuous transition), such as a financial market crash, economic depression or a catastrophic climate change. An important result produced by the study is the identification of “strategic islands” created or destroyed in the strategy space in response to different perturbations to the underlying incentives. These islands are shown to be isolated by disruptive transitions between strategies, making more optimal strategic regions not smoothly attainable from the

current strategy. However, under some conditions, there are possibilities of generating alternative smooth paths to globally better outcomes, guided by both individually incentivised choices and macro-economic adjustments. As argued by the authors, game theory provides a simplified representation of what is often a very complex strategic space, providing key insights into the consequences of both actions and inactions in very dynamic environments.

Gogolev and Marcenaro [60] study the problem of consensus [61], considering faulty nodes with random and persistent failures. Computer simulations are used to show that, counter-intuitively, different randomisations can actually increase the robustness of consensus, as the effect of faulty nodes can be reduced with noise, message loss or topology. They also show that random failures inhibit consensus less than persistent failures, while in some cases, random failures can even promote consensus. This can be seen as an example of the “order from noise” principle [62]. Systems that are too rigid can benefit from variation [63], so that self-organisation can be guided not only by restricting entropy, but also by promoting it when necessary.

What is the optimal amount of entropy of a system? This question is explored by Zubillaga *et al.* [64] in the context of self-organising traffic lights [65,66]. Measures of emergence, self-organisation, complexity and autopoiesis based on information theory [30] are applied to different traffic scenarios and controllers. The variations in the measures reflect different dynamical phases and show that each regime requires different entropy values. Self-organising traffic lights reach an optimal or close to optimal performance, because they are able to increase their complexity as the complexity of the traffic flows increases for different densities.

3. Conclusion

The contributions to this special issue show that research on guided self-organisation is advancing along several theoretical and practical dimensions. The proposed formal theories and measures promise to bring us closer to a unifying theory of GSO with important implications for numerous research fields.

Acknowledgments

We would like to thank all of the reviewers for this special issue for their timely responses and useful comments. We are also grateful for the support provided by the organizers of the 2013 European Conference on Complex Systems (ECCS'2013) for facilitating the organisation of the GSO-2013 Workshop, as well as to our co-chair, Daniel Polani. We also appreciate the effort of all of the authors who submitted to the special issue and/or the GSO-2013.

References

1. Prokopenko, M. Guided self-organization. *HFSP J.* **2009**, *3*, 287–289.
2. Ay, N.; Der, R.; Prokopenko, M. Guided self-organization: Perception-action loops of embodied systems. *Theory Biosci.* **2011**, *131*, 1–3.
3. Polani, D.; Prokopenko, M.; Jaeger, L.S. Information and self-organization of behavior. *Adv. Complex Syst.* **2013**, *16*, 1303001.

4. Prokopenko, M., Ed. *Guided Self-Organization: Inception*; Emergence, Complexity and Computation Series, Volume 9; Springer: Berlin/Heidelberg, Germany, 2014.
5. Gershenson, C. *Design and Control of Self-organizing Systems*; CopIt Arxiv: Mexico, 2007. Available online: <http://tinyurl.com/DCSOS2007> (accessed on 9 October 2014).
6. Shannon, C.E. A mathematical theory of communication. *Bell Syst. Tech. J.* **1948**, *27*, 379–423, 623–656.
7. Linsker, R. Self-organization in a perceptual network. *Computer* **1988**, *21*, 105–117.
8. Lungarella, M.; Sporns, O. Mapping Information Flow in Sensorimotor Networks. *PLoS Comput. Biol.* **2006**, *2*, e144.
9. Polani, D.; Sporns, O.; Lungarella, M. How Information and Embodiment Shape Intelligent Information Processing. In *50 Years of Artificial Intelligence: Essays Dedicated to the 50th Anniversary of Artificial Intelligence*, Proceedings of the 50th Anniversary Summit of Artificial Intelligence, Monte Verità, Ascona, Switzerland, 9–14 July 2006; Lecture Notes in Computer Science, Volume 4850; Lungarella, M., Iida, F., Bongard, J., Pfeifer, R., Eds.; Springer: Berlin/Heidelberg, Germany, 2007; pp. 99–111.
10. Polani, D. Information: Currency of life? *HFSP J.* **2009**, *3*, 307–316.
11. Lizier, J.T.; Prokopenko, M.; Zomaya, A.Y. Coherent information structure in complex computation. *Theory Biosci.* **2012**, *131*, 193–203.
12. Lizier, J.T.; Prokopenko, M.; Zomaya, A.Y. A Framework for the Local Information Dynamics of Distributed Computation in Complex Systems. In *Guided Self-Organization: Inception*; Prokopenko, M., Ed.; Emergence, Complexity and Computation Series, Volume 9; Springer: Berlin/Heidelberg, Germany, 2014; pp. 115–158.
13. Prokopenko, M.; Polani, D.; Ay, N. On the Cross-Disciplinary Nature of Guided Self-Organisation. In *Guided Self-Organization: Inception*; Emergence, Complexity and Computation Series, Volume 9; Prokopenko, M., Ed.; Springer: Berlin/Heidelberg, Germany, 2014; pp. 3–15.
14. Crooks, G. Measuring thermodynamic length. *Phys. Rev. Lett.* **2007**, *99*, 100602.
15. Prokopenko, M.; Lizier, J.T.; Obst, O.; Wang, X.R. Relating Fisher information to order parameters. *Phys. Rev. E* **2011**, *84*, 041116.
16. Barnett, L.; Lizier, J.T.; Harré, M.; Seth, A.K.; Bossomaier, T. Information Flow in a Kinetic Ising Model Peaks in the Disordered Phase. *Phys. Rev. Lett.* **2013**, *111*, 177203.
17. Prokopenko, M.; Lizier, J.T.; Price, D.C. On Thermodynamic Interpretation of Transfer Entropy. *Entropy* **2013**, *15*, 524–543.
18. Prokopenko, M.; Lizier, J.T. Transfer Entropy and Transient Limits of Computation. *Sci. Rep.* **2014**, *4*, 5394.
19. Newman, M.E.J. The structure and function of complex networks. *SIAM Rev.* **2003**, *45*, 167–256.
20. Newman, M., Barabási, A.L., Watts, D.J., Eds. *The Structure and Dynamics of Networks* (Princeton Studies in Complexity); Princeton University Press: Princeton, NJ, USA, 2006.
21. Gershenson, C. The Implications of Interactions for Science and Philosophy. *Found. Sci.* **2013**, *18*, 781–790.
22. Gershenson, C. Computing Networks: A General Framework to Contrast Neural and Swarm Cognitions. *Paladyn. J. Behav. Rob.* **2010**, *1*, 147–153.

23. Piraveenan, M.; Prokopenko, M.; Zomaya, A.Y. Assortativeness and information in scale-free networks. *Eur. Phys. J. B* **2009**, *67*, 291–300.
24. Fuentes, M.A. Complexity and Emergent Properties. *Entropy* **2014**, *16*, 4489–4496.
25. Gell-Mann, M.; Lloyd, S. Information Measures, Effective Complexity, and Total Information. *Complexity* **1996**, *2*, 44–52.
26. Crutchfield, J. The Calculi of Emergence: Computation, Dynamics, and Induction. *Physica D* **1994**, *75*, 11–54.
27. Shalizi, C. Causal Architecture, Complexity and Self-Organization in Time Series and Cellular Automata. Ph.D. Thesis, University of Michigan, MI, USA, 2001.
28. Shalizi, C.R.; Crutchfield, J.P. Computational Mechanics: Pattern and Prediction, Structure and Simplicity. *J. Stat. Phys.* **2001**, *104*, 819–881.
29. Prokopenko, M.; Boschiatti, F.; Ryan, A.J. An Information-Theoretic Primer on Complexity, Self-Organization, and Emergence. *Complexity* **2009**, *15*, 11–28.
30. Fernández, N.; Maldonado, C.; Gershenson, C. Information Measures of Complexity, Emergence, Self-organization, Homeostasis, and Autopoiesis. In *Guided Self-Organization: Inception; Emergence, Complexity and Computation Series, Volume 9*; Prokopenko, M., Ed.; Springer: Berlin/Heidelberg, Germany, 2014; pp. 19–51.
31. Griffith, V.; Chong, E.K.P.; James, R.G.; Ellison, C.J.; Crutchfield, J.P. Intersection Information Based on Common Randomness. *Entropy* **2014**, *16*, 1985–2000.
32. Williams, P.L.; Beer, R.D. Nonnegative Decomposition of Multivariate Information **2010**. arXiv:abs/1004.2515.
33. Harder, M.; Salge, C.; Polani, D. Bivariate measure of redundant information. *Phys. Rev. E* **2013**, *87*, 012130.
34. Lizier, J.T.; Flecker, B.; Williams, P.L. Towards a Synergy-based Approach to Measuring Information Modification. In IEEE Symposium Series on Computational Intelligence (SSCI 2013) — IEEE Symposium on Artificial Life, Singapore, April 2013.
35. Griffith, V.; Koch, C. Quantifying synergistic mutual information. In *Guided Self-Organization: Inception*; Prokopenko, M., Ed.; Emergence, Complexity and Computation Series, Volume 9; Springer: Berlin/Heidelberg, 2014; pp. 159–190.
36. Wolf, S.; Wulschleger, J. Zero-error information and applications in cryptography. In Proceedings of Information Theory Workshop, San Antonio, TX, USA, 24–29 October 2004; pp. 1–6.
37. Ivancevic, V.; Reid, D.; Scholz, J. Action-Amplitude Approach to Controlled Entropic Self-Organization. *Entropy* **2014**, *16*, 2699–2712.
38. Salge, C.; Glackin, C.; Polani, D. Changing the Environment Based on Empowerment as Intrinsic Motivation. *Entropy* **2014**, *16*, 2789–2819.
39. Klyubin, A.S.; Polani, D.; Nehaniv, C.L. All Else Being Equal Be Empowered. In *Advances in Artificial Life*, Proceedings of 8th European Conference on Artificial Life (ECAL 2005), Canterbury, UK, 5–9 September 2005; Lecture Notes in Artificial Intelligence, Volume 3630; Springer: Berlin/Heidelberg, Germany, 2005; pp. 744–753.

40. Klyubin, A.S.; Polani, D.; Nehaniv, C.L. Empowerment: A universal agent-centric measure of control. In Proceedings of the 2005 IEEE Congress on Evolutionary Computation, Edinburgh, UK, 2–4 September 2005; Volume 1, pp. 128–135.
41. Capdepuy, P.; Polani, D.; Nehaniv, C. Maximization of Potential Information Flow as a Universal Utility for Collective Behaviour. In Proceedings of 2007 IEEE Symposium on Artificial Life, Hawaii, HI, USA, 1–5 April 2007; pp. 207–213.
42. Jung, T.; Polani, D.; Stone, P. Empowerment for Continuous Agent-Environment Systems. *Adapt. Behav.* **2011**, *19*, 16–39.
43. Salge, C.; Glackin, C.; Polani, D. Approximation of Empowerment in the Continuous Domain. *Adv. Complex Syst.* **2012**, *16*, 1250079.
44. Ristic, B.; Skvortsov, A.; Walker, A. Autonomous Search for a Diffusive Source in an Unknown Structured Environment. *Entropy* **2014**, *16*, 789–813.
45. Doucet, A.; Freitas, N.D.; Murphy, K.P.; Russell, S.J. Rao-Blackwellised Particle Filtering for Dynamic Bayesian Networks. In Proceedings of the 16th Conference on Uncertainty in Artificial Intelligence, Stanford University, Stanford, CA, USA, 30 June–3 July 2000; Morgan Kaufmann Publishers Inc.: San Francisco, CA, USA, 2000; pp. 176–183.
46. Lo Iacono, G. A Comparison of Different Searching Strategies to Locate Sources of Odor in Turbulent Flows. *Adapt. Behav.* **2010**, *18*, 155–170.
47. Nurzaman, S.G.; Yu, X.; Kim, Y.; Iida, F. Guided Self-Organization in a Dynamic Embodied System Based on Attractor Selection Mechanism. *Entropy* **2014**, *16*, 2592–2610.
48. Adaptive Response of a Gene Network to Environmental Changes by Fitness-Induced Attractor Selection. *PLoS ONE* **2006**, *1*, e49.
49. Der, R.; Steinmetz, U.; Pasemann, F. Homeokinesis—A New Principle to Back Up Evolution with Learning. In *Computational Intelligence for Modelling, Control, and Automation*; IOS Press: Amsterdam, The Netherlands, 1999; *Concur. Syst. Eng. Series*, Volume 55, pp. 43–47.
50. Gros, C. *Complex and Adaptive Dynamical Systems: A Primer*; Springer: Berlin/Heidelberg, Germany, 2008.
51. Der, R.; Martius, G. *The Playful Machine—Theoretical Foundation and Practical Realization of Self-Organizing Robots*; Springer: Berlin/Heidelberg, Germany, 2012.
52. Ay, N.; Bernigau, H.; Der, R.; Prokopenko, M. Information-driven self-organization: the dynamical system approach to autonomous robot behavior. *Theory Biosci.* **2012**, *131*, 161–179.
53. Beer, R., Dynamical systems and embedded cognition. In *The Cambridge Handbook of Artificial Intelligence*; Frankish, K., Ramsey, W., Eds.; Cambridge University Press: Cambridge, UK, 2013; Chapter 12.
54. Beer, R., Dynamical analysis of evolved agents: A primer. In *The Horizons for Evolutionary Robotics*; Vargas, P., Di Paolo, E., Harvey, I., Husbands, P., Eds.; MIT Press: Cambridge, MA, USA, 2014.
55. Der, R. On the Role of Embodiment for Self-Organizing Robots: Behavior As Broken Symmetry. In *Guided Self-Organization: Inception; Emergence, Complexity and Computation Series*, Volume 9; Prokopenko, M., Ed.; Springer: Berlin/Heidelberg, Germany, 2014; pp. 193–221.

56. Martius, G.; Der, R.; Herrmann, J.M. Robot learning by guided self-organization. In *Guided Self-Organization: Inception*; Prokopenko, M., Ed.; Emergence, Complexity and Computation Series, Volume 9; Springer: Berlin/Heidelberg, Germany, 2014; pp. 223–260.
57. Gros, C. Generating functionals for guided self-organization. In *Guided Self-Organization: Inception*; Prokopenko, M., Ed.; Emergence, Complexity and Computation Series, Volume 9; Springer: Berlin/Heidelberg, Germany, 2014; pp. 53–66.
58. Guckelsberger, C.; Polani, D. Effects of Anticipation in Individually Motivated Behaviour on Survival and Control in a Multi-Agent Scenario with Resource Constraints. *Entropy* **2014**, *16*, 3357–3378.
59. Harré, M.; Bossomaier, T. Strategic islands in economic games: Isolating economies from better outcomes. *Entropy* **2014**, *16*, 5102–5121.
60. Gogolev, A.; Marcenaro, L. Randomized Binary Consensus with Faulty Agents. *Entropy* **2014**, *16*, 2820–2838.
61. Barborak, M.; Dahbura, A.; Malek, M. The consensus problem in fault-tolerant computing. *ACM Computing Surveys (CSUR)* **1993**, *25*, 171–220.
62. von Foerster, H. On self-organizing systems and their environments. In *Self-organizing systems*; Yovits, M., Cameron, S., Eds.; Pergamon Press: Oxford, UK, 1960.
63. Gershenson, C. Guiding the self-organization of random Boolean networks. *Theory Biosci.* **2012**, *131*, 181–191.
64. Zubillaga, D.; Cruz, G.; Aguilar, L.D.; Zapotécatl, J.; Fernández, N.; Aguilar, J.; Rosenblueth, D.A.; Gershenson, C. Measuring the Complexity of Self-Organizing Traffic Lights. *Entropy* **2014**, *16*, 2384–2407.
65. Gershenson, C. Self-Organizing Traffic Lights. *Complex Syst.* **2005**, *16*, 29–53.
66. Gershenson, C.; Rosenblueth, D.A. Self-organizing traffic lights at multiple-street intersections. *Complexity* **2012**, *17*, 23–39.