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*Article*

# The Dark Energy Properties of the Dirac–Born–Infeld Action

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**Abstract:** Introducing a new potential, we deduce a general Lagrangian for Dirac–Born–Infeld (DBI) inflation, in which the determinant of the induced metric naturally includes the kinetic energy and the potential energy. In particular, the potential energy and kinetic energy can convert into each other at any same order, which is in agreement with the limit of classical physics. We also present a general sound speed in the evolutions of the universe, and the exact expressions of energy-momentum tensor, pressure and density. Furthermore, from the results we obtain the new equation of states. The analytic form of the action that is consistent with data turns out to be surprisingly simple and easy to categorize. Finally, we examine properties of the dark energy and introduce a novel mechanism for realizing either quintessence or phantom dark energy dominated phases within a string theoretical context.

**Keywords:** DBI Lagrangian; dark energy; sound speed

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## 1. Introduction

High energy physics theories for dark energy leading the accelerated expansion of the Universe face issues of naturalness. One problem is that the current dark energy density measured so different from the initial conditions of the high energy and early universe. The other problem is that the current low energy form of the potential energy is related to the initial high-energy form that should receive quantum corrections.

It has been known for a long time now that adiabatic $\Lambda$CDM (constant Lambda, non-evolving dark energy) appears to be not inconsistent with the observational data, see, e.g., [1–4] for early discussions. In particular, the cosmological constant suffers both problems. In order to solve the amplitude problem, one prefers an attractor solution, where the present behavior is largely insensitive to the exact initial conditions. The attractor solution, first found by Peebles [5,6], does eventually drive the quintessence model to the $\Lambda$CDM model with $\omega = 1$. To improve the problem one would like a symmetry or geometric quantity that protects the potential, or predict it from a fundamental theory such as string theory. Even the attractor solutions have difficulty in naturally reaching a dark energy equation of state $\omega = -1$ [7] as indicated by cosmological observations.

It found not only the attractor solutions accessible to quintessence, but also three new classes that could achieve or approach $\omega = -1$, the cosmological constant state. String theory can impose a specific non-trivial kinetic behavior through the Dirac–Born–Infeld (DBI) action that arises naturally in consideration of $D_3$-brane motion within a warped compactification. The field properties are related to the geometric position of a three-dimensional brane within higher dimensions, and the brane tension and potential functions are (in principle) given by string theory, through the AdS/CFT correspondence.

Field theories of the DBI type have attracted much attention in recent years, which is due to their critical role in inflationary models based on string theory [8–12]. These scenarios indicate that the inflation relates with the D-brane moving on a 6-dimensional compact submanifold of spacetime, which means that the inflation is interpreted as an open string mode. This interpretation of the inflation implies that the effective field theory is rather distinct and well motivated by string computations.

Since the dynamics of a D-brane is described by a DBI action in string theory and characterized by a nonstandard kinetic term, inflation could turn out with steep potentials, in contrast with usual slow-roll inflation. Many models of inflation are based on the motion of D-branes in a higher-dimensional spacetime with DBI actio, which is the so-called DBI inflation [13–16]. The DBI inflation concludes a more general class, e.g., k-inflation models [17–19].

Inflation provides a natural mechanism for creating the homogeneity and flatness of our observable universe [20–23]. It presents an elegant way of generating the perturbations that seeds the structure formation of the galaxies. In order for the inflation to last long enough and then successfully exit reheating the universe, the inflation has to be held up on a potential for a long enough time. Such a mechanism is achieved most commonly by a potential that is very flat on the top. The required flatness is summarized by the slow-roll conditions. One critical problem in the inflation is how to realize such flat potential in a fundamental theory naturally. Many years of investigations in supergravity and superstring theory indicates that although such flat potentials may arise in many occasions, they are not generic.

DBI inflation shows that string theory motivates the mechanism for the inflation, where the inflation appears to be well studied. Furthermore, DBI inflation has the additional nice feature of a natural ending when the branes collide, where the collision itself is useful for reheating and the possible production of cosmic strings. String theory dictates both the dynamics of the inflation and its potential, so that one can make precise cosmological predictions from a given set of background parameters [24]. Furthermore, given the potential energy from current and future measurements of the power spectrum including tensor perturbation, the string theory parameters can eventually be over-constrained by the data. The calculability and the limited numbers of the parameters make brane inflation an interesting arena to explore the possibilities for cosmology in string theory. Therefore, it is possible to set up a cosmic evolution model, which satisfies the data and is coincident with the fundamental theory. There has been a great deal of work done in a variety of inflationary scenarios and in understanding the inflationary fields [25–31].

However, with so many successful and well-motivated inflationary scenarios in string theory, there are still some problems. In the light of Dvali and Tyes model [32], branes and antibranes or branes without the same supersymmetry are both present in different parts of the compact space M. The candidate inflation is the distance between the branes and antibranes on M [33], while the inflationary potential is generated by interbrane Ramond-Ramond (RR) and gravitational forces. On the other hand, the exit from inflation can occur when the brane and antibrane reach a distance as the length of string from one another, where the lightest stretched string becomes tachyonic. Such brane inflation models were generalized and explored by many studies [34–41].

In the review of [42], the brane inflation should concern the problem of moduli stabilization to make sense. This is an improvement on what is usually done, however, the right 4D Einstein frame potential is not quite the formula $V(r){=}2T_3[1{-}T_3/(2\pi)^3 M_{10}^8 r^4]$ ($T_3$ is the brane tension, $r$ is related to a canonically normalized scalar field via $\phi = \sqrt{T_3}r$). In this case, the potential in that paper must undergo a Weyl rescaling to reach 4D Einstein frame, and this multiplies the potential by an overall factor of $1/R^{12}$ [33]. Thus, achieving slow-roll inflation requires stabilizing the radion and the dilaton. Regardless of the interbrane potential, the system of the moduli in the early universe will undergo rapid decompactification [42]. In fact, even if the problem is solved in flux compactification, one still must engineer a flat interbrane potential to satisfy the standard slow-roll conditions. In this case, we find a natural way to guarantee the potential energy in the DBI Lagrangian could convert into kinetic energy in any same order, and improve the work in [43–47].

In our previous work [48], we show that the Taylor expansion of the DBI action can be reduced into the form in the non-linear classical physics. These investigations are the support for the statement that the results of string theory are consistent with quantum mechanics and classical physics.

In this paper, we present a new general DBI Lagrangian and consider the evolution of the universe, as well as the properties of the dark energy. In Section 2, we show that there always exists a general scalar function potential in the DBI action, and deduce the new general DBI Lagrangian, where the determinant of the induced metric naturally includes the kinetic energy and the potential energy. In particular, the potential energy and kinetic energy can convert into each other at any same order. In Section 3, we examine the new expressions of the exact energy-momentum tensor, pressure, density and sound speed. We show in Section 4 how to construct the required potential for a given equation of state. Generalizing the discuss of DBI theory to multiple branes adds a degree of freedom. We deduce the equation of states, examine the exact evolution of the universe and properties of the dark energy. We have introduced a novel mechanism for realizing either quintessence or phantom dark energy dominated phases within a string theoretical context.

## 2. The New General Lagrangian of DBI Inflation

In the usual DBI inflation scenario, the Lagrangian of the system is in the case of single field DBI inflation, where the determinant of the induced metric only contains kinetic energy. The DBI Lagrangian that presents in previous works ignores the possibility that the potential energy can convert into kinetic energy in the determinant. In this section, we present the Lagrangian for a general scenario, and discover that the determinant of the induced metric may naturally include the kinetic energy and potential energy.

In a general field DBI inflation, the field $\phi$ responsible for inflation is relative to the degree of freedom associated with a $(3+1)$-dimensional world volume with metric $\mathrm{d}s_4^2$ moving in a six-dimensional throat, where the $(3+1)$-dimensional volume looks like a particle moving along the radial $r$ and compacted by a 5-dimensional orbifold, the corresponding metric is [46]

$$\mathrm{d}s_{10}^2 = h^2(r)\mathrm{d}s_4^2 + h^{-2}(r)(\mathrm{d}r^2 + r^2\mathrm{d}s_{x_5}^2) \tag{1}$$

In principle, our universe may exist in various parts of compactification, including other warped throats. The construction involves wrapped D-branes and orientifold planes [8,30,35].

In the presence of a non-Bogomolnyi–Prasad–Sommerfield (BPS) brane or multiple $D_3$-branes, the DBI action acquires an additional potential $U$ multiplying the DBI term [49,50]. $U$ can arise in different places within the theory. First, if the brane is actually a non-BPS one, then the scalar field mode is actually tachyonic and the potential is therefore of the usual runaway form [51,52]. If there are $N$ multiple coincident branes, then the world-volume field theory is a $U(N)$ non-Abelian gauge theory and the potential term is simply a reflection of the additional degrees of freedom [53,54].

We now generalize Equation (1) to a more general case

$$\mathrm{d}s_{10}^2 = h^2(r)U(r)\mathrm{d}s_4^2 + h^{-2}(r)U(r)(\mathrm{d}r^2 + r^2\mathrm{d}s_{x_5}^2) \tag{2}$$

In the effective field theory, the induce metric on $D_3$-brane is

$$G_{\alpha\beta} = h^2(r)U(r)g_{\mu\nu}\frac{\partial x^\mu}{\partial\sigma^\alpha}\frac{\partial x^\nu}{\partial\sigma^\beta} + h^{-2}(r)U(r)\frac{\partial r}{\partial\sigma^\alpha}\frac{\partial r}{\partial\sigma^\beta} \tag{3}$$

where

$$h^{-2}(r)r^2 g_{ab}\frac{\partial x^a}{\partial\sigma^\alpha}\frac{\partial x^b}{\partial\sigma^\beta} = 0 \tag{4}$$

because $x^b$ is not dependent on $\sigma^\beta$.

Therefore, we get the DBI action on $D_3$-brane as follows

$$S_{DBI} = -T\int \mathrm{d}^4\sigma\sqrt{-\det[h^2(r)g_{\mu\nu}+h^2(r)g_{\mu\nu}U(r)+h^{-2}(r)\frac{\partial r}{\partial\sigma^\mu}\frac{\partial r}{\partial\sigma^\nu}]}$$

$$= -T\int \mathrm{d}^4\sigma\sqrt{-\det[h^2(r)g_{\mu\alpha}]}\sqrt{\det[\delta^\alpha_\nu(1+U(r))+h^{-2}(r)(h^{-2}(r)g^{\alpha\beta})\frac{\partial r}{\partial\sigma^\beta}\frac{\partial r}{\partial\sigma^\nu}]} \quad (5)$$

in which $U$ is an arbitrary function of $r$. The form of DBI action is typically given by the first line of Equation (5). We would like to point out that, if the background flux is turned on, this form could be generalized by taking into account the flux effects, as shown in [55]. However, Equation (5) is a toy model which is only expected to qualitatively reproduce some aspects of a more realistic theory. Then we neglect the effects of background flux in the warped geometry as [23,50].

We define the scalar field $\phi=\sqrt{T_{P_3}}r$ and the brane tension $T_{P_3}$ is a function of the string scale $m_s$ and the string coupling $g_s$, then we have [29]

$$T_{D_p}=\frac{1}{g_s(2\pi)^p(\alpha')^{(p+1)/2}}$$

$$\frac{m_s}{g_s}=T_{D_0}=\frac{1}{g_s\sqrt{\alpha'}} \quad (6)$$

and then we obtain

$$T_{P_3}=\frac{m_s^4}{(2\pi)^3 g_s} \quad (7)$$

Therefore, we have the new DBI action

$$S_{DBI}=-T_{P_3}\int \mathrm{d}^4\sigma h^4(\phi)\sqrt{-\det g_{\mu\alpha}}\sqrt{\det[\delta^\alpha_\nu(1+U(\phi))+h^{-4}(\phi)T^{-1}_{P_3}g^{\alpha\beta}\partial_\beta\phi\partial_\nu\phi]} \quad (8)$$

and the new DBI Lagrangian

$$L_{DBI} = -T_{P_3}h^4(\phi)\sqrt{-\det g_{\mu\alpha}}\sqrt{\det[\delta^\alpha_\nu(1+U(\phi))+h^{-4}(\phi)T^{-1}_{P_3}g^{\alpha\beta}\partial_\beta\phi\partial_\nu\phi]}$$

$$= -f^{-1}(\phi)\sqrt{-\det g_{\mu\alpha}}\sqrt{\det[\delta^\alpha_\nu(1+U(\phi))+f(\phi)T^{-1}_{P_3}g^{\alpha\beta}\partial_\beta\phi\partial_\nu\phi]} \quad (9)$$

where the inverse brane tension $f(\phi)$ is relative to Equation (7) and the warp factor $h$ by $f(\phi)=\frac{1}{T_{P_3}h^4(\phi)}$.

We consider $\sqrt{-\det g_{\mu\alpha}}\mathrm{d}^4\sigma$ as the invariant volume element of the integral Equation (8) and add an integral constant term $T_{p_3}\int \mathrm{d}^4\sigma h^4(\phi)\sqrt{-\det g_{\mu\alpha}}=\int \mathrm{d}^4\sigma\sqrt{-\det g_{\mu\alpha}}f^{-1}(\phi)$ into Equation (8), we finally achieve the general DBI Lagrangian

$$L_{DBI}= = -f^{-1}(\phi)\sqrt{U(\phi)}\sqrt{1+f(\phi)g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi}+f^{-1}(\phi) \quad (10)$$

the final line expression neglects the higher order term of $\partial_\alpha\phi\partial_\beta\phi$, and is equivalent to the DBI Lagrangian in [56]. In the present of a non-BPS brane, $\sqrt{U(\phi)}$ is relative to an additional potential multiplying the special DBI term.

For $\sqrt{U(\phi)}=1$, we have

$$L_{DBI}=-f^{-1}(\phi)\sqrt{1+f(\phi)g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi}+f^{-1}(\phi) \tag{11}$$

Equation (11) is the same as the Lagrangian in [23], without potential energy. Adding a potential energy $V$ ($V$ is relative to $\phi$) into Equation (11), it follows that

$$L_{DBI}=-f^{-1}(\phi)\sqrt{1+f(\phi)g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi}+f^{-1}(\phi)+V(\phi) \tag{12}$$

Equation (12) is a DBI action that arises in type IIB string theory in terms of the volume swept out by a D3-brane in a warped geometry, coupled to gravity [56]. The origin of the $V$ term is less explicit, but is a sum of the terms. One expects open or closed string interactions to generate a scalar potential $V$; however, the precise form of such an interaction depends upon many factors such as the number of additional branes and geometric moduli, the number of nontrivial cycles in the compact space, and the choice of embedding for branes on these cycles. Typically, one can only compute this in special cases in the full string theory. There are also additional terms coming from coupling of the brane to any background Ramond-Ramond form fields. Since, in the presence of multiple $D_3$-branes or a non-BPS brane, the DBI action acquires an additional potential $U$ multiplying the DBI term.

The potential energy in Equation (12) obeys the basic principles of physics on any order, while Equation (10) obeys the basic principles of physics on any order, *i.e.*, the potential energy can convert into kinetic energy in the determinant in any same order. Equation (10) reveals the potential energy in another way, which is quite different from previous work.

Without loss of generality, we define $U(\phi)=constant+V_0(\phi)$ in Equation (10), where $U(\phi)$ is an arbitrary function of $\phi$, and $V_0(\phi)=f(\phi)\xi V(\phi)$ ($\xi$ is an arbitrary parameter). Then we have

$$\begin{aligned}L_{DBI}&=-f^{-1}(\phi)\sqrt{\det[\delta^\alpha_\nu(1+V_0(\phi))+f(\phi)U(\phi)g^{\alpha\beta}\partial_\beta\phi\partial_\nu\phi)]}+f^{-1}(\phi)\\ &=-f^{-1}(\phi)\sqrt{1+U(\phi)[f(\phi)g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi+\frac{V_0(\phi)}{U(\phi)}]}+f^{-1}(\phi)\end{aligned} \tag{13}$$

Thus, Equation (13) can be rewritten as

$$L_{DBI}=-f^{-1}(\phi)\sqrt{1+U(\phi)f(\phi)[g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi+\xi\frac{V(\phi)}{U(\phi)}]}+f^{-1}(\phi) \tag{14}$$

By defining the new inverse brane tension $F(\phi)=f(\phi)U(\phi)$ and $f^{-1}=F^{-1}(\phi)/U^{-1}(\phi)$, we have

$$L_{DBI}=-\frac{F^{-1}(\phi)}{U^{-1}(\phi)}\sqrt{1+F(\phi)[g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi+\xi\frac{V(\phi)}{U(\phi)}]}+\frac{F^{-1}(\phi)}{U^{-1}(\phi)} \tag{15}$$

Equation (15) is a new general DBI Lagrangian. Taking linear approximation of Equation (15), we have

$$\begin{aligned}L_{DBIL}&=-\frac{F^{-1}(\phi)}{U^{-1}(\phi)}[1+\frac{1}{2}F(\phi)(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi+\xi\frac{V(\phi)}{U(\phi)})]+\frac{F^{-1}(\phi)}{U^{-1}(\phi)}\\ &=-U(\phi)(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi+\frac{1}{2}\xi\frac{V(\phi)}{U(\phi)})\end{aligned} \tag{16}$$

For $U(\phi)=1$ and $\xi=1$, we have

$$L_{DBI\ L}{=}{-}\frac{1}{2}g^{\alpha\beta}\partial_{\alpha}\phi\partial_{\beta}\phi{-}\frac{1}{2}V(\phi) \tag{17}$$

which means that Equation (17), a useful expression in [23], is a special case of the new general DBI Lagrangian Equation (15). Equation (15) is more general than Equation (17), and Equation (15) reveals that the potential energy can convert into kinetic energy in the determinant in any same order, and the potential energy emerges from the Lagrangian naturally.

In this section, we deduce a new general Lagrangian Equation (10) for DBI action with a general potential, where the determinant of induced metric naturally includes the kinetic energy and potential energy. In addition, we obtain a new linear approximation of Lagrangian Equation (15) for DBI action corresponding to Equation (10). Furthermore, Equation (15) guarantees that the kinetic energy and the potential energy can transform into each other in any same order, and never suffers from the problem of not corresponding in high order. The forms of the new Lagrangian Equations (10) and (15) will play an important role in the following sections of this paper.

## 3. Energy-Momentum Tensor and Sound Speed of the New DBI Action

Sound speed depends on the media, which is sensitive to the pressure from the density perturbation. In the early universe with general relativity, primordial fluctuation propagates by the flux with a relativistic sound speed $C_s=c/\sqrt{3}$ [30,31]. The most important fact is that the baryonic matter and radiation are closely coupled throughout the pre-recombination era. With the expanding of the universe, baryonic matter is decoupled from radiation. Meanwhile, their corresponding sound speed will decline rapidly.

On the other hand, the decline of sound speed is greatly postponed by a new-coupled mechanism [43–46]. In high temperature, high pressure and high density of the early universe, baryonic matter is ionized, which strongly interacts with photon, *i.e.*, scattering of photon and electron, fusing of proton and electron. Finally, all the baryonic matter is ionized by photon. Therefore, we can consider baryonic matter and radiation as a uniform flux, since there is extremely strong couple between them, named baryonic-photon flux (baryonic matter is mainly composed of proton and neutron). The radiation pressure is a large number, which is a domain term in the baryonic-photon flux. Thus, under the situation, the sound speed is decreased very slowly. The initial spectrum of fluctuations from Big Bang is propagating in such flux with a sound speed close to light.

From Lagrangian Equation (13) we can obtain the new energy-momentum tensor

$$T_{\mu\nu}{=}2\frac{[-f^{-1}(\phi)\sqrt{1{+}U(\phi)[f(\phi)g^{\alpha\beta}\partial_{\alpha}\phi\partial_{\beta}\phi{+}\frac{V(\phi)}{U(\phi)}]}]}{g^{\mu\nu}}$$

$$+(-g_{\mu\nu})[-f^{-1}(\phi)\sqrt{1{+}U(\phi)[f(\phi)g^{\alpha\beta}\partial_{\alpha}\phi\partial_{\beta}\phi{+}\frac{V(\phi)}{U(\phi)}]}{+}f^{-1}(\phi)] \tag{18}$$

In order to simplify the energy-momentum tensor, we can rewrite Equation (18) by using Equation (15)

$$T_{\mu\nu}{=}2\frac{[-\frac{F^{-1}(\phi)}{U^{-1}(\phi)}\sqrt{1{+}F(\phi)(g^{\alpha\beta}\partial_{\alpha}\phi\partial_{\beta}\phi{+}\xi\frac{V(\phi)}{U(\phi)})}}{g^{\mu\nu}}$$

$$+(-g_{\mu\nu})[-\frac{F^{-1}(\phi)}{U^{-1}(\phi)}\sqrt{1+F(\phi)(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi+\xi\frac{V(\phi)}{U(\phi)})}+\frac{F^{-1}(\phi)}{U^{-1}(\phi)}]$$

$$=U(\phi)(-\frac{\partial_\mu\phi\partial_\nu\phi}{\sqrt{1+F(\phi)(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi+\xi\frac{V(\phi)}{U(\phi)})}}$$

$$+g_{\mu\nu}F^{-1}(\phi)\sqrt{1+F(\phi)(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi+\xi\frac{V(\phi)}{U(\phi)})}-g_{\mu\nu}F^{-1}(\phi)) \quad (19)$$

Similar to the discussion in [29], we define

$$\gamma(\phi)=\frac{1}{\sqrt{1+F(\phi)(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi+\xi\frac{V(\phi)}{U(\phi)})}}=\frac{1}{\sqrt{1+\xi F(\phi)v(\phi)-2F(\phi)X}} \quad (20)$$

where we define a function $X=-\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi$ and $v(\phi)=V(\phi)/U(\phi)$. Using Equation (19), we obtain an energy-momentum tensor

$$T_{\mu\nu}=U(\phi)[-\gamma(\phi)\partial_\mu\phi\partial_\nu\phi-g_{\mu\nu}F^{-1}(\phi)(1-\gamma^{-1}(\phi))] \quad (21)$$

For $U(\phi)=1$ and $\xi=1$, Equation (20) is reduced into the expression in [23,56]. In general, using $U_\mu=\{1,0,0,0\}$, $T_{\mu\nu}=(\rho+P)U_\mu U_\nu-Pg_{\mu\nu}$, and Friedmann–Robertson–Walker metric $g_{\mu\nu}=diag\ (1,-\frac{R^2(t)}{1-Kr^2},-r^2R^2(t),-r^2R^2(t)\sin^2\theta)$, we have

$$\rho=T_{00}=U(\phi)[-\gamma(\phi)\partial_0\phi\partial_0\phi-g_{00}f^{-1}(\phi)[1-\gamma^{-1}(\phi)]] \quad (22)$$

$$P=-\frac{1}{3}T^j_j=U(\phi)[\frac{1}{3}\gamma(\phi)\partial^j\phi\partial_j\phi+f^{-1}(\phi)[1-\gamma^{-1}(\phi)]] \quad (23)$$

Using Equations (22), (23) and the condition $T_{ik}=-Pg_{ik}$, we obtain the density $\rho$, which is relative to $P$, $V$, $X$, $\gamma$ as follows

$$\rho=3P+2X\gamma(\phi)-4U(\phi)f^{-1}(\phi)[1-\gamma^{-1}(\phi)] \quad (24)$$

For a general DBI Lagrangian, see Equation (10), we can obtain the general sound speed

$$C_s^2=\frac{dP}{d\rho}=\frac{P_{,X}}{3P_{,X}+U(\phi)[2\gamma(\phi)+2X\gamma(\phi)_{,X}-4[\frac{\partial f^{-1}(\phi)}{\partial X}-\gamma^{-1}(\phi)\frac{\partial f^{-1}(\phi)}{\partial X}-f^{-1}(\phi)\frac{\partial\gamma^{-1}(\phi)}{\partial X}]]} \quad (25)$$

Expanding Equation (20) and taking the one order terms of $g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi$, we rewrite Equations (21)–(25) as the corresponding new expressions

$$T_{\mu\nu}=U(\phi)[-\gamma(\phi)\partial_\mu\phi\partial_\nu\phi-g_{\mu\nu}F^{-1}(\phi)(1-\gamma^{-1}(\phi))]$$

$$=U(\phi)[-\partial_\mu\phi\partial_\nu\phi+\frac{1}{2}F(\phi)g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi\partial_\mu\phi\partial_\nu\phi$$

$$+\frac{1}{2}F(\phi)\xi v(\phi)\partial_{\mu}\phi\partial_{\nu}\phi+\frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\partial_{\alpha}\phi\partial_{\beta}\phi+\frac{1}{2}g_{\mu\nu}\xi v(\phi)] \tag{26}$$

$$\rho=T_{00}=U(\phi)[-\gamma(\phi)\partial_0\phi\partial_0\phi-g_{00}F^{-1}(\phi)(1-\gamma^{-1}(\phi))]$$

$$=U(\phi)[-\partial_0\phi\partial_0\phi+\frac{1}{2}F(\phi)g^{\alpha\beta}\partial_{\alpha}\phi\partial_{\beta}\phi\partial_0\phi\partial_0\phi$$

$$+\frac{1}{2}F(\phi)\xi v(\phi)\partial_0\phi\partial_0\phi+\frac{1}{2}g_{00}g^{\alpha\beta}\partial_{\alpha}\phi\partial_{\beta}\phi+\frac{1}{2}g_{00}\xi v(\phi)] \tag{27}$$

$$P=-\frac{1}{3}T^j_j=U(\phi)[\frac{1}{3}\gamma(\phi)\partial^j\phi\partial_j\phi+F^{-1}(\phi)(1-\gamma^{-1}(\phi))]$$

$$=U(\phi)[\frac{1}{3}\partial^j\phi\partial_j\phi-\frac{1}{6}F(\phi)g^{\alpha\beta}\partial_{\alpha}\phi\partial_{\beta}\phi\partial^j\phi\partial_j\phi$$

$$-\frac{1}{6}F(\phi)\xi v(\phi)\partial^j\phi\partial_j\phi-\frac{1}{2}g^{\alpha\beta}\partial_{\alpha}\phi\partial_{\beta}\phi-\frac{1}{2}\xi v(\phi)] \tag{28}$$

$$\rho=3P+U(\phi)[2X\gamma(\phi)-4F^{-1}(\phi)(1-\gamma^{-1}(\phi))]$$

$$=3P+U(\phi)[2X-XF(\phi)(g^{\alpha\beta}\partial_{\alpha}\phi\partial_{\beta}\phi+\xi v(\phi))+2g^{\alpha\beta}\partial_{\alpha}\phi\partial_{\beta}\phi+2\xi v(\phi)] \tag{29}$$

$$C_s^2=\frac{P_{,X}}{3P_{,X}+\frac{U(\phi)[2X-XF(\phi)(g^{\alpha\beta}\partial_{\alpha}\phi\partial_{\beta}\phi+\xi v(\phi))+2g^{\alpha\beta}\partial_{\alpha}\phi\partial_{\beta}\phi+2\xi v(\phi)]}{X}} \tag{30}$$

Regardless of the higher derivative terms, Equations (26)–(30) can be simplified as

$$T_{\mu\nu}=U(\phi)[-\frac{1}{2}\partial_0\phi\partial_0\phi+\frac{1}{2}\xi v(\phi)] \tag{31}$$

$$\rho=U(\phi)[-\frac{1}{2}\partial_0\phi\partial_0\phi+\frac{1}{2}\xi v(\phi)] \tag{32}$$

$$P=U(\phi)[-\frac{1}{2}g^{\alpha\beta}\partial_{\alpha}\phi\partial_{\beta}\phi-\frac{1}{2}\xi v(\phi)] \tag{33}$$

$$\rho=3P+U(\phi)[\partial_0\phi\partial_0\phi+2\xi v(\phi)+\frac{1}{2}\partial_0\phi\partial_0\phi F(\phi)\xi v(\phi)] \tag{34}$$

$$C_s^2 = \frac{P_{,X}}{3P_{,X} + U(\phi)[-2 - 2F(\phi)\partial_0\phi\partial_0\phi - \xi F(\phi)v(\phi)]} \quad (35)$$

Defining $X = -\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi$ and $Y = -\frac{1}{2}g^{00}\partial_0\phi\partial_0\phi = -\frac{1}{2}\partial_0\phi\partial_0\phi$, and neglecting the space derivative part $\partial^j\phi\partial_j\phi$ due to the homogeneous property of space in the early universe, we can rewrite Equations (31)–(35) as

$$T_{\mu\nu} = U(\phi)[Y + \frac{1}{2}\xi v(\phi)] \quad (36)$$

$$\rho = U(\phi)[Y + \frac{1}{2}\xi v(\phi)] \quad (37)$$

$$P = U(\phi)(Y - \frac{1}{2}\xi V(\phi)) \quad (38)$$

$$\rho = 3P + U(\phi)[-2Y + 2\xi v(\phi) - YF(\phi)\xi v(\phi)] \quad (39)$$

$$C_s^2 = \frac{P_{,X}}{3P_{,X} + U(\phi)[-2 + 4F(\phi)Y - \xi F(\phi)v(\phi)]} \quad (40)$$

For $U(\phi) = 1$, $\xi = 2$, we can rewrite Equations (37) and (38) as the simplified expressions that are obtained in the past from the usual form of DBI Lagrangian [30,31]

$$\rho = Y + V(\phi) \quad (41)$$

$$P = Y - V(\phi) \quad (42)$$

and we consider the case that the kinetic energy should be greater than the potential in Equation (20), *i.e.*, the relativity case, $4\left|-\frac{1}{2}g^{00}\partial_0\phi\partial_0\phi\right| \gg 2v(\phi)$, it follows

$$4\left|-\frac{1}{2}g^{00}\partial_0\phi\partial_0\phi\right| \gg 2v(\phi) \quad (43)$$

Therefore, Equations (41) and (42) that appear in [25–29] is a special case of our work. Thus, this work not only keeps the potential energy in Equation (15) to convert into kinetic energy in any same order, but also is more general than the former work of the others. The sound speed Equation (25) is a new general expression for DBI inflation, where we consider the influence of kinetic energy and potential energy, *i.e.*, the conversion between them.

In summary, we achieve the general energy-momentum tensor, pressure, density and speed sound of the new general DBI Lagrangian, and deduce their special cases, e.g., Equations (36)–(40). When we choose the convention in [25], the energy-momentum tensor, pressure, density and speed sound of the new general DBI Lagrangian will be reduced into the usual forms [25–27].

## 4. Dark Energy with DBI Inflation

In the previous sections, we introduced the concept of D-branes, and we will study the dark-energy equation-of-state parameter of this version of DBI scenario, and further investigate the various cosmological possibilities with general properties. We desire to explore the general features of the equation of states for possible forms of the involved tension and potentials, and even examining in detail the equations of motion. In recent years, there have been many works around the issue of multiple fields, such as [57–59], and we refer to [60,61], which provide some ideas and support the analysis in this section.

In order to discuss the evolutions of the universe, we use the Friedmann equations [33]

$$H^2 = \frac{\rho}{3M_{pl}^2} + \frac{\Lambda}{3} - \frac{K}{a^2} \tag{44}$$

where $H = \dot{a}/a$ is the Hubble parameter. The constant $K$ is related to the spatial geometry of the universe. The Universe is flat (Euclidean) for $K = 0$, finite or closed for $K > 0$, and infinite or open for $K < 0$. And

$$\rho = 3M_{pl}^2 H^2 \tag{45}$$

$$\dot{\rho} = -3H(\rho + P) \tag{46}$$

and then we can have

$$-(\rho + P) = 2M_{pl}^2 \dot{H} \tag{47}$$

Now accelerating expansion $(\ddot{a} > 0)$ requires smallness of the variation of the Hubble parameter $H \equiv \partial_t Ina$, as defined by the parameter [34]

$$\varepsilon \equiv -\frac{\dot{H}}{H^2} = \frac{3}{2}(1+\omega) < 1 \tag{48}$$

thus

$$\omega < -\frac{1}{3} \tag{49}$$

Using the Equation (24), we have

$$\omega = \frac{P}{\rho} = \frac{P}{3P + U(\phi)[2X\gamma(\phi) - 4F^{-1}(\phi)(1-\gamma^{-1}(\phi))]} \tag{50}$$

Combining the expressions for the energy-momentum tensor components with the continuity equation, we obtain the general DBI version of the Klein–Gordon equation from Friedmann equations

$$[-2 - \gamma^2(\phi)U(\phi)]\ddot{\phi}$$

$$+ \frac{\frac{1}{2}\gamma^2(\phi)U(\phi) - 1 + 2U^{-1}(\phi)\gamma^{-2}(\phi) - U^{-1}(\phi)\gamma^{-1}(\phi)}{F(\phi)}\frac{dU(\phi)}{d\phi}$$

$$+\frac{-\frac{1}{2}\gamma^2(\phi)U^2(\phi)+\frac{1}{2}U(\phi)+\gamma^{-1}(\phi)-\gamma^{-2}(\phi)}{F^2(\phi)}\frac{dF(\phi)}{d\phi}$$

$$=3H\dot{\phi} \tag{51}$$

For $U(\phi)=1$, the Equation (51) can be reduced into the Klein–Gordon equation in [49].

Let us consider the scenario by using Equations (22), (23) and the general DBI inflation Lagrangian Equation (10), we deduce the equation of states

$$\omega=\frac{P}{\rho}=\frac{\frac{1}{3}\gamma(\phi)\partial^j\phi\partial_j\phi+F^{-1}(\phi)[1-\gamma^{-1}(\phi)]}{-\gamma(\phi)\partial_0\phi\partial_0\phi-F^{-1}(\phi)[1-\gamma^{-1}(\phi)]} \tag{52}$$

Substituting the expression of $\gamma$ into Equation (52), it follows

$$\omega=\frac{\frac{1}{3}\frac{1}{\sqrt{1+F(\phi)(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi+\xi\frac{V(\phi)}{U(\phi)})}}\partial^j\phi\partial_j\phi+F^{-1}(\phi)[1-\sqrt{1+F(\phi)(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi+\xi\frac{V(\phi)}{U(\phi)})}]}{-\frac{1}{\sqrt{1+F(\phi)(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi+\xi\frac{V(\phi)}{U(\phi)})}}\partial_0\phi\partial_0\phi-F^{-1}(\phi)[1-\sqrt{1+F(\phi)(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi+\xi\frac{V(\phi)}{U(\phi)})}]} \tag{53}$$

With the linear approximation of $\gamma$, it follows that

$$\gamma(\phi)=1-\frac{1}{2}F(\phi)(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi+\xi v(\phi))=1-\frac{1}{2}F(\phi)(-\dot{\phi}^2+\frac{\xi V(\phi)}{U(\phi)})$$

$$\gamma^{-1}(\phi)=1+\frac{1}{2}F(\phi)(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi+\xi v(\phi))=1+\frac{1}{2}F(\phi)(-\dot{\phi}^2+\frac{\xi V(\phi)}{U(\phi)}) \tag{54}$$

and we obtain the new expression of the equation of states

$$\omega=\frac{F^{-1}(\phi)[1-\sqrt{1+F(\phi)(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi+\xi\frac{V(\phi)}{U(\phi)})}]}{-\frac{1}{\sqrt{1+F(\phi)(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi+\xi\frac{V(\phi)}{U(\phi)})}}\partial_0\phi\partial_0\phi-F^{-1}(\phi)[1-\sqrt{1+F(\phi)(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi+\xi\frac{V(\phi)}{U(\phi)})}]}$$

$$=\frac{\dot{\phi}^2-\xi v(\phi)}{-2-F(\phi)\dot{\phi}^4+F(\phi)\xi v(\phi)\dot{\phi}^2-\dot{\phi}^2+\xi v(\phi)} \tag{55}$$

Using $U(\phi)=1+f(\phi)\xi V(\phi)$, $(U^2(\phi)-U(\phi))/F(\phi)=\xi V(\phi)$ and $\xi V(\phi)/U(\phi)=(U(\phi)-1)/F(\phi)$, we achieve a new form of the equation of states

$$\omega=\frac{F(\phi)\dot{\phi}^2-(U(\phi)-1)}{-2F(\phi)-F^2(\phi)\dot{\phi}^4+F(\phi)(U(\phi)-2)\dot{\phi}^2+(U(\phi)-1)} \tag{56}$$

The standard energy conditions accepted in cosmology are [62]:
Null Energy Condition (NEC): $\rho+P\geq 0$ ;
Weak Energy Condition (WEC): $\rho\geq 0$ and $\rho+P\geq 0$ ;
Strong Energy Condition (SEC): $\rho+3P\geq 0$ and $\rho+P\geq 0$ ;

Dominant Energy Condition (DEC): $\rho \geq 0$ and $\rho \pm P \geq 0$ .
In this paper, we focus on NEC, which is the case of:

$$U(\phi) + \frac{1}{2}F(\phi)U(\phi)\dot{\phi}^2 - \frac{1}{2}F(\phi)\xi V(\phi) \leq 0 \tag{57}$$

For a general $\omega$, the preliminary phenomenology suggests that $\omega$ could cross the $-1$ bound. We identify several limits of interest, focusing on the behavior of $\omega$:

(i) Let us first consider the scenario where no scalar potential is present, *i.e.*, $U(\phi) = 1$, $\xi V(\phi) = 0$, $\xi V(\phi)/U(\phi) = 0$, that is to study solely the brane action. In this case, the equation of state is rewritten in

$$\omega = \frac{\dot{\phi}^2}{-2 - F(\phi)\dot{\phi}^4 - \dot{\phi}^2} \tag{58}$$

In particular, the equation of state would become phantom with

$$\omega = \frac{\dot{\phi}^2}{-2 - F(\phi)\dot{\phi}^4 - \dot{\phi}^2} < -1$$

$$-2 > F(\phi)\dot{\phi}^4 \tag{59}$$

(ii) Let us now turn on the scalar potential term $V(\phi)$. A first simple solution subclass would be to consider $F(\phi) = 0$, where we obtain $\omega = -1$ that recovers the case of pure de Sitter expansion.

(iii) In the general case of nonzero potential $V(\phi)$ and tension terms $f(\phi)$, but with $V(\phi) \gg f(\phi)$ (corresponds to $U(\phi) \gg F(\phi)$ in the new DBI Lagrangian), and then we have

$$\omega = \frac{-1 + \frac{1}{U(\phi)}}{F(\phi)\dot{\phi}^2 + 1 - \frac{1}{U(\phi)}} < -1$$

$$\dot{\phi}^2 > 0 \tag{60}$$

which will stand only with $F(\phi)\dot{\phi}^2 + 1 - \frac{1}{U(\phi)} > 0$. Therefore, in the relativistic regime equation of states will be negative definite, leading to the realization of the phantom phase. This phantom realization is obtained naturally from a large solution subclass of the model.

(iv) Another class of solutions will occur when we have nonzero potential $V(\phi)$ and tension terms $f(\phi)$, but with $V(\phi) \ll f(\phi)$(corresponds to $U(\phi) \ll F(\phi)$ in the new DBI Lagrangian), and then we have

$$\omega = \frac{\dot{\phi}^2 + \frac{1}{F}}{-2 - F(\phi)\dot{\phi}^4 + (U(\phi) - 2)\dot{\phi}^2 - \frac{1}{F}} < -1$$

$$3U(\phi) + F(\phi)U(\phi)\dot{\phi}^2 - U(\phi)^2 \ll 0 \tag{61}$$

Once, the Equation (61) changes into $F(\phi) < 0$ and $U(\phi)\dot{\phi}^2 < 2 + F(\phi)\dot{\phi}^4 + \dot{\phi}^2$, the case of the regime $V(\phi) \ll f(\phi)$ presents a quintessence behavior with $\omega > -1$.

(v) The last class of solutions will occur when we have nonzero potential $V(\phi)$ and tension terms $f(\phi)$, but with $V(\phi) \approx f(\phi)$ ($U(\phi) \approx F(\phi)$), and then we have

$$\omega = \frac{\dot{\phi}^2 - 1 + \frac{1}{U(\phi)}}{-1 - \frac{1}{U(\phi)} - F(\phi)\dot{\phi}^4 + (U(\phi) - 2)\dot{\phi}^2} < -1 \tag{62}$$

In this case, we suppose that $\xi = 1$, and

$$\gamma(\phi) = 1 - \frac{1}{2}F(\phi)(-\dot{\phi}^2 + \frac{\xi V(\phi)}{U(\phi)}) \approx 1 + \frac{1}{2}F(\phi)(\dot{\phi}^2 - 1) \tag{63}$$

In order to make sense of physics, the value of $\gamma(\phi)$ should be positive and $\dot{\phi}^2 > 0$.

To study the solution with more details, we first consider $F(\phi) > 0$, in which the parameters $F(\phi)$ and $U(\phi)$ should satisfy inequations $(-U(\phi) + 1)^2 - 8F(\phi) > 0$ and $-(-U(\phi) + 1) - \sqrt{(-U(\phi) + 1)^2 - 8F(\phi)} > 0$. Thus, for $F(\phi) > 0$, the region of the parameter is $-U(\phi) + 1 > 2\sqrt{2F(\phi)} > 0$. On the other hand, in the case of $F(\phi) < 0$, the corresponding inequations are $(-U(\phi) + 1)^2 - 8F(\phi) > 0$ and $-(-U(\phi) + 1) - \sqrt{(-U(\phi) + 1)^2 - 8F(\phi)} < 0$. It is obvious to found out that if $F(\phi) < 0$, the equation of state will pass through $-1$ definitely. By providing a more transparent picture of the obtained cosmological behavior, we present the solution for the simple scenario of Equation (56), corresponding to different kind brane models (comparing all the lines with the bound of $-1$).

These features in Table 1 reveal that the use of D-branes does lead to quintessence and phantom realization, depending on the specific forms of the potential terms and of the tension in the new DBI action.

In summary, from the solution subclasses, we reveal an interesting $\omega(\phi)$ behavior. We obtain that $\omega(\phi)$ are determined by the $U(\phi)$ and $F(\phi)$. Clearly, considering more general scenarios, with various $U(\phi)$ and $F(\phi)$, the resulting cosmological behaviors can be significantly richer.

**Table 1.** Behavior of $V$, $U$, $F$ and $\omega$.

| $V, U, F$ | quintessence $\omega > -1$ | de-Sitter $\omega = -1$ | phantom $\omega < -1$ |
|---|---|---|---|
| (i) $V = 0, U = 1$ | $-2 < F(\phi)\dot{\phi}^4$ | $-2 = F(\phi)\dot{\phi}^4$ | $-2 > F(\phi)\dot{\phi}^4$ |
| (ii) $V \neq 0$, $F = 0$ | – | $\omega(\phi) = -1$ | – |
| (iii) $V \neq 0$, $U \gg F$ | $F(\phi)\dot{\phi}^2 + 1 - \frac{1}{U(\phi)} < 0$ | – | $F(\phi)\dot{\phi}^2 + 1 - \frac{1}{U(\phi)} > 0$ |
| (iv) $V \neq 0$, $U \ll F$ | $U(\phi)\dot{\phi}^2 > 2 + F(\phi)\dot{\phi}^4 + \dot{\phi}^2$ | – | $U(\phi)\dot{\phi}^2 < 2 + F(\phi)\dot{\phi}^4 + \dot{\phi}^2$ |
| (v) $V \neq 0$, $U \approx F$ | $F(\phi) < 0$ | – | $-U(\phi) + 1 > 2\sqrt{2F(\phi)} > 0$ |

## 5. Discussion and Conclusion

In this paper, we have deduced a new general Lagrangian for DBI-Inflation, where the determinant of the induced metric naturally includes the kinetic energy and potential energy. The motivation to study the new general potential with DBI action is to find a natural way to guarantee that the potential energy in the DBI Lagrangian of [43–45] could convert into kinetic energy in any same order. On the other hand, our new Lagrangian (see Equation 15) for DBI action naturally includes potential energy. The technical difference between this work and the previous is that we present a more general induced metric, which gives rise to the potential naturally. Meanwhile, the novel feature in our treatment is that kinetic energy and potential energy varies in the same order, and never suffers from the problem of not corresponding in any high order. Therefore, Equation (15) keeps the basic physical principle in any same order.

We demonstrate the new general Lagrangian for DBI inflation in scenario, and represent the energy-momentum tensor, pressure, density and sound speed of the new general DBI Lagrangian. We deduce the equation of states, and examine the exact evolution of the universe and properties of the dark energy. We introduce a novel mechanism for realizing either quintessence or phantom dark energy dominated phases within a string theoretical context. It is very interesting, since the more accurate cosmological data can restrict fundamental string parameters.

Although these features arise from the different model subclasses, it is clear that more complicated behavior can be revealed by considering more general $F(\phi)$, $U(\phi)$ and $\omega(\phi)$, with a natural realization of quintessence and phantom behavior, of the crossing $-1$ and of a big rip.

One remaining issue pertains to the quantum stability of such a phantom model. In this case, we conclude that the usual phantom models are robust only for small momenta, since for larger momenta the higher derivative terms dominate.

There is a long way to completely discover the natures of string theory and to build a perfect model for describing the universe. This work is just the current improvement in the brane inflationary model with DBI action. This paper presents a generally useful DBI action. In addition, we analyzed in detail on how the dark energy can constrain some aspects of fundamental string theory within the DBI framework. The connections between string theory and astrophysical data offer exciting prospects for revealing the nature of the cosmological constant and the accelerating universe.

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