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Article

Entropy Generation During the Interaction of Thermal Radiation with a Surface

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Abstract: The entropy calculation for radiation fluxes is reviewed and applied to simple radiation-surface interactions. A plate interacting with radiation from a hot object in the zenith of the hemisphere surrounded by a colder atmosphere is analyzed in detail. The entropy generation rate upon absorption and reflection of the incoming radiation is calculated and discussed. The plate is adiabatic in a first version (thermal equilibrium), then its temperature is fixed by allowing a heat flux to or from the plate. This analysis prepares the way towards an entropy generation minimization analysis of more complex radiation settings.

Keywords: thermal radiation; reflection; absorption; entropy production

PACS Codes: 44.40 +a

Nomenclature

A	area, m^2
B	geometry parameter related to solid angle [see Equation (17)]; sr
B_{in}	geometry parameter of the inner radiation source; sr
B_{at}	geometry parameter of the (atmospheric) environment;

c	speed of light in vacuum, c = 299,792,458 m/s;
D	overall hemispheric radiation entropy fluxes, W/m²/K;
D_{at}	incoming radiation entropy flux from the atmosphere, W/m²/K;
D^b	blackbody radiation entropy flux, W/m²/K;
D_{in}	incoming radiation entropy flux from the inner source, W/m²/K;
$D_{pl,overall}$	entropy flux of overall outgoing radiation, W/m²/K;
$D_{irr,cond.}$	entropy production rate by heat conduction;
E	overall hemispheric radiation energy flux, W/m²;
E_{at}	incoming radiation energy flux from the atmosphere, W/m²;
E^b	blackbody radiation energy flux, W/m²;
E_{in}	incoming radiation energy flux from the inner radiation source, W/m²;
E_{pl}	radiation energy flux, emitted from the plate, W/m²;
$E_{refl,at}$	reflected radiation energy flux from the atmosphere, W/m²;
$E_{refl,in}$	reflected radiation energy flux from the inner source, W/m²;
h	Planck's constant, $h = 6.6261 \times 10^{-34}$ J·s;
K^b	overall radiation entropy intensity of blackbody radiation, W/K/m²/sr;
K_λ	spectral radiation entropy intensity, W/K/m²/µm/sr;
K_λ^b	spectral entropy flux of blackbody radiation, W/K/m²/µm;
k	Boltzmann's constant, $k = 1.3806 \times 10^{-23}$ J/K;
L^b	overall energy intensity of blackbody radiation, W/m²/sr
L_λ	spectral radiation intensity, W/m²/µm/sr
L_λ^b	spectral intensity of blackbody radiation, W/m²/µm/sr
N_λ	density of number of photons, 1/m³;
N_λ^{eq}	density of number of photons (equilibrium), 1/m³;
$\vec{n}$	normal vector of a surface;
p	pressure, N/m²;
$\dot{Q}$	heat conduction flow to or from the plate, W/m²;
S	entropy, J/K;
s^{eq}	volume specific radiation entropy in equilibrium, J/m³/K;
$\dot{S}_{irr}$	entropy production rate, W/K;
s	volume specific entropy, J/m³/K;
s_λ	volume specific spectral radiation entropy, J/m³/K;
T	absolute temperature, K;
T_{at}	temperature of (atmospheric) environment, K;
T_{pl}	temperature of the plate, K;
T^{eq}	equilibrium temperature, K;
T_{in}	temperature of inner radiation source, K;
T_s	formal radiation flux temperature, K;
T_λ	spectral radiation temperature, K;
t	time, s;
U	internal energy, J;
U_λ^{eq}	spectral energy of cavity radiation in equilibrium, J/µm;

u	volume specific internal energy, J/m³;
u^{eq}	volume specific overall energy of cavity radiation in equilibrium, J/m³;
u_{λ}^{eq}	volume specific spectral energy of cavity radiation in equilibrium, J/m³/μm;
u_{λ}	volume specific spectral energy of cavity radiation, J/m³/μm;
V	Volume, m³;
x	average occupation number of the photon state in equilibrium;
$X(\varepsilon)$	grey body entropy function
Ω	solid angle, sr;
ε	emissivity coefficient;
ε_{in}	emissivity coefficient of inner radiation source;
ε_{at}	emissivity coefficient of outer (atmosphere) radiation source;
ε_{pl}	emissivity coefficient of the plate;
ε_{re}	real part of complex dielectrical constant;
ε_{λ}	energy of a photon with wavelength λ;
θ	polar angle measured from normal of surface, °;
θ_{at}	polar angle of radiation from environment, °;
θ_{in}	upper limit of polar angle of inner radiation source, °;
φ	azimuth angle, °;
λ	wavelength in vacuum, μm;
σ	Stefan–Boltzmann constant, $\sigma = 5.67 \times 10^{-8}$ W/m²/K^4

1. Introduction

Within thermal engineering society, the topic of thermal radiation is not well integrated. Its energetic contribution to heat transfer apparatus is well known, but in most cases the calculation procedure is quite involved [1]. The dependency of radiation fluxes from the solid angle as well as from wavelength makes the radiative transfer equation somewhat complicated and the material property research on emissivities, for example, is somewhat demanding. Application within the field of solar energy conversion, high temperature heat transfer and combustion engineering has kept research in thermal radiation alive, but still there are deficiencies. This is especially true for the thermodynamic basics of topics in radiation energy transfer and conversion. Even though solar radiation is an incoming energy flux which is free of cost, the efficiency of solar energy conversion, for example, is of interest. Photovoltaic cells as well as thermal solar power plants have a high material expenditure, which can be reduced only if the plant conversion efficiency is increased. The study of the basic mechanisms which influence the entropy generation upon reflection, absorption and transmission of thermal radiation interacting with technical surfaces can help to increase the radiation conversion efficiency. Some research efforts have been devoted to the entropy generation in atmospheric physics, see e.g., [2]; less is known about the entropy generation on radiation-surface interactions. Even though the validity of a general extremum principle on behalf of the entropy production is discussed controversially in literature [3] and outcomes are not clear at all in the moment, the calculation of entropy has to be clear to begin with. As a first step on the way to benefit from a possible entropy production extremum principle [4,5], the paper will recall the calculation procedure for arbitrary radiation entropy and derive the entropy production rate on the occasion of reflection of radiation on a diffuse grey surface. It is shown that already this simple situation with simplifying assumptions gets quite involved on behalf of the role of the parameters of influence. Only after gaining a thorough understanding of the entropy production during basic radiation situations like reflection, absorption and transmission, more complex situations in radiative transfer shall be tackled.

2. Radiation Entropy

The calculation and handling of radiation entropy seems to be well known. This is certainly true for isotropic equilibrium radiation in an isothermal cavity (Hohlraum). But leaving this theoretical solid island to more practical nonequilibrium radiation fluxes with spectral and directional dependencies, the situation is less clear. Before entropy generation problems involving radiation fluxes are addressed and before entropy generation minimization is used as a design tool, the problems and possible uncertainties in calculating the entropy fluxes of arbitrary radiation are analysed in this chapter.

2.1. Equilibrium Situation

The volume specific spectral energy of cavity radiation as given in Equation (1) is the product of the number of photons N_λ^{eq} in the wavelength interval $\lambda, \lambda + \mathrm{d}\lambda$ and the energy of one such photon, ε_λ [6]:

$$u_\lambda^{eq}(\lambda,T) = \frac{U_\lambda^{eq}(\lambda,T)}{V} = N_\lambda^{eq} \cdot \varepsilon_\lambda = 8 \cdot \pi \cdot \frac{1}{\lambda^4} \cdot \frac{1}{\exp\left[\dfrac{hc}{k\lambda T}\right] - 1} \cdot \frac{h \cdot c}{\lambda} \tag{1}$$

The number of photons is a product of the volumetric density of states, which reads $2 \cdot 4\pi / \lambda^4$ and the average occupation of those states given by the Bose–Einstein statistics, which is $(\exp[hc/k\lambda T] - 1)^{-1}$.

The factor of 2 in the density of states takes account of the two possible planes of polarization. As the density of states used in the Planck formula Equation (1) is an integral approximation, the Planck formula holds true only if $V/\lambda^3 >> 1$. For extremely small cavities or very low temperatures a summation of the quantum states has to be performed instead of an integration [7].

The radiation within a cavity originates, as in all other situations, from the material of the cavity surface. All matter with $T > 0$ K radiates. The reason for cavity radiation having a characteristic spectrum which does not show the specific fingerprint of the boundary material is the entropy of the closed radiation system, which takes a maximum value in the equilibrium situation. To calculate this radiation entropy the fundamental thermodynamic equation:

$$\mathrm{d}U = T \cdot \mathrm{d}S - p \cdot \mathrm{d}V \tag{2}$$

is used [8]. If the volume V is constant $(\mathrm{d}V = 0)$, derivation of the volumetric version of Equation (2) on behalf of the wavelength λ gives the differential of the spectral radiation entropy:

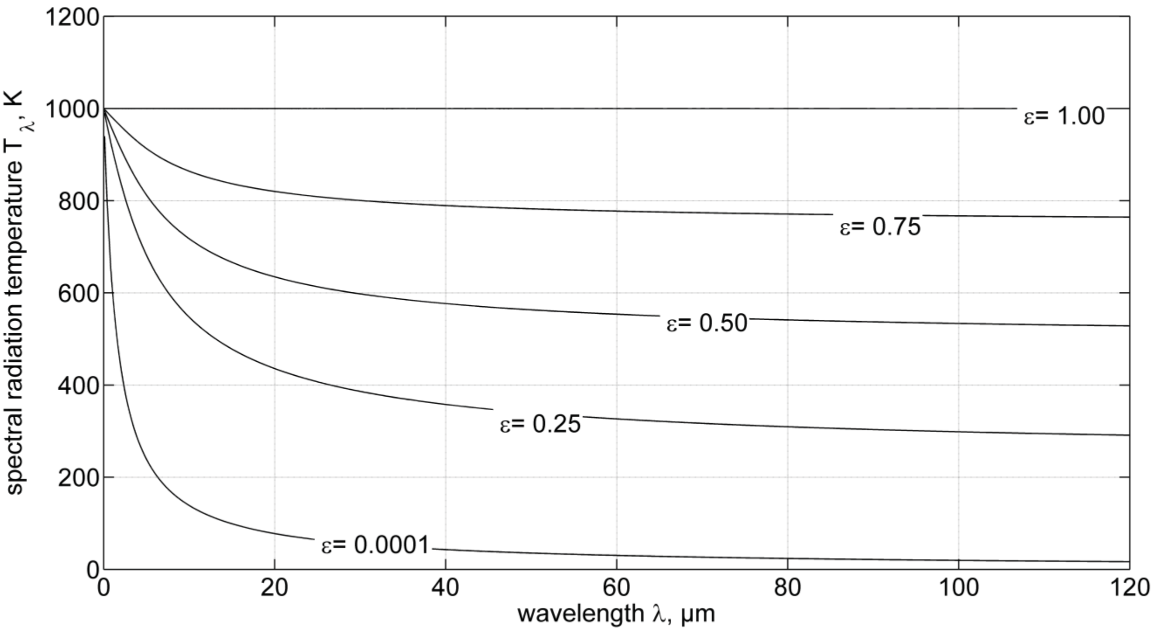
$$\mathrm{d}u_\lambda = \frac{\partial(\mathrm{d}u)}{\partial\lambda} = \left(\frac{\partial T}{\partial\lambda}\right) \cdot \mathrm{d}s + T\frac{\partial(\mathrm{d}s)}{\partial\lambda} = T^{eq}\mathrm{d}s_\lambda^{eq} \tag{3}$$

The temperature showing up in Equation (2) is the thermodynamic absolute temperature of the system under consideration, the "gas of photons" in this case. It is the temperature of the system in the moment in which there is an infinitely small change of its energy content as described by Equation (2). If an "equation of state" is known for this system as is true for the equilibrium photon gas, Equation (1), the temperature can be given as:

$$T_\lambda = \frac{h \cdot c}{k \cdot \lambda \cdot \ln\left[8\pi hc / \lambda^5 u_\lambda + 1\right]} \tag{4}$$

where T_λ is a spectral temperature valid for radiation in the wavelength interval $\lambda, \lambda + \mathrm{d}\lambda$. For equilibrium radiation in a cavity, Equation (4) gives, fortunately, one uniform temperature only for all wavelengths, *i.e.*, there is a special thermal equilibrium situation were $T_\lambda = \mathrm{const} = T^{eq}$. This is no surprise, as Equation (1) holds for equilibrium radiation only. So one basic question is whether Equation (4) can be used for non-equilibrium situations as well. The spectral temperature T_λ is shown as a function of wavelength λ for different non-equilibrium situations in Figure 1.

Figure 1. The dependency of the spectral radiation temperature T_λ on wavelength λ for grey body radiation. The parameter is the emissivity $\varepsilon = const$, the grey body temperature is $T = 1000\ K$.



A non-equilibrium situation is introduced by means of a dilution factor ε in the density of states for photons, leading to the model of grey radiation, as will be discussed later. It is seen that grey radiation is no longer isothermal. Equation (3) can be integrated to give the volume specific spectral entropy of cavity radiation as:

$$s_\lambda = \int \frac{1}{T} \mathrm{d}u_\lambda = \frac{8\pi k}{\lambda^4} \int \ln\left(1 + \frac{1}{x}\right) \mathrm{d}x = \frac{8\pi k}{\lambda^4} \left[(1+x)\ln(1+x) - x \cdot \ln x\right] \tag{5}$$

Here the substitution $x := [\exp(hc/k\lambda T) - 1]^{-1}$ has been used, which is the average occupation number of the photon state in equilibrium. Integrating Equation (5) over wavelength gives:

$$s^{eq} = \frac{4}{3}\frac{u^{eq}(T)}{T} \tag{6}$$

which is the same as if Equation (2) would have been used not for spectral values but for the Stefan–Boltzmann equation directly:

$$s^{eq} = \int \frac{\mathrm{d}u}{T} = \frac{4}{3} \cdot \frac{8}{\lambda^5} \frac{\pi^5 \cdot k^4}{h^3 c^3} \cdot T^3 = \frac{4}{3}\frac{u^{eq}(T)}{T} .$$

Thus cavity radiation, a photon gas in equilibrium, is all consistent with (classical) thermodynamics. The thermodynamic temperature T plays a major role in this derivation, so special care has to be taken if there is no equilibrium situation any more, *i.e.*, if $T^{eq} \neq T_\lambda$.

2.2. Fluxes

Problems arise when the theoretically safe and well founded equilibrium status is abandoned and open flux situations are considered. The basic and very central idea here is the definition of a black body, which tries to freeze some results known from cavity radiation and transfer these results to non-equilibrium situations. So a black body of temperature T has, by definition, the same radiation energy spectrum as an isothermal cavity at this same temperature. As the radiation emitted from an arbitrary surface is a result of quantum processes regarding adjustments in the occupation of energy levels which are very specific to individual species of molecules, there is no reason why a real body should radiate like a blackbody. The radiation spectrum is not automatically supplemented to give the characteristic cavity radiation equilibrium spectrum, there is no entropy maximization. The radiation emitted is a fingerprint of these oscillators. The radiation becomes anisotropic *i.e.*, directional, thus there is now an additional variable, the solid angle Ω. The spectral radiation energy intensity L_λ is analogous to the volume specific spectral energy of cavity radiation u_λ^{eq}, they are connected by Equation (7):

$$L_\lambda = \frac{\mathrm{d}U}{\mathrm{d}\lambda \cdot \mathrm{d}\Omega \cdot \cos\theta \cdot dt \cdot \mathrm{d}A} = \frac{c}{4\pi} u_\lambda \tag{7}$$

The speed of radiation in vacuum, c, takes care of changing a volume to an area specific flux, the solid angle of an unit sphere, 4π, takes care in changing an isotropic value to a directional value. The derivation is given in [9]. The cosine of the polar angle θ seen in Equation (7) approaches unity for radiation normal to a surface. The spectral directional entropy accompanying the ray of radiation is defined in analogy to Equation (7):

$$K_\lambda = \frac{\mathrm{d}S^{rad}}{\mathrm{d}\lambda \cdot \mathrm{d}\Omega \cdot \cos\theta \cdot \mathrm{d}t \cdot \mathrm{d}A} = \frac{c}{4\pi} s_\lambda \tag{8}$$

The corresponding spectral and integral values for the special case of a black body are:

$$L_\lambda^b = \frac{2hc^2}{\lambda^5} \frac{1}{\exp[hc/k\lambda T]-1}\ ;\ L^b = \frac{\sigma}{\pi} T^4 \tag{9}$$

$$K_\lambda^b = \frac{2kc}{\lambda^4}\left[(1+x)\ln(1+x) - x\ln x\right];\ K^b = \frac{4}{3}\frac{\sigma}{\pi} T^3 \tag{10}$$

with $x = \dfrac{L_\lambda \cdot \lambda^5}{2hc^2} = \dfrac{1}{\exp\left[hc/k\lambda T\right]-1}$ and $\sigma = \dfrac{2\pi^5 k^4}{15c^2 h^3}$.

The black body spectrum is the only thermal radiation spectrum which can be calculated. All spectra radiated from real bodies are modifications from Equation (9) using additional factors like the spectral directional emissivity.

In a nice and valuable recent contribution, Feistel [10] has shown that the entropy flux derived from Equation (10) for a black body is perfectly consistent with the Second Law of Thermodynamics. The radiation entropy flux as an area specific value is designated as D and is retrieved by integrating Equation (10) over the hemispheric solid angle:

$$D^b = \int_\Omega K^b \cdot \cos\theta \cdot \mathrm{d}\Omega = \frac{4}{3}\sigma T^3 = \frac{4}{3}\frac{E^b}{T} \tag{11}$$

where $E^b = \int_\Omega L^b \cos\theta\, \mathrm{d}\Omega = \sigma T^4$.

It should be kept in mind that even if black body radiation is assumed for two bodies having different temperatures, there is a non-equilibrium situation resulting in a net energy flux between these bodies appearing as an anisotropic photon gas. Feistel suggests a formal flux temperature T_s associated for example with the radiation exchange between a plan parallel pair of black body plates having the temperature T_{hot} and T_{cold} respectively. T_s is defined as:

$$T_s = \frac{3}{4}\frac{\left(T_{hot}^4 - T_{cold}^4\right)}{\left(T_{hot}^3 - T_{cold}^3\right)} \text{ with } T_{cold} < T_s < T_{hot} \tag{12}$$

This is also known as a contact temperature of the stationary radiation field between a pair of black bodies. Using this contact temperature allows to keep the universal bilinear form seen in all entropy production formulas of generalized fluxes times generalized forces. It will be used later in the context of entropy generation minimization.

Changing from the black body assumption to real body radiation profiles, one has to devote more attention to the temperatures. For arbitrary radiation spectra it is generally assumed that each spectral ray behaves thermodynamically independent as an individual phase. This is because the photons, once they leave the emitting surface, do not interact. This is not true for the surface molecules from where the photons originate. Each spectral directional radiation energy intensity $L_\lambda(\lambda,\Omega)$ can be assigned to have an individual temperature:

$$T_\lambda = \frac{hc}{k\cdot\lambda\cdot\ln\left[2hc^2/\lambda^5\cdot L_\lambda\right]+1} \tag{13}$$

and a radiation entropy intensity K_λ calculated as:

$$K_\lambda = \frac{2kc}{\lambda^4}\left[\left(1+\frac{\lambda^5\cdot L_\lambda}{2hc^2}\right)\ln\left(1+\frac{\lambda^5\cdot L_\lambda}{2hc^2}\right)-\frac{\lambda^5\cdot L_\lambda}{2hc^2}\ln\frac{\lambda^5\cdot L_\lambda}{2hc^2}\right] \tag{14}$$

for unpolarized radiation. For polarized or partially polarized radiation additional constraints for the radiation entropy has to be taken into account, see for example [11], but only unpolarized radiation shall be considered here. The temperature given in Equation (13) is the equivalent black body temperature for this given spectral intensity L_λ as the Planck equation (1) is used.

Please note that $T_\lambda \neq (\partial K_\lambda / \partial L_\lambda)$, also the term $(\partial T / \partial \lambda)$ appearing in Equation (3) will not be equal to zero for a real radiating flux from real bodies. Thus there is still some doubt on how to calculate thermodynamically consistent radiation entropy fluxes, because if the temperature is not a thermodynamic safe variable the fundamental thermodynamic formula, Equation (2), may not be correct. In this paper, the calculation procedure for the overall radiation energy flux E and radiation entropy flux D:

$$E = \int_\lambda \int_\Omega L_\lambda \cos\theta \cdot \mathrm{d}\Omega \mathrm{d}\lambda \tag{15}$$

$$D = \iint K_\lambda \cos\theta \cdot \mathrm{d}\Omega \mathrm{d}\lambda \tag{16}$$

will be used with the simplifying assumption that the radiation is unpolarized.

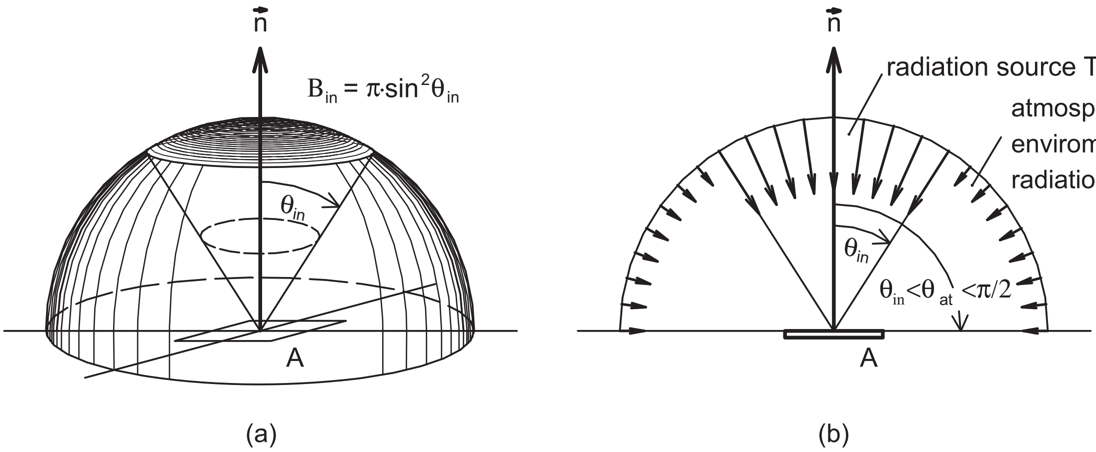
Other impact on designing the radiation entropy behaviour of a surface will come from the surface plasmon theory, which shows how to create partial coherent thermal radiation. When a surface is microstructured in a specific way, the normaly unseen surface waves can interfere with the far field waves which are responsible for the technical radiative emission of a body. The surface waves show up for materials for which the real part of the complex dielectric constant obeys $\varepsilon_{re} < -1$ [12]. These surface waves travel in the surface plane of a body and are dampened to zero in less than one wavelength distance away from the surface plane. Thus in a standard radiation situation these surface waves do not play any role. For specific surface materials and a specific surface structure however, the surface waves interfere with the electromagnetic waves leaving the surface in normal direction. This interference can leave a fingerprint on the macroscopic emission spectrum of this surface in both directional and spectral behaviour [13]. If these electrophysical mechanisms are understood to a larger extent the specific design of surface properties to achieve a specific entropy characteristic comes into realm. This could lead to designer type surfaces for example in solar energy conversion to give favourable energy conversion results [14].

3. Entropy Production upon Absorption and Emission of Radiation

In this section the entropy production rate $\dot{S}_{irr}$ will be analysed for simple absorption and emission situations with radiating surfaces involved. Absorption and emission are basic steps in all solar energy conversion situations and many other technical applications as well. Thus it is of interest to understand the mechanism of entropy generation when radiation is reflected, absorbed or transmitted.

A standard geometric situation will be analysed. The simple setup is a grey body with a flat surface A looking into a hemispheric environment, see Figure 2. The hemisphere is divided into two concentric spherical segments, one for representing the incoming radiation from some hot radiating source (the sun for example), the other segment representing the ambient. The radiation source segment is concentric to the normal vector $\vec{n}$ of the surface A and it is characterized by a polar (opening) angle θ_{in}, by a source temperature T_{in} and by a constant emissivity ε_{in}. The second segment around the first one is considered as something like an atmosphere environment having a temperature T_{at} and a polar angle range $\theta_{in} < \theta_{at} < \pi/2$. It is considered to be unpolarized grey body radiation homogeneously distributed across the spherical segment shown shaded in Figure 2. The concentric arrangement for this incoming radiation is chosen because of its simplicity, the radiation situation is shown again in Figure 2b for clarity.

Figure 2. (**a**) The geometric setting for the (small) plate A used in the example calculations. Shown are the two concentric regions of the hemisphere irradiating the plate. (**b**) Cross section of Fig. 2a to clarify the situation under consideration. The plate area A is infinitesimally small as compared to the hemisphere envelope.



The solid angle Ω can be combined with the cosine of θ in Equation (6) to give a simple single geometry parameter $B = B(\theta)$, which allows to decompose the integration procedure:

$$B(\theta) = \int_0^{\theta}\int_0^{2\pi} \cos(\theta)\cdot\sin(\theta)\cdot \mathrm{d}\theta\cdot\mathrm{d}\varphi = \pi\cdot\sin^2(\theta) \tag{17}$$

For the special case of a whole homogeneous hemisphere $\theta = \pi/2$ we get $B = \pi$.

For the more general case $\theta \le \pi/2$ the incoming grey radiation energy fluxes can be calculated as:

$$E = \int_0^{\infty}\int_0^{2\pi}\int_0^{\theta} L_\lambda\cdot\cos(\theta)\cdot\sin(\theta)\cdot\mathrm{d}\theta\cdot\mathrm{d}\varphi\cdot\mathrm{d}\lambda = B(\theta)\cdot\int_\lambda L_\lambda \mathrm{d}\lambda = \frac{B(\theta)}{\pi}\varepsilon\sigma T^4 \tag{18}$$

The entropy formula for grey body radiation is given by Landsberg and Tonge [15] as:

$$D = \int_0^{\infty}\int_0^{2\pi}\int_0^{\theta} K_\lambda\cdot\cos(\theta)\cdot\sin(\theta)\cdot\mathrm{d}\theta\cdot\mathrm{d}\varphi\cdot\mathrm{d}\lambda = B(\theta)\cdot\int_\lambda K_\lambda \mathrm{d}\lambda = \frac{4}{3}\frac{B(\theta)}{\pi}\varepsilon X(\varepsilon)\sigma T^3 \tag{19}$$

with the grey body entropy function (dilution function):

$$X(\varepsilon) = \frac{45}{4\pi^4}\frac{1}{\varepsilon}\int z^2\left[(1+x)\ln(1+x) - x\ln x\right]\cdot\mathrm{d}z \tag{20}$$

and $x := \dfrac{\varepsilon}{\exp(z)-1}$ and $z := \dfrac{hc}{k\lambda T}$.

An approximation of this function (20) for an error less than 2% value of reads $X(\varepsilon) \approx 0,965157 + 0,2776566 \ln\left(\dfrac{1}{\varepsilon}\right) + 0,05115\varepsilon$.

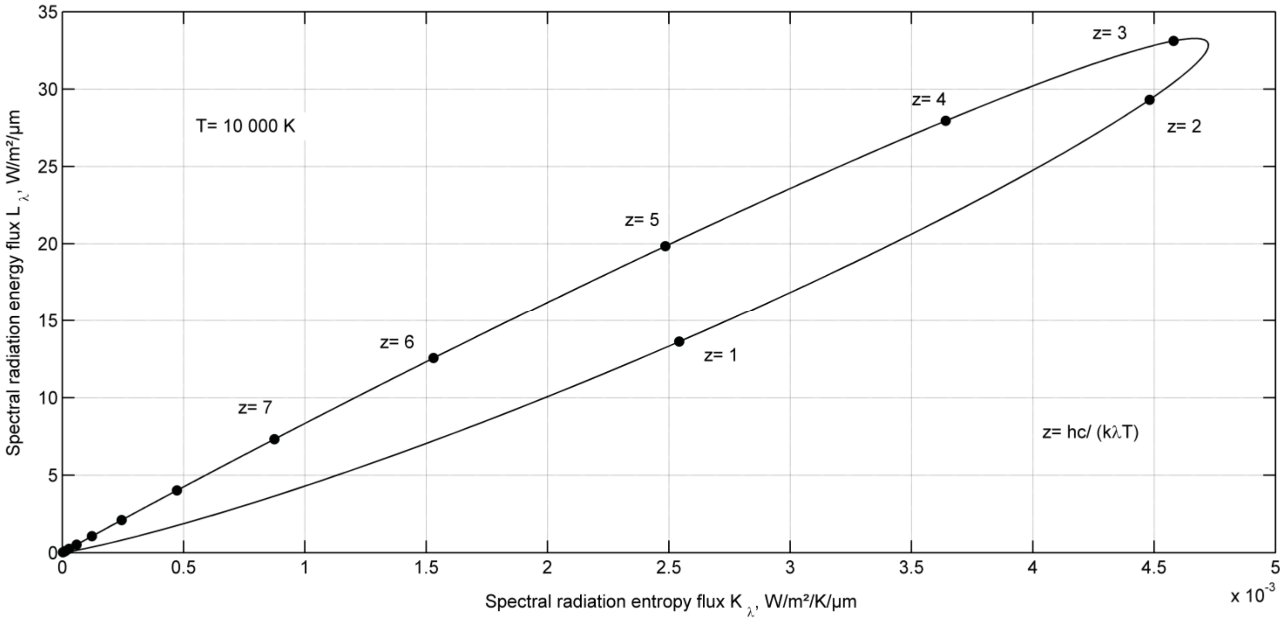
Landsberg and Tonge introduce this type of radiation as "diluted black body" radiation, because the factor ε dilutes the occupation number N_λ in the Bose–Einstein statistics for photons, see Equation (1).

Using a transformation $L_\lambda \mathrm{d}\lambda = L_z \mathrm{dz}$ and $K_\lambda \mathrm{d}\lambda = K_z \mathrm{dz}$, an interesting relation between the spectral radiation intensity L_λ and the spectral radiation entropy intensity K_λ for grey body radiation given by Equation (14) is shown in Figure 3 for different values of z. This relation reads:

$$K_z = \frac{L_z}{T} + z^2 \cdot T^3 \cdot \ln(1+x) \cdot \frac{2k^4}{c^2 h^3}$$

and it shows that from a thermodynamic viewpoint arbitrary radiation is not a heat flux, nor is it corresponding to Equation (11).

Figure 3. A graphic presentation of the relation $L_\lambda = L_\lambda(K_\lambda)$ using the variable $z = hc/(k\lambda T)$.

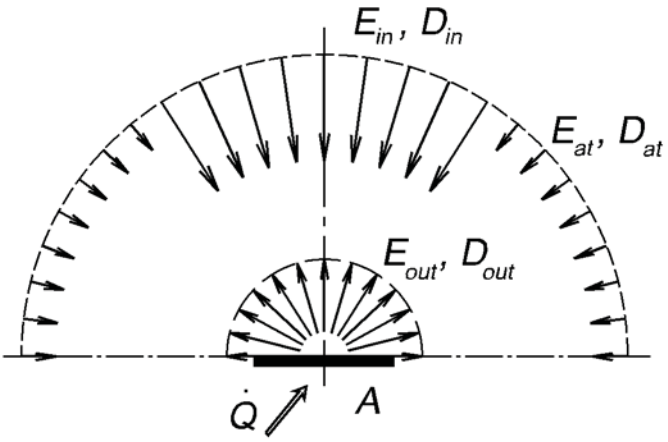


Using the grey body approximation for both the plate with surface area A and the incoming radiation, the entropy production rate for the plate due to the radiation interaction will be calculated.

The energy balance equation for the plate which has an emissivity ε_{pl} in a stationary state, see Figure 4, reads:

$$0 = A \cdot \left[E_{in} + E_{at} + \frac{\dot{Q}}{A} - E_{pl} - E_{refl,in} - E_{refl,at} \right]$$

Figure 4. The setting for the energy and entropy balance equation. The system boundary is around the plate A, the outgoing radiation energy is $E_{out} = E_{pl} + E_{refl,in} + E_{refl,at}$.



The energy radiation fluxes E for the grey body assumption are given by Equation (18):

$$\frac{\dot{Q}}{A\sigma} = \varepsilon_{pl} T_{pl}^4 + \frac{B_{in}}{\pi}\left(1-\varepsilon_{pl}\right)\varepsilon_{in} T_{in}^4 + \frac{\pi - B_{in}}{\pi}\left(1-\varepsilon_{pl}\right)\varepsilon_{at} T_{at}^4 - \frac{B_{in}}{\pi}\varepsilon_{in} T_{in}^4 - \frac{\pi - B_{in}}{\pi}\varepsilon_{at} T_{at}^4 .$$

Adiabatic Plate

Assuming the grey plate on the ground to be adiabatic $(\dot{Q} \overset{!}{=} 0)$ for the first round and letting the environmental atmosphere be black ($\varepsilon_{at} = 1$) we get $0 = \varepsilon_{pl} T_{pl}^4 - \varepsilon_{pl} \cdot \varepsilon_{in} \frac{B_{in}}{\pi} T_{in}^4 - \varepsilon_{pl} \frac{\pi - B_{in}}{\pi} T_{at}^4$.

We also assume that the reflection of the incoming radiation from the plate is diffuse. With these settings the temperature T_{pl} of the plate can be calculated from the energy balance equation.

The entropy balance equation reads accordingly with the same simplifying assumptions (adiabatic grey plate, black atmosphere in the outer spheric segment):

$$0 = A \cdot \left[D_{in} + D_{at} + D_{irr} - D_{out}\right] \tag{21}$$

Inserting the entropy radiation fluxes for the grey body assumption from Equation (19) we get, using the index pl for the outgoing radiation flux:

$$0 = D_{out} - \frac{4}{3}\frac{B_{in}}{\pi}\varepsilon_{in} X(\varepsilon_{in})\sigma T_{in}^3 - \frac{4}{3}\frac{\pi - B_{in}}{\pi}\sigma T_{at}^3 - D_{irr}$$

or:

$$D_{irr} = D_{out} - \frac{4}{3}\sigma\left[\frac{B_{in}}{\pi}\varepsilon_{in} X(\varepsilon_{in}) T_{in}^3 + \frac{\pi - B_{in}}{\pi} T_{at}^3\right] \overset{!}{>} 0 \tag{22}$$

with $D_{out} = \iint K_\lambda\left(L_\lambda^{out}\right)\cos\theta \cdot \mathrm{d}\Omega \mathrm{d}\lambda$ using the outgoing intensity L_λ^{out}:

$$L_\lambda^{out} = \varepsilon_{pl} L_\lambda^b\left(T_{pl}\right) + \left(1-\varepsilon_{pl}\right)\left[\varepsilon_{in}\frac{B_{in}}{\pi} L_\lambda^b\left(T_{in}\right) + \varepsilon_{at}\frac{\pi - B_{in}}{\pi} L_\lambda^b\left(T_{at}\right)\right]$$

The entropy of the two incoming radiation fluxes D_{in} and D_{at} are additive, as they are not correlated to any extent. The entropy of the overall outgoing radiation flux from the plate into the hemisphere contains a reflection part and an own emission part. These must be calculated in one step, as its constituents are not additive. This is seen when the radiation energy fluxes leaving the plate are regarded, the addition of the own emission of the plate and the reflection does not result in grey body radiation any more. The radiation entropy flux leaving the plate has to be calculated by using Equations (10) and (11).

Figure 5 shows the spectral radiation *energy* fluxes $E_\lambda = B(\theta)\cdot L_\lambda$ involved for the special case of an adiabatic plate ($\dot{Q}$ = 0). For this specific incoming radiation situation (T_{in} = 1000 K confined to $B_{in} = 0.01\cdot\pi$), the rest of the hemisphere at T_{at} = 300 K) the plate temperature adjusts to T_{pl} = 371 K. Because the (shortwave) reflected part adds to the (longwave) own emission, the outgoing overall radiation energy flux from the plate is not grey anymore.

Figure 5. The spectral radiation energy flux $E_\lambda = B(\theta)\cdot L_\lambda$ as a function of wavelength λ for one specific, but arbitrary setting. The incoming radiation flux can be extrapolated from the shown reflected radiation.

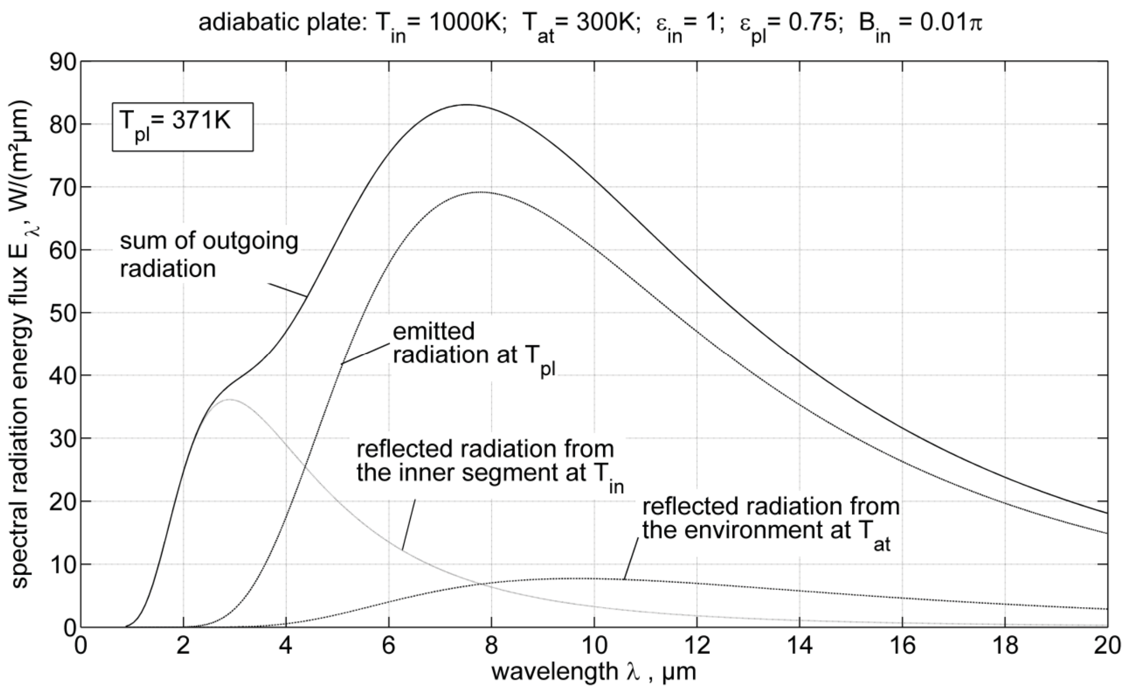
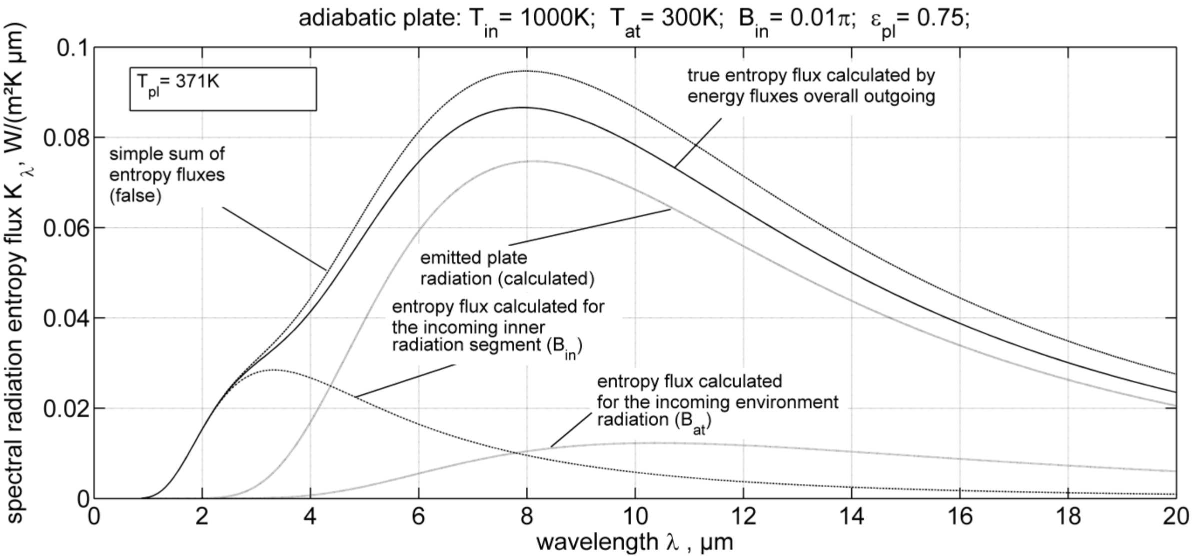


Figure 6 shows the corresponding radiation entropy fluxes $D_\lambda = B(\theta)\cdot K_\lambda$. The simple addition of these outgoing entropy fluxes, which is shown by the dotted line in Figure 6, is erroneous, as both result from the same surface molecules and are thus correlated. If the plate is assumed to be specular reflecting instead of diffusive, the emitted radiation entropy would be lower. Reflection from a diffusive surface produces more entropy as from a specular surface, as the incoming radiation is diluted into the whole hemisphere. As a result of these radiation energy and radiation entropy fluxes upon emission and reflection from the plate there is an irreversible entropy production which will be discussed below.

Figure 6. The spectral radiation entropy flux $D_\lambda = B(\theta) \cdot K_\lambda$ as a function of wavelength, corresponding to Figure 5. The true outgoing entropy flux is less than the simple addition of the flux components.



The entropy production for an *adiabatic* plate is calculated by Equation (22). Results are shown in Figure 7 and in Figure 8 as a function of the plate emissivity ε_{pl} and as a function of the geometric solid angle parameter B_{in} of the incoming radiation, which is given in Equation (17).

Figure 7. The entropy production rate D_{irr} for an adiabatic plate shown as a function of the geometry parameter B_{in} and the emissivity of the plate ε_{pl}. The incoming radiation is black.

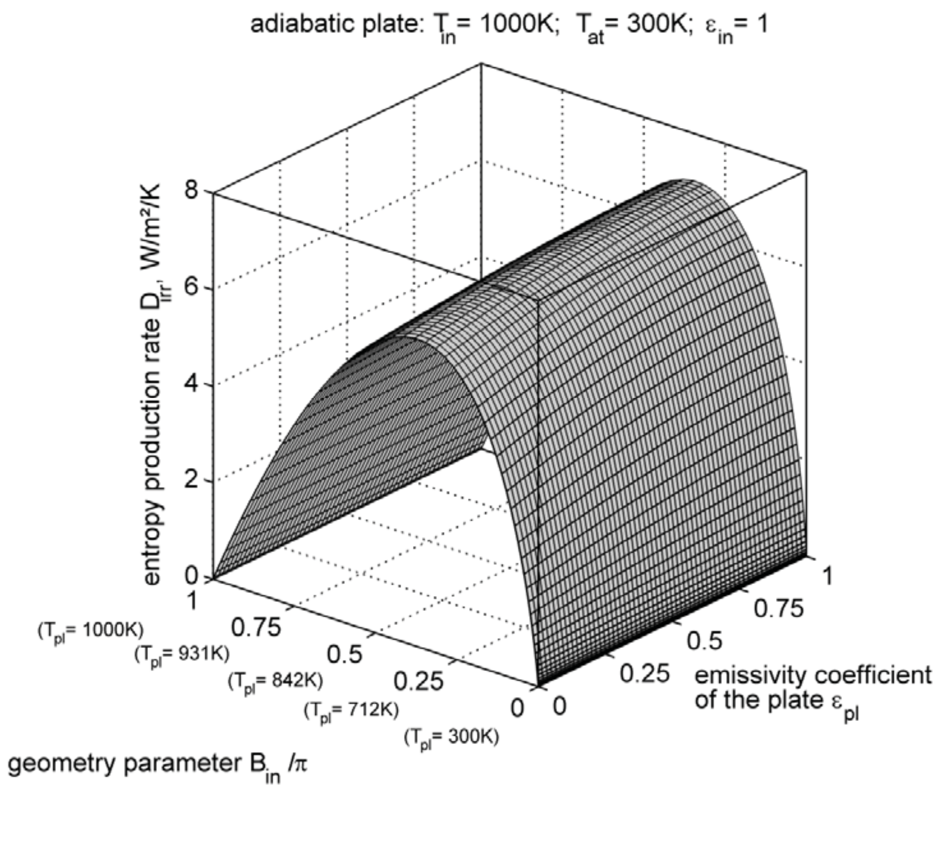
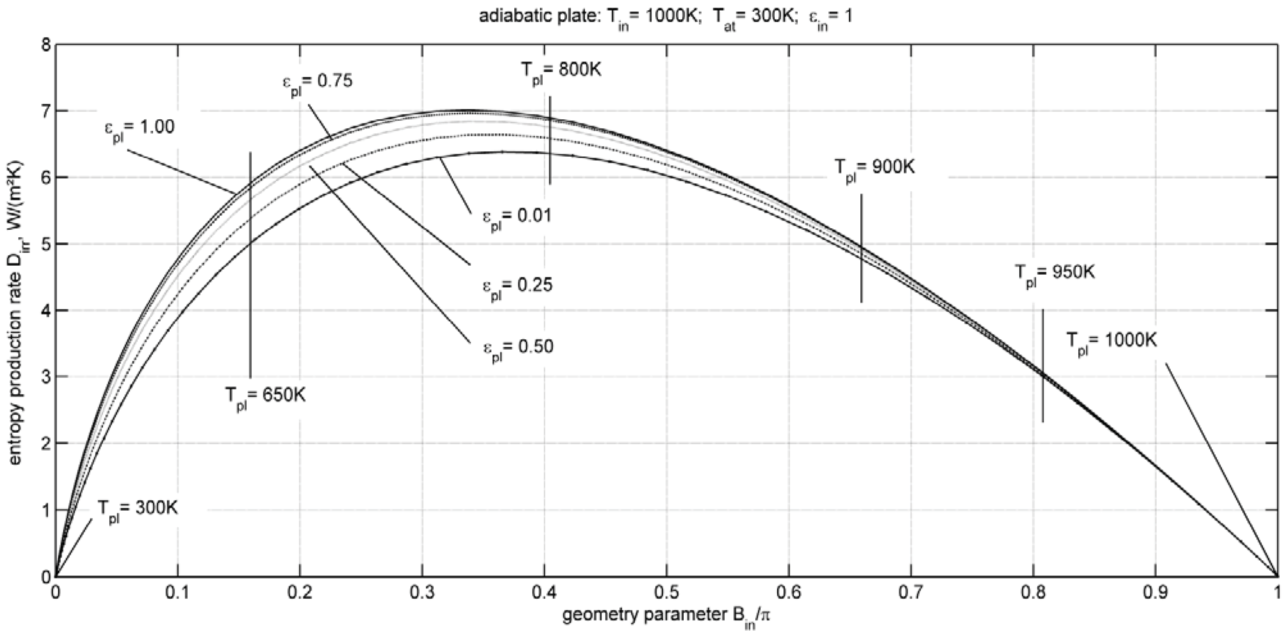


Figure 8. Cross sections of the 3-dim. plot in Figure 7. Some equilibrium temperatures of the plate are given in addition.



So first a grey adiabatic plate will be analysed for diffuse reflection. Figures 7 and 8 have the following setting:

The incoming radiation comes from two regions of the hemisphere as described before. The central hot radiation source is black radiation with a temperature of T_{in} = 1000 K, the surrounding atmospheric radiation is black at T_{at} = 300 K. The polar angle for the radiation source, expressed by the geometry function B_{in}, is one variable varying from $0 < B_{in} < \pi$.

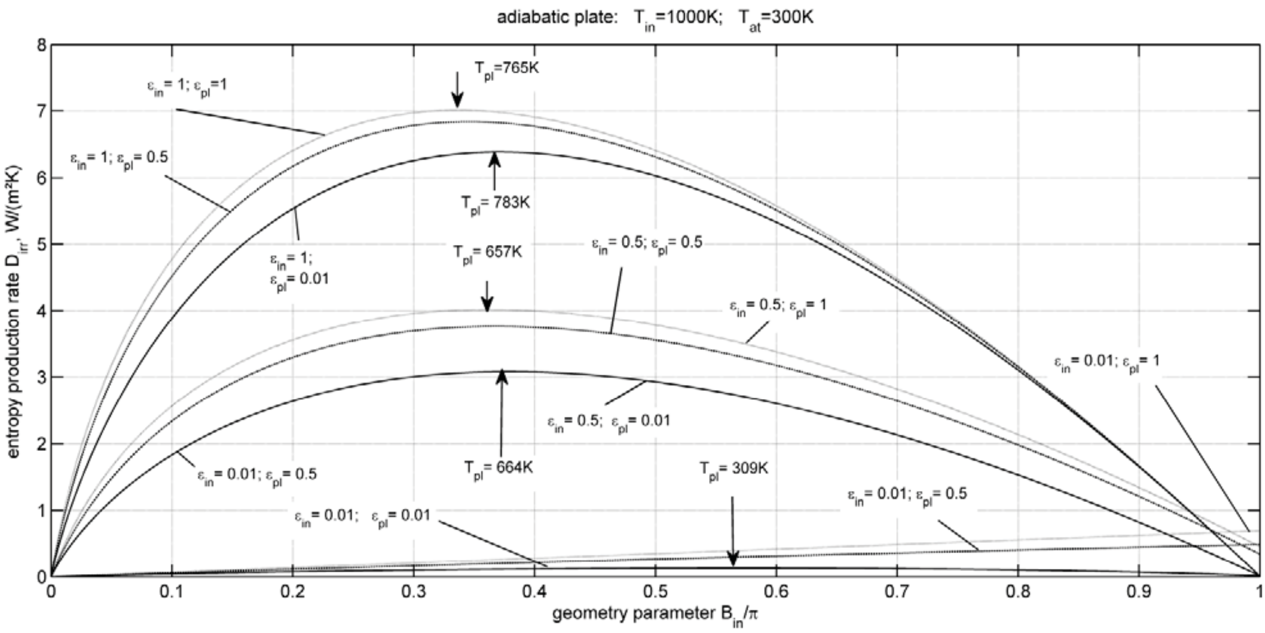
The plate is a grey radiator, thus showing a diffuse reflection. The overall outgoing radiation leaving the plate is non-grey because of the addition of the reflection part to the own (grey) emission.

The emissivity of the plate is the second variable in those figures, it varies between $0.01 < \varepsilon_{pl} < 1.0$. It is shown in Figure 7 that the polar angle of the incoming radiation has a pronounced effect on the entropy production rate. The emissivity has almost no influence. The analysis of the entropy production for this simple case hopefully leads to an understanding of the physical basics in radiation entropy production. The maximum values in Figure 7 result when the temperature difference between incoming radiation and plate as well as the difference between atmospheric temperature and plate are largest. The temperature of the (adiabatic) plate T_{pl} is given for some different geometry values B_{in} in Figure 8, which is a cross section of Figure 7 at 5 different values for ε_{pl}. The entropy production rate is zero for $B_{in} = 0$ and for $B_{in} = \pi$, because then there is a "Hohlraum" cavity situation. The maximum in the entropy production rate is reached when the plate temperature is closest to a "flux temperature" as adapted from Equation (12):

$$T_{pl} = \lim_{E_{pl} \to E_{in} + E_{at}} \left\{ \frac{(E_{in} + E_{at}) - E_{pl}}{(D_{in} + D_{at}) - D_{out}} \right\}$$

As the plate is diffuse reflecting in this case the geometry of the rays (the parallelism) is distorted (diluted) in the greatest possible way. More specular reflecting plates will give less entropy production. A similar story is true for the dependency of the entropy production in an adiabatic plate on the spectral distribution of the incoming radiation. If the incoming radiation would be quasi-monochromatic laser radiation and the plate would still be grey the entropy production is larger as compared to the case of incoming grey radiation, *i.e.*, a matching between the incoming and outgoing type of radiation has to be done for entropy production minimization. The entropy production rate for cases where both the incoming radiation and the plate radiation is grey body radiation is shown in Figure 9.

Figure 9. Extension of Figure 7 to cases were ε_{in} varies. The plate temperature at some maximum points is shown.



It is seen again that the solid angle function B_{in} has a much larger influence on the entropy production than the emissivity of the plate ε_{pl}. When the solid angle function B_{in} of the radiation source approaches $B_{in} \to 0$ or $B_{in} \to \pi$, the entropy production rate becomes zero as this again is the cavity radiation situation. The entropy production rate increases when the temperature of the plate differs more and more from the incoming radiation temperature. If B_{in} is small, the incoming radiation is dominated by the atmospheric radiation at 300 K and the plate temperature is also close to 300 K, thus the entropy production is small. Similar arguments hold true for large B_{in}. Some plate temperatures are again assigned in Figure 9 to underline this argument. So, similar to other heat transfer situations, the temperature difference between the source and the recipient of the thermal energy is steering the entropy production.

4. Entropy Production Minimization

The aim of this analysis is to seek the minimum in entropy production for a given set of incoming radiation. If the incoming radiation is fixed, what specific feature and properties of the plate would result in the smallest possible entropy production? The adiabatic case has been analysed above for simplicity, now the more interesting cases of withdrawal or the input of heat $\dot{Q}$ into the plate shall be analyzed. The conversion of internal energy of a body into radiation energy or *vice versa* is one of the most important energy conversion situations. The properties of the plate which one could adapt for optimization are the emissivity ε_{pl} and possibly the diffuse or specular reflection property. Later also the degree of polarization could be varied, but this out of scope of this study. In the non-adiabatic case the temperature of the plate can be adjusted by governing the heat input or heat or heat withdrawal of the plate.

There are two main factors of influence on the radiation entropy flux, the spectral distribution of the radiation energy and the distribution within the solid angle. Parallel rays give less entropy than diverging rays.

The entropy production rate in the non-adiabatic case is given for the case T_{in} = 1000 K in Figure 10. There truly is a minimum in the entropy production rate n as a function of the plate parameters. There is only a small dependence of the minimum entropy production rate on the emissivity of the plate, the geometry parameter B_{in} again has more effect, as this parameter dominates the plate temperature as long as T_{in} and T_{at} are fixed.

The incoming radiation is typically fixed, *i.e.*, the parameters T_{in}, B_{in}, ε_{in} and T_{at} are not free to choose. The engineer can take influence on the emissivity of the plate ε_{pl} and on the temperature T_{pl} to minimize the entropy production rate. The temperature of the plate is varied by controlling the heat flux to or from the plate, as done in Figure 10. The equation corresponding to this Figure is derived from Equation (21), now written for the non-adiabatic case, where heat $\dot{Q}$ is supplied or with drawn by conduction:

$$D_{irr} = D_{out} - D_{in} - D_{at} - \frac{\dot{Q}}{A \cdot T_{pl}}$$

or, with the radiation entropy fluxes from Equation (19):

$$D_{irr} = D_{out} - \frac{4}{3}\sigma\left[\frac{B_{in}}{\pi}\varepsilon_{in}X(\varepsilon_{in})T_{in}^3 + \frac{\pi - B_{in}}{\pi}\varepsilon_{at}X(\varepsilon_{at})T_{at}^3\right] - \frac{\dot{Q}}{A \cdot T_{pl}}$$

The minimum entropy production rate seen in Figure 10 is evaluated by taking the derivative $\left(\partial D_{irr}/\partial T_{pl}\right) = 0$. For simplicity, all radiation fluxes are assumed to be black. This results in:

$$\frac{\partial D_{irr}}{\partial T_{pl}} = \frac{\sigma}{\pi}\left\{\pi T_{pl}^2 - \frac{1}{T_{pl}^2}\left[B_{in} \cdot T_{in}^4 + (\pi - B_{in}) \cdot T_{at}^4\right]\right\}$$

and:

$$\left(\frac{\partial D_{irr}}{\partial T_{pl}}\right) \overset{!}{=} 0 = \pi T_{pl}^4 - B_{in} \cdot T_{in}^4 - (\pi - B_{in})T_{at}^4$$

which is true for $\dot{Q} = 0$. So as to be expected all types of heat fluxes, positive or negative, which keep the plate away from its thermal equilibrium create an additional entropy production. The influence of a second parameter on the entropy production rate is analysed by taking the derivative $(\partial D_{irr} / \partial B_{in}) = 0$. This results, when again taking all radiation as black radiation, *i.e.*, taking $\varepsilon_{in} = \varepsilon_{at} = \varepsilon_{pl} = 1$:

$$\left(\frac{\partial D_{irr}}{\partial B_{in}}\right)_{T_{pl}} = \sigma\left[-\frac{4}{3}T_{in}^3 + \frac{4}{3}T_{at}^3 + \frac{T_{in}^4}{T_{pl}} - \frac{T_{at}^4}{T_{pl}}\right]$$

and:

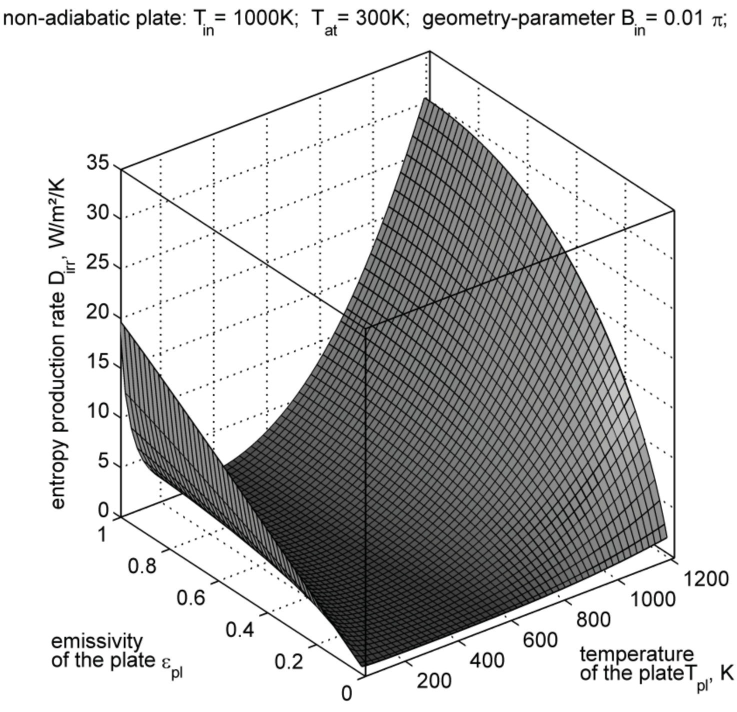
$$\left(\frac{\partial D_{irr}}{\partial B_{in}}\right)_{T_{pl}} \overset{!}{=} 0 = 4 \cdot T_{pl}\left(T_{at}^3 - T_{in}^3\right) + 3\left(T_{in}^4 - T_{at}^4\right)$$

which gives:

$$T_{pl} = \frac{T_{in}^4 - T_{at}^4}{\frac{4}{3}\left(T_{in}^3 - T_{at}^3\right)} = \frac{E_{in}(T_{in}) - E_{at}(T_{at})}{D_{in}(T_{in}) - D_{at}(T_{at})}.$$

With $T_{at} < T_{pl} < T_{in}$ this corresponds to the flux temperature in Equation (12) as suggested by Feistel [10]. The dependency of the entropy production rate on the emissivity of the plate ε_{pl} is, for a constant temperature T_{pl}, a continuously decreasing function.

Figure 10. The entropy production rate D_{irr} for a non-adiabatic plate as a function of plate emissivity ε_{pl} and plate temperature T_{pl}.



5. Conclusions

The main contributors to the entropy production in radiation applications are radiation entropy, heat flux and temperature, as these are the constituents of the entropy balance equation. The radiation entropy itself has the spectral and the directional dependency of the radiant intensity, so the characteristics of the radiation spectrum and its geometric distribution across the hemisphere are of indirect influence. The influence of the degree of polarization has not been taken into account in this paper.

The temperature difference between, for example, the incoming radiation and the surface onto which this radiation is impinging is of pronounced influence on the entropy production during absorption. The upper limit for this entropy production rate is given by heat conduction situation:

$$D_{irr,cond.} = \dot{Q}\left(\frac{1}{T_{pl}} - \frac{1}{T_{in}}\right)$$

Only black body radiation carries the same content in entropy as a heat flux $\dot{Q}$. Real radiation has less entropy, down to monochromatic laser spotlight radiation with quasi zero entropy content. Reflection of radiation itself is free of entropy production, only if directed incoming radiation is spread into a broader solid angle, entropy will be produced. This is true, for example, when laser radiation is reflected by a diffuse surface.

So the temperature of the surface is the key to minimum entropy production. When the temperature of the surface approaches the radiation temperature of the incoming radiation, the entropy production vanishes (cavity equilibrium situation). If this is not possible, the flux temperature:

$$T = \left\{\frac{3}{4}\frac{\left(T_{hot}^4 - T_{cold}^4\right)}{\left(T_{hot}^3 - T_{cold}^3\right)}\right\}$$

gives the best choice, at least in the case of black body radiation. This result will probably be extendable to arbitrary radiation by:

$$T_\lambda = \lim_{E_{out} \to E_{in}} \left\{\frac{E_{in} - E_{out}}{D_{in} - D_{out}}\right\}$$

for each wave length, but this has not been proven yet.

6. Summary

The simple case of grey incoming radiation being reflected and absorbed by a grey plate is the first step in learning about the entropy production when radiation is interacting with some material. Before calculating entropy production rates, the calculation procedure for radiation entropy must be clear. In the first part of the paper the radiation entropy calculation has been reviewed. The second part of the paper analyses the entropy production rate for a simple setting, where radiation is absorbed, reflected and emitted by a grey body surface. The entropy production rate is at an extremum when the plate is in a thermal equilibrium and attains a "flux temperature" given by $T_s = lim\{(E_{in} - E_{out})/(D_{in} - D_{out})\}$. The simple radiation interaction analysed in this paper can be extended to more complex radiation transfer situations, where the extremum principle for the entropy production rate can help to optimize radiation heat transfer problems.

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