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Article

Entropy Generation at Natural Convection in an Inclined Rectangular Cavity

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Abstract: Natural convection in an inclined rectangular cavity filled with air is numerically investigated. The cavity is heated and cooled along the active walls whereas the two other walls of the cavity are adiabatic. Entropy generation due to heat transfer and fluid friction has been determined in transient state for laminar natural convection by solving numerically: the continuity, momentum and energy equations, using a Control Volume Finite Element Method. The structure of the studied flows depends on four dimensionless parameters which are: the thermal Grashof number, the inclination angle, the irreversibility distribution ratio and the aspect ratio of the cavity. The obtained results show that entropy generation tends towards asymptotic values for lower thermal Grashof number values, whereas it takes an oscillative behavior for higher values of thermal Grashof number. Transient entropy generation increases towards a maximum value, then decreases asymptotically to a constant value that depends on aspect ratio of the enclosure. Entropy generation increases with the increase of thermal Grashof number, irreversibility distribution ratio and aspect ratio of the cavity. Bejan number is used to measure the predominance of either thermal or viscous irreversibility. At local level, irreversibility charts show that entropy generation is mainly localized on bottom corner of the left heated wall and upper corner of the right cooled wall.

Keywords: natural convection; rectangular cavity; entropy generation; numerical methods

PACS Codes: 05.70-a

Nomenclature

A	aspect ratio of the cavity
A_x; A_y	aspect ratios along x and y, respectively
g	gravitational acceleration (m·s^{-2})
Gr	thermal Grashof number
H' (L')	length (width) of the cavity (m)
J_z	dimensionless flux vector (z = u, v, T)
L^*	characteristic length (m)
P	dimensionless pressure
P'	pressure (kg·m^{-1}·s^{-2})
Pr	Prandtl number
Ra	Rayleigh number
$\dot{S}_L$	dimensionless local entropy generation
S	dimensionless total entropy generation
T	dimensionless temperature
T'	temperature (K)
T'_c	cold temperature (K)
T_h'	hot temperature (K)
T'_o	reference temperature (K)
t	dimensionless time
t'	time (s)
U^*	characteristic velocity (m·s^{-1})
V	dimensionless velocity vector
V'	velocity vector (m·s^{-1})
u, v	dimensionless velocity components
u', v'	velocity components (m·s^{-1})
x, y	dimensionless Cartesian coordinate system
x', y'	cartesian coordinates (m)

Greek Letters

α	thermal diffusivity (m^2·s^{-1})
β_T	thermal bulk expansion coefficient (K^{-1})
$\Delta T'$	temperature difference (K)
υ	kinematic viscosity (m^2·s^{-1})
μ	dynamic viscosity (kg·m^{-1}·s^{-1})
ϕ	inclination angle of the cavity (°)
φ_D	irreversibility distribution ratio

ρ fluid density ($kg \cdot m^{-3}$)
λ thermal conductivity ($J \cdot m^{-1} \cdot s^{-1} \cdot K^{-1}$)
$\dot{\sigma}_L$ volumetric local entropy generation ($J \cdot m^{-3} \cdot s^{-1} \cdot K^{-1}$)

1. Introduction

Today, thermodynamic studies are not limited to the knowledge of exchanges between work and heat. The field of investigation is being extended, namely in applications of exchanges between mechanical, chemical and electromagnetic forms of energy. Thermodynamics is not restricted to equilibrium state studies but domains near and far from equilibrium which describe thermodynamics of irreversible processes have also become of great practical importance. Thermodynamic systems submitted to thermal gradients and friction effects are subjected to energy loss, which induces entropy generation in the system. Many studies have been published concerning entropy generation. Bejan [1] investigated entropy generation phenomena by considering a small 2D element of the fluid as an open thermodynamic system submitted to mass and energy fluxes. Poulikakos and Bejan [2] were interested in rectangular flasks in laminar flow; they showed that entropy generation is proportional to work loss in the system. To reduce irreversibility origins of entropy generation, three geometrical parameters should be selected: flask length (L), its width (b) and its thickness (δ) which seems only in thermal contribution term to entropy generation. Optimal values of these parameters are calculated from correlations established by thermal flux, flow structures and flask thickness, therefore optimum length (L_{opt}) increases with (δ) but the optimum width (b_{opt}) decreases, that means the increase of Minceur ratio:

$$(\gamma_{opt} = \frac{L_{opt}}{b_{opt}})$$

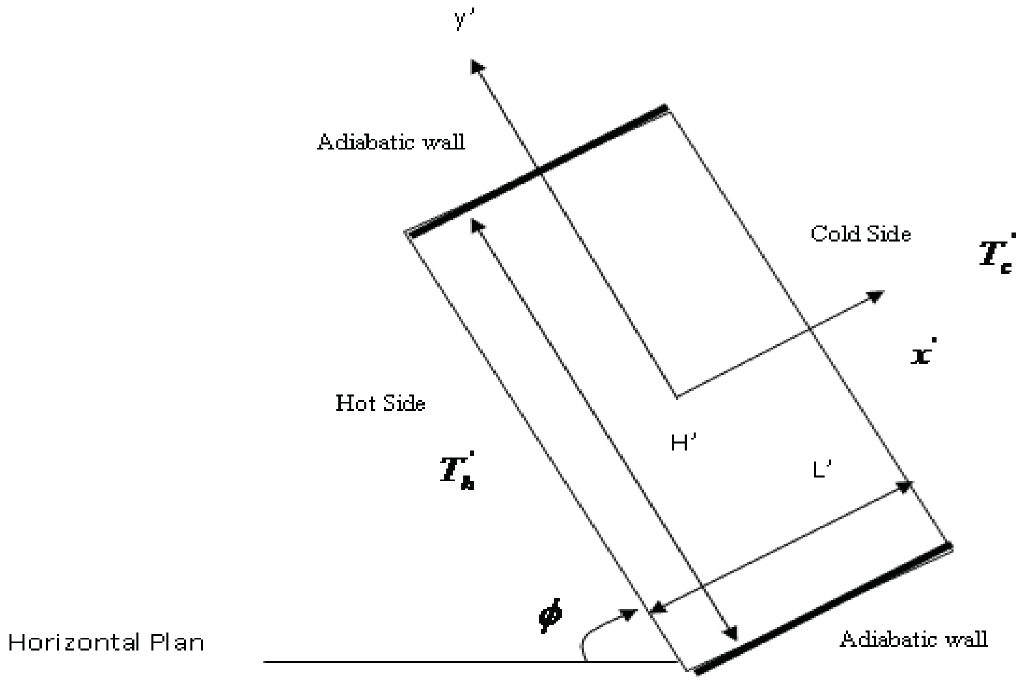
Similar results are obtained for turbulent flow by changing heat transfer coefficient (h) by friction coefficient (C_f) using correlations appropriate to heat transfer and friction in turbulent regime. An analytical study of entropy generation's problem was established by Sahin [3] who consider a viscous fluid's turbulent flow in a duct. Results show that entropy generation initially decreases then increases along the duct, it is proportional to the dimensionless temperature difference:

$$(\tau = \frac{T_w - T_0}{T_w})$$

where T_w and T_o represent wall temperature and fluid temperature input, respectively. Baytas [4,5] determined entropy generation in an inclined square cavity with two isotherm walls and two adiabatic walls. He firstly [4] determined optimum angles for which energy losses are reduced. It was shown that entropy generation decreases with the inclination angle of the cavity for low external Rayleigh number (Ra_E), maximum values are obtained for angles between 35° and 55°. Secondly, entropy generation in a porous cavity was studied [5]. It was found that entropy generation is the result of a continuous exchange of energy between fluid and enclosure's walls. Demirel and Kahraman [6] showed that irreversibility distributions are not continuous through the horizontal walls of a rectangular enclosure that it was differentially heated from its upper side. Magherbi *et al.* [7] numerically studied entropy generation at the onset of natural convection in a square cavity. They showed that entropy generation depends on thermal Rayleigh number. The effect of irreversibility distribution ratio on entropy generation was also analyzed.

This paper presents a numerical study about entropy generation in transient state for natural convection concerning an incompressible fluid enclosed in an inclined heated rectangular cavity as shown in Figure 1. This study concerns steady-unsteady states where the effects of the aspect ratio of the cavity, the Grashof number, the inclination angle and the irreversibility distribution ratio on entropy generation are investigated. The behaviours of the Bejan number and local irreversibility are also studied.

Figure 1. Schematic view of 2D inclined rectangular cavity.



2. Governing Equations

The conservative equations of continuity, momentum and energy in dimensionless form are given as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t}+div\ J_u = -\frac{\partial P}{\partial y}+ Gr{\cdot}T{\cdot}\cos\phi \tag{2}$$

$$\frac{\partial v}{\partial t}+div\ J_v = -\frac{\partial P}{\partial y}+ Gr{\cdot}T{\cdot}\sin\phi \tag{3}$$

$$\frac{\partial T}{\partial t}+ div\ J_T = 0 \tag{4}$$

with:

$$J_u = u \cdot V - \Pr \cdot grad\ (u) \tag{5}$$

$$J_v = v \cdot V - \Pr \cdot grad\ (v) \tag{6}$$

$$J_T = T \cdot V - grad\ (T) \tag{7}$$

The dimensionless used variables are defined by:

$$x = \frac{x'}{L^*};\ y = \frac{y'}{L^*};\ u = \frac{u'}{U^*};\ v = \frac{v'}{U^*};\ t = \frac{t'U^*}{L^*};\ P = \frac{P'}{\rho_0 U^{*2}};\ T = \frac{T' - T'_0}{\Delta T'};\ L^* = \text{L' if A} \geq 1$$

and:

$$L^* = \text{H' if A} < 1\ (\text{A} = \frac{H'}{L'});\ \Pr = \frac{\nu}{\alpha};\ \text{Gr} = \frac{\text{g}\,\beta_t\,\Delta\text{T'}\,\text{L}^{*3}}{\alpha^2};\ \Delta\text{T'} = \text{T'}_h - \text{T'}_c \tag{8}$$

The appropriate boundary and initial conditions of the considered problem are given as follows:

* For all walls:

$$u = v = 0 \tag{9}$$

* Adiabatic walls:

$$y = \pm \frac{A_y}{2};\ \frac{\partial T}{\partial y} = 0 \tag{10}$$

* Active walls:

$$\begin{aligned} x &= \frac{A_x}{2};\ T = -0.5 \\ x &= -\frac{A_x}{2};\ T = 0.5 \end{aligned} \tag{11}$$

where:

$$\begin{gathered} A_x = 1 \text{ and } A_y = \text{A if A} \geq 1 \text{ and } A_x = \frac{1}{A} \text{ and } A_y = 1 \text{ if A} < 1 \\ \text{at } t = 0 \text{ (for all the cavity); } u = v = 0;\ P = 0 \text{ and } T = 0.5 - x \end{gathered} \tag{12}$$

3. Entropy Generation

The existence of thermal gradients between the active walls of the inclined rectangular cavity sets the fluid in a non-equilibrium state which causes entropy generation in the system. According to local thermodynamics of equilibrium with linear transport theory, the local volumetric entropy generation is given by [7]:

$$\dot{\sigma}_l = \frac{\lambda}{T_0'^2}\left(\overrightarrow{grad}\ T'\right)^2 + \frac{\overline{\overline{\tau}} : \nabla\vec{v}}{T'_0} \tag{13}$$

In the case of two dimensional Cartesian systems, Equation (13) can be written as:

$$\dot{\sigma}_l = \frac{\lambda}{T'^2_0}\left[\left(\frac{\partial T'}{\partial x'}\right)^2 + \left(\frac{\partial T'}{\partial y'}\right)^2\right] + \frac{\mu}{T'_0}\left[2\left(\frac{\partial u'}{\partial x'}\right)^2 + 2\left(\frac{\partial v'}{\partial y'}\right)^2 + \left(\frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'}\right)^2\right] \tag{14}$$

The dimensionless local entropy generation is obtained by using the dimensionless variables listed in Equation (8), it is given by:

$$\dot{S}_l = \dot{S}_{l,a,h} + \dot{S}_{l,a,f} \tag{15}$$

where:

$$\dot{S}_{l,a,h} = \left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] \tag{16}$$

$$\dot{S}_{l,a,f} = \phi_D\left[2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right] \tag{17}$$

$$\phi_D = \frac{\mu T'_0}{\lambda}\left(\frac{\alpha}{L^*(\Delta T')}\right)^2 \tag{18}$$

The first term on the right-hand side of Equation (15) is the local entropy generation due to heat transfer ($\dot{S}_{l,a,h}$) while the second term is the local entropy generation due to fluid friction ($\dot{S}_{l,a,f}$), φ_D is the irreversibility distribution coefficient related to fluid friction irreversibility. The dimensionless total entropy generation is the integral over the volume (Ω) of the dimensionless local entropy generation:

$$S = \int_v \dot{S}_l \, d\Omega \tag{19}$$

An alternative irreversibility distribution parameter called Bejan number (Be) is given in dimensionless form as follows [8]:

$$\mathrm{Be} = \frac{\dot{S}_{l,a,h}}{\dot{S}_l} \tag{20}$$

When Be >> 1/2, the irreversibility due to heat transfer dominates. For Be << 1/2, the irreversibility due to viscous effects dominates. For Be = 1/2, heat transfer and fluid friction irreversibilities are equal. Heat transfer irreversibility is the only origin of entropy generation when Be = 1. When Be = 0, the fluid friction irreversibility is the only origin of entropy generation. Entropy generation is calculated by using Equation (15) after solving the system of the dimensionless Equations (1)–(4).

4. Numerical Procedure

Energy and Navier-Stokes's equations are solved by determination of the temperature and the velocity scalar fields which depend on the choice of numerical support of resolution. In this study a modified version based on Control Volume Finite Element Method (CVFEM) of Saabas and Baliga [9] is used. A standard grid in which diagonals are added to form triangular elements around each node where velocity components are calculated is considered. A staggered grid is used to calculate pressure around each nodal point. Pressure and velocity components are calculated at different points to avoid numerical oscillations. To resolve pressure-velocity components, the SIMPLE algorithm (Semi Implicit Method for Pressure Linked Equations) of Patankar [10] is firstly applied, then the SIMPLER algorithm (SIMPLE Revised) and the SIMPLEC approximation of Van Doormal and Raithby in which addition terms of pressure and their relative to velocity are considered, respectively in conjunction with an Alternating Direction Implicit (ADI) scheme for performing the time evolution. The used numerical code written in FORTRAN language was described and validated in details in Abbassi *et al.* [11,12].

Nusselt number (Nu) that describes heat transfer can be considered as a good criterion for validation of our code by comparison of the obtained results with those given in literature. Table 1 gives comparison of these results for the case of a square cavity (A = 1).

Table 1. Code validation test.

	Ra = Gr × Pr	**10^3**	**10^4**	**10^5**
	Present Study	1.0987	2.279	4.618
Nu	De Vahl Davis [13]	1.118	2.243	4.519
	Baliga and Patankar [14]	1.114	2.218	4.371

From Table 1, our results are in good agreements with those obtained in [13] and [14].

5. Results and Discussions

The aim of the present study is to investigate the influence of the operating parameters such as the thermal Grashof number, the irreversibility distribution and the aspect ratios and the inclination angle of a rectangular enclosure on entropy generation behavior for the case of an incompressible viscous fluid. The Prandtl number is fixed at 0.71, the Thermal Grashof number, the irreversibility distribution ratio, the aspect ratio and the inclination angle are in the following ranges: $10^3 \leq Gr \leq 10^5$; $10^{-4} \leq \varphi_D \leq 10^{-2}$; $1 \leq A \leq 5$ and $0° \leq \phi \leq 180°$, respectively.

For fixed values of the inclination angle of the enclosure ($\phi = 90°$) and the irreversibility coefficient ($\varphi_D = 10^{-2}$), transient entropy generation for $Gr = 10^4$ and 10^5 at different aspect ratio values is illustrated in Figures 2 and 3. As can be seen, entropy generation increases at the beginning of the transient state where the conduction is the dominant mode of heat transfer, reaches a maximum value which is more important as the aspect ratio of the cavity is more important. As time proceeds, entropy generation decreases and tends towards a constant value at the steady state which depends also on the aspect ratio. For low thermal Grashof number, the decrease of entropy generation is asymptotically showing that the system's evolution follows the linear branch of thermodynamics for irreversible process according to Prigogine's theorem. Oscillations of entropy generation are observed for the high values of thermal Grashof number as seen in Figure 4. That is the oscillation behavior obtained before the steady state, corresponds to non linear branch of irreversible processes. In steady state, entropy generation tends towards an asymptotic value which increases with the increase of aspect ratio of the enclosure.

Figure 2. Dimensionless total entropy generation *versus* time for: Gr = 10^4; $\varphi_D = 10^{-2}$; ϕ = 90° (a) A = 1; (b) A = 2; (c) A = 3; (d) A = 4; (e) A = 5.

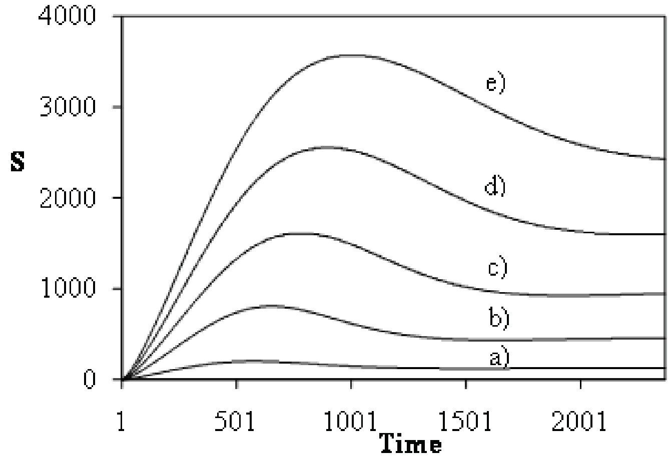


Figure 3. Dimensionless total entropy generation *versus* time for Gr = 10^5; $\varphi_D = 10^{-2}$; ϕ = 90°; (a) A = 1; (b) A = 2; (c) A = 4; (d) A = 5.

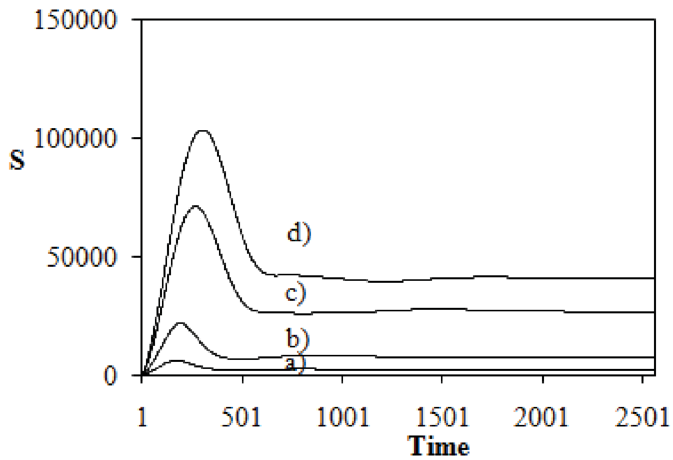
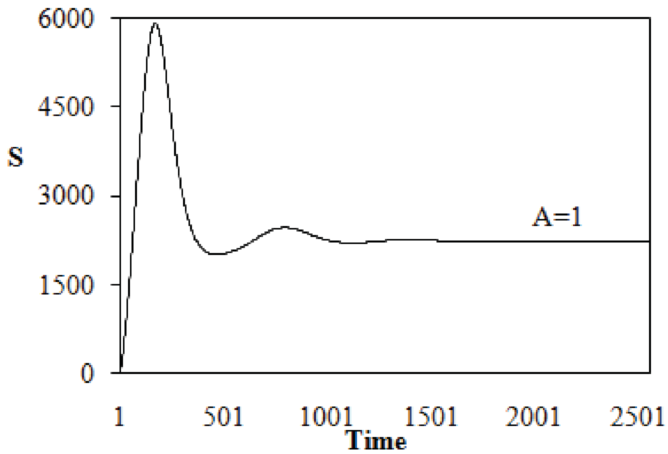


Figure 4. Dimensionless total entropy generation *versus* time for Gr = 10^5; $\varphi_D = 10^{-2}$; ϕ = 90°; A = 1 (oscillations).



Figures 5, 6 and 7 show the effect of the inclination angle on entropy generation for fixed values of irreversibility distribution ratio (10^{-4}) and thermal Grashof number (10^3, 10^4 and 10^5) and for different aspect ratio values of the cavity. As it can be seen, for any fixed aspect ratio and thermal Grashof number values, entropy generation increases with the inclination angle, reaches a maximum value then decreases. Maximum value depends on aspect ratio. This value is obtained at $\phi \approx 50°$ for A = 2 and at $\phi \approx 80°$ for A = 5. The increase of both aspect ratio and thermal Grashof number induces an increase of entropy generation value. It could be noticed that for the two studied limiting inclination angle values (*i.e.*, ϕ = 0°, 180°), entropy generation value is the same for any fixed aspect ratio.

Figure 5. Dimensionless total entropy generation *versus* inclination angle for Gr = 10^3; $\varphi_D = 10^{-4}$; (a) A = 1; (b) A = 2; (c) A = 3; (d) A = 4; (e) A = 5.

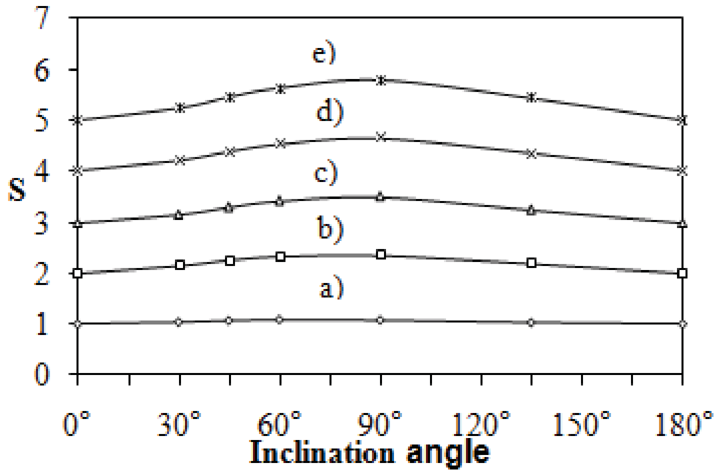


Figure 6. Dimensionless total entropy generation *versus* inclination angle for Gr = 10^4; $\varphi_D = 10^{-4}$; (a) A = 1; (b) A = 2; (c) A = 3; (d) A = 4; (e) A = 5.

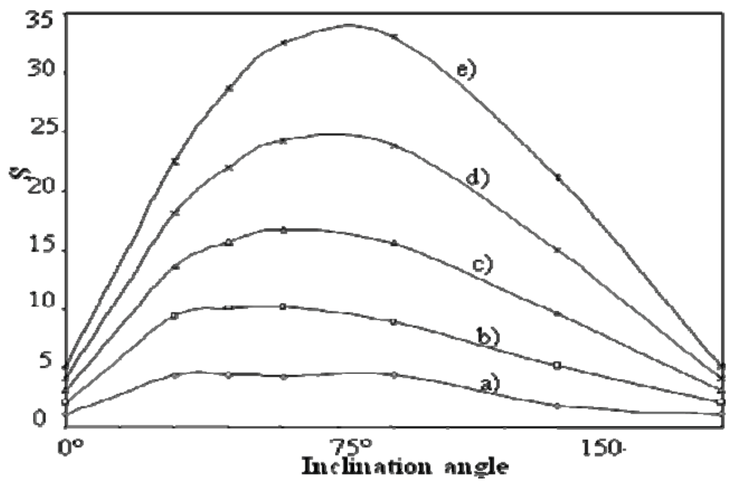
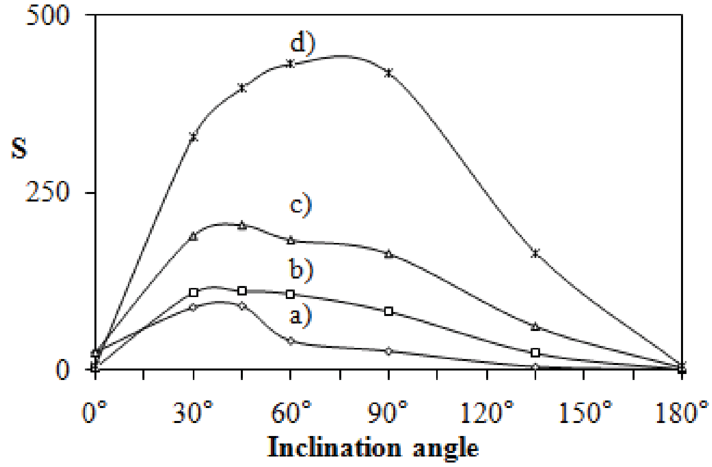


Figure 7. Dimensionless total entropy generation *versus* inclination angle for Gr = 10^5; $\varphi_D = 10^{-4}$; (a) A = 1; (b) A = 2; (c) A = 3; (d) A = 5.



The influence of the irreversibility distribution ratio on entropy generation is depicted on Figure 8. As it can be seen, for lower aspect ratio values (*i.e.*, A = 1: square cavity), no important variation of entropy generation is observed, that is the square enclosure gives the lowest value of irreversibility. On increasing the aspect ratio, entropy generation value increases especially when $\varphi_D \geq 10^{-3}$. This is due to the predominance of convective irreversibility inside the enclosure, which results from the increase of both thermal and velocity gradients of the fluid. The aspect ratio of the cavity has a considerable effect on entropy generation as illustrated in Figure 9. Obtained results show that entropy generation increases with aspect ratio and Grashof number. As it can be seen, a linear behavior of entropy generation *versus* aspect ratio is obtained for A ≥ 3.

Figure 8. Dimensionless total entropy generation *versus* irreversibility distribution ratio for: Gr = 10^3; ϕ = 90°; (a) A = 1; (b) A = 2; (c) A = 3; (d) A = 4; (e) A = 5.

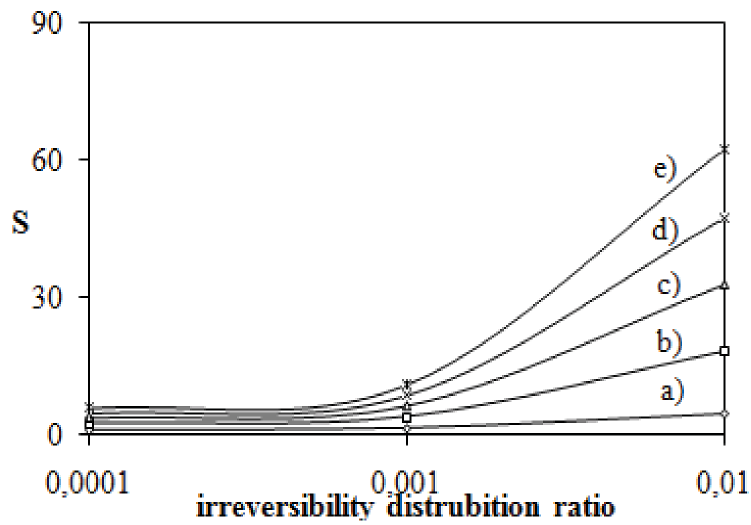
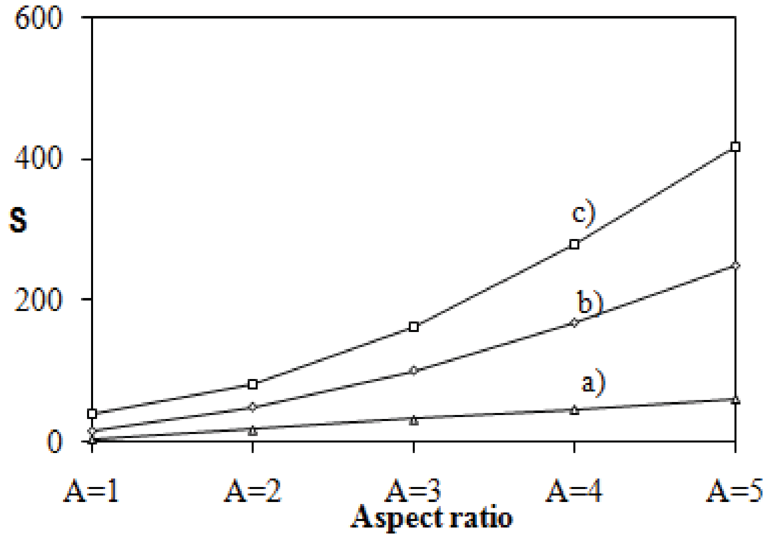
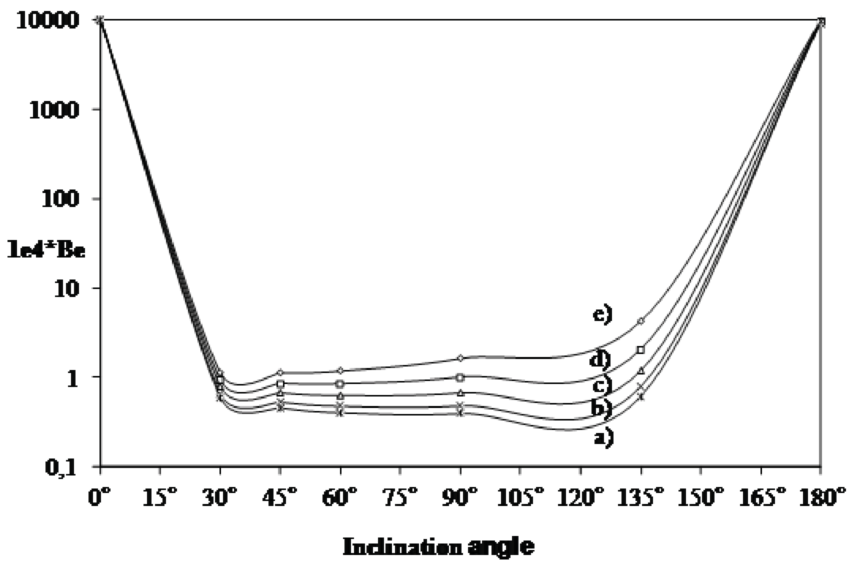


Figure 9. Dimensionless total entropy generation *versus* aspect ratio for ϕ = 90°; (a) Gr = 10^3 and $\varphi_D = 10^{-2}$; (b) Gr = 10^4 and $\varphi_D = 10^{-3}$; (c) Gr = 10^5 and $\varphi_D = 10^{-4}$.



Contribution of either thermal or viscous irreversibility on total entropy generation is characterized by the dimensionless number called Bejan number (Be) which is defined as the ratio of entropy generation due to heat transfer by total entropy generation. For Be = 1, irreversibility is due to heat transfer. When Be = 0, irreversibility is due to viscous effect. For Be = ½, contributions of the two terms are equal. For Be > ½ thermal irreversibility dominates and for Be < ½ friction one dominates. As it can be seen in Figure 10, variation of the inclination angle of the cavity from 0° to 40° induces a decrease of Bejan number value from unit value observed at ϕ = 0° corresponding to pure conduction regime which is relative to dominance of thermal irreversibility to a first minimum at 40°.

Figure 10. Variation of Bejan number *versus* inclination angle for Gr = 10^4; $\varphi_D = 10^{-4}$; (a) A = 1; (b) A = 2; (c) A = 3; (d) A = 4; (e) A = 5.



It could be remarked that the aspect ratio has no significant influence on Bejan number for $0° \leq \varphi \leq 30°$. A second minimum of Bejan number is obtained at $\phi = 125°$. It is important to notice that for $40° \leq \varphi \leq 125°$, viscous irreversibility dominates since convective mode is the dominant mode. In this case, viscous irreversibility increases with the aspect ratio value as velocity gradients increase. For $\varphi > 125°$, Bejan number again increases showing that the thermal irreversibility dominates and the aspect ratio has no practical effect on Bejan number value.

At the local level, Figure 11 shows that entropy generation is mainly located on the lower corner of the heated wall and on the upper corner of the cooled wall for $10^3 \leq Gr \leq 10^4$. This is due to thermal and velocity gradients in the above mentioned regions as indicated by isothermal lines and stream lines. For relatively higher Grashof number value (*i.e.*, $Gr \geq 10^5$), entropy generation chart shows that lines of irreversibility are practically located through the active sides (heated and cooled walls). This is due to considerable thermal and velocity gradients as described by isothermal and stream lines for $Gr = 10^5$.

Figure 11. Isotherm, stream and isentropic lines for A = 2; $\varphi_D = 10^{-2}$; $\phi = 90°$ (a) Gr = 10^3; (b) Gr = 10^4; (c) Gr = 10^5.

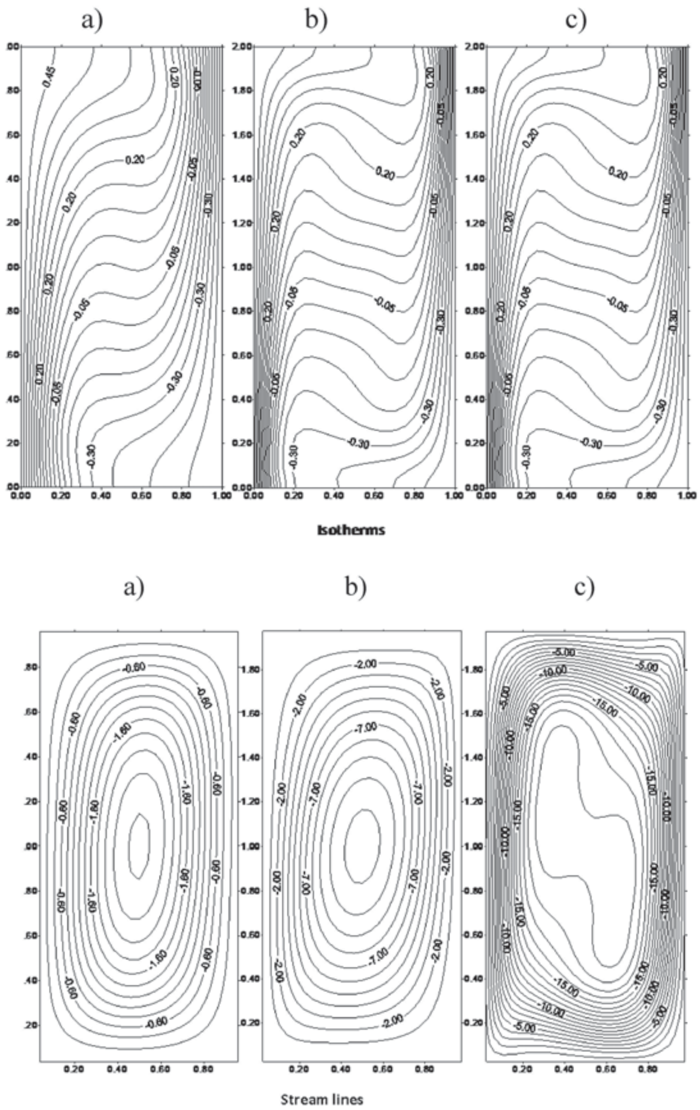
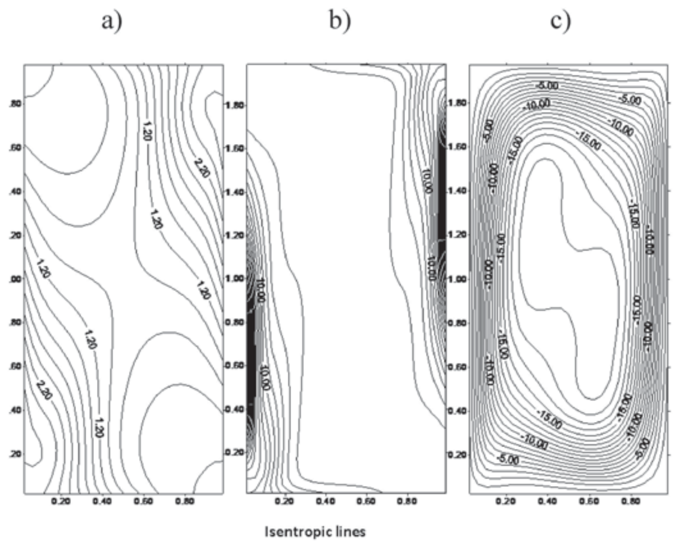


Figure 11. *Cont.*



6. Conclusions

Entropy generation in natural convection through an inclined rectangular cavity is numerically calculated using the Control Volume Finite Element Method (CVFEM). Results show that total entropy generation increases with the aspect ratio of the cavity for high thermal Grashof number, for any fixed irreversibility distribution ratio and with the last parameter at constant Grashof number.

The transient state study shows that entropy generation increases at the beginning of this regime, reaches a maximum value, then decreases asymptotically for low Grashof number and with oscillations at high Grashof number towards a constant value at the steady state. At ϕ = 0° and ϕ = 180°, entropy generation takes a constant value depending on aspect ratio of the cavity and corresponding to pure conduction regime by heat transfer. Contributions of thermal and viscous irreversibility on entropy generation are investigated using dimensionless number called Bejan number which takes unity value for the two limit angles: 0° and 180° and it decreases between them according to dominance of fluid velocity or heat transfer. Bejan number increases as the aspect ratio increases at fixed values of thermal Grashof number and irreversibility distribution ratio. At local level, entropy generation is located on low corner of the heated side and upper corner of the cooled side of the enclosure.

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