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Article

Superstatistics and Gravitation

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Abstract: We suggest to consider the spacetime as a non-equilibrium system with a long-term stationary state that possess as a spatio-temporally fluctuating quantity β. These systems can be described by a superposition of several statistics, "superstatistics". We propose a Gamma distribution for $f(\beta)$ that depends on a parameter p_l. By means of it the corresponding entropy is calculated, p_l is identified with the probability corresponding to this model. A generalized Newton's law of gravitation is then obtained following the entropic force formulation. We discuss some of the difficulties to try to get an associated theory of gravity.

Keywords: superstatistics; entropy; modified gravity

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1. Introduction

Several years ago Beck and Cohen [1,2] considered nonequilibrium systems with a long-term stationary state that possess a spatio-temporally fluctuating intensive quantity. They have shown that after averaging over the fluctuations one can obtain not only non-extensive statistical mechanics [3,4] but an infinite set of more general statistics that they called "superstatistics". In their work they selected the temperature as fluctuating quantity among various possible intensive quantities (e.g., chemical potential or energy dissipation) and showed for general distributions $f(\beta)$, how to get a kind of effective Boltzmann factor

$$B(E) = \int_0^\infty d\beta \; f(\beta) e^{-\beta E} \tag{1}$$

where E is the energy of a microstate associated with each of the considered cells. The ordinary Boltzmann factor is recovered for $f(\beta) = \delta(\beta - \beta_0)$ and $B(E)$, corresponding to non-extensive statistical mechanics [3,4], is obtained assuming a Gamma (or χ^2) inverse temperature β distribution, depending on a parameter q. Also in [1,2] the uniform, bimodal, log-normal and F-distributions were assumed for $f(\beta)$. Later in [5], considering the examples in [1,2] it was nicely shown how from the corresponding Boltzmann factors the associated entropies $S(x)$ could be obtained. The Boltzmann-Gibbs entropy and the non-extensive statistical mechanics entropy can be calculated from their Boltzmann factors analytically. For other $f(\beta)$ distributions [1,2] it is not possible to get a closed analytic expression for $S(x)$ and in [5] the calculations have been performed numerically using the corresponding $B(E)$ for each of these cases.

From the classical Einstein equations, the laws of black hole mechanics, analogues to those of thermodynamics, were originally derived [6]. It was understood that the analogy is in fact an identity with the discovery of quantum Hawking radiation [7]. In an insightful work, Jacobson [8] was able to turn this logic by deriving the Einstein equations from the proportionality of entropy and horizon area together with the fundamental Clausius relation $\delta Q = TdS$. Thus the Einstein equations can be viewed as an equation of state. Jacobson procedure presumes the existence of local equilibrium conditions in which the relation $\delta Q = T\ dS$ only applies between near states of local thermodynamic equilibrium. Recently, Verlinde in a remarkable paper [9] proposed a framework for gravity as a kind of entropic force. His proposal, while related with Jacobson's approach, showed that Newtonian gravity could be obtained by using entropic and holographic arguments [10,11]. The assumption was then that space is emergent and that the holographic principle holds. The Einstein equations were also derived under this hypothesis.

More recently nonequilibrium thermodynamics of spacetime has been studied [12]. Here the Clausius relation is modified to $dS = \delta Q/T + d_iS$, where d_iS is a bulk viscosity entropy production term that is determined by imposing energy-momentum conservation. General Relativity can still be the resulting theory and it has been shown [13] that then the internal entropy production term is identical to the so called tidal heating term of Hartle-Hawking [14,15]. It is also possible to make the entropy density proportional to a function of the Ricci scalar leading to a breakdown of the local thermodynamical equilibrium, emerging then a $f(R)$ gravity theory.

In [16] an analogy is made between the description of a solid and that of spacetime. In the case of solids three different levels are distinguished; elasticity for the macroscopic one, statistical mechanics at the microscopic level and an intermediate thermodynamical description. Moving on from a solid to the spacetime; the macroscopic level is the smooth spacetime continuum with the metric tensor and the equations governing the metric. At the microscopic level, we expect a quantum description in terms of the "atoms of spacetime" and some associated degrees of freedom which are still elusive. But as Boltzmann taught us, for matter, heat is a form of motion and we will not have the thermodynamic description layer of matter if it were not constituted by atoms. Just like a solid cannot exhibit thermal phenomena if it does not have microstructure, horizons cannot exhibit thermal behavior if spacetime has no microstructure. Superstatistics is a procedure that, in principle, allows us to generate several possible entropies and their associated statistical mechanics, among them non-extensive statistical mechanics.

We do not know gravitation at the level of the "atoms of spacetime", we can then assume that spacetime could be described as a nonequilibrium system of the kind generated by superstatistics. It is then possible to obtain different entropies by assuming the appropriate $f(\beta)$ distribution. From this distribution we get the associated Boltzmann factor (1) and by means of it the entropy [5], in general not in an analytical closed form.

What kind of modified Newton's laws of gravitation will we get as a consequence of these generalized entropies? Which is the expected structure of the corresponding gravity theories?

In this work we will assume a particular Gamma distribution for β, this is similar to the one associated with non-extensive statistical mechanics [1–4], that depends on a parameter q. However the one presented in this work does not have any free parameter, it depends on the probability p_l corresponding to this model and, clearly, on β. For other distributions like the log-normal distribution in [1,2] that depends also on a free parameter q, one can similarly propose a distribution depending also only on the associated probability p_l to the model of interest. The relation between these p_l, and the product $\beta_0 E$ differs from the standard one of the Boltzmann-Gibbs statistics. This is, however, a certain limit of these models, as the same happens with the F-distribution which depends on two parameters [1,2], its dependence on one of them can also be understood in terms of its corresponding probability p_l. A comprehensive presentation of these models and their physical implications will be presented elsewhere [17]. In this work we will be interested in a $f(\beta)$ Gamma distribution depending on the p_l resulting in this model [3,4] and will analyze its consequences in connection with gravity. It should be mentioned that the Gamma, the log-normal and the F-distributions depending on q, all have the same approximated Boltzmann factor $B(E)$ [1,2], up to the first correction term. For the same three distributions depending on p_l [17] also their approximated corresponding Boltzmann factors coincide and consequently their associated entropies will coincide to the corresponding order of approximation by following the procedure in [5].

In section II the Gamma $f(\beta)$ distribution depending on p_l is introduced and from it the Boltzmann factor and the entropy are calculated. In section III the entropy is analyzed in connection with the usual Boltzmann-Gibbs entropy and with the entropy-area law. From it, following [9] the generalized Newton's law is obtained and discussed. Section IV is devoted to discussion and outlook.

2. The Entropy

Let us begin by assuming a Gamma distribution for β depending on an associated probability p_l

$$fp_l(\beta) = \frac{1}{\beta_0 p_l \Gamma\left(\frac{1}{p_l}\right)} \left(\frac{\beta}{\beta_0}\frac{1}{p_l}\right)^{\frac{1-p_l}{p_l}} e^{-\beta/\beta_0 p_l} \tag{2}$$

integration over β gives the generalized Boltzmann factor (1),

$$Bp_l(E) = (1 + p_l\beta_0 E)^{-\frac{1}{p_l}} \tag{3}$$

this expression can be expanded for small $p_l\beta_0 E$ to get

$$Bp_l(E) = e^{-\beta_0 E}\left[1 + \frac{1}{2}p_l\beta_0^2 E^2 - \frac{1}{3}p_l^2\beta_0^3 E^3 + ...\right] \tag{4}$$

Up to this point if one assumes a fixed $p_l = q - 1$ expressions (2,3,4) coincide with those corresponding to non-extensive statistical mechanics [1–4]. As shown in [1,2] for the Gamma, the log-normal and F-distributions, depending now on p_l the second term in (4) is the same for these three cases.

As nicely shown in [5] one can deduce the entropy $S(x)$ from the Boltzmann factor. $S(x)$ is defined as $S = \sum_{l=1}^{\Omega} s(p_l)$, in terms of a generic $s(p_l)$; as known $s(x) = -x \ln x$ for the Boltzmann-Gibbs entropy S_{BG}. The authors in [5] expressed $s(x)$ as well as a generic $u(x)$ in terms of integrals on a function $E(y)$ that can be directly calculated from the Boltzmann factor $B(E)$ of interest. They considered the functional

$$\Phi = S - \alpha \sum_{l=1}^{\Omega} p_l - \beta U \tag{5}$$

where $U = \dfrac{\sum_{l=1}^{\Omega} u(p_l) E_l}{\sum_{l=1}^{\Omega} u(p_l)}$ is the constraint associated with the energy and α and β are Lagrange parameters. They imposed the condition $\frac{\partial \Phi}{\partial p_l} = 0$ and assuming a particular form for the function $u(x)$ in terms of x and $s(x)$ they were able to arrive to the integrals

$$s(x) = \int_0^x dy \frac{\alpha + E(y)}{1 - E(y)/E^*} \tag{6}$$

and

$$u(x) = (1 + \alpha/E^*) \int_0^x \frac{dy}{1 - E(y)/E^*} \tag{7}$$

where $E(y)$ is to be identified with the inverse function of $Bp_l(E)/\int_0^\infty dE'\ Bp_l(E')$. One proceeds first by selecting the $f(\beta)$ of interest, then $B(E)$ is calculated and the integral $\int_0^\infty B(E')\ dE'$ is performed. Inverting the axes of the variables, $E(y)$ of the Beck-Cohen superstatistics is found, and from it E^*. In our case, the starting point is the $fp_l(\beta)$ distribution (2), $E(y)$ results in

$$E(y) = \frac{y^{-x} - 1}{x} \tag{8}$$

$E^* = -1/x$. A straightforward calculation gives for $u(x)$

$$u(x) = x^{x+1} \tag{9}$$

where α has been already determined by means of the condition $u(1) = 1$

$$s(x) = 1 - x^x \tag{10}$$

These expressions fulfill the conditions $s(0) = 1, u(0) = 0$ and $u(1) = 1, s(1) = 0$. By this means the desired entropy results in

$$S = k \sum_{l=1}^{\Omega} (1 - p_l^{p_l}) \tag{11}$$

where k is the conventional constant and $\sum_{l=1}^{\Omega} p_l = 1$. The expansion of expression (11) gives

$$-\frac{S}{k} = \sum_{l=1}^{\Omega} p_l \ln p_l + \frac{(p_l \ln p_l)^2}{2!} + \frac{(p_l \ln p_l)^3}{3!} + \cdots \tag{12}$$

being the first term the usual entropy expression in information theory.

One should remember that the above condition on the functional (5), $\frac{\partial \Phi}{\partial p_l} = 0$ allows for $S = -k \sum_{l=1}^{\Omega} p_l \ln p_l$ to obtain $p_l \sim e^{-\beta E}$. This nonrenormalized probability coincides in this case with the corresponding Boltzmann factor $B(E) \sim e^{-\beta E}$. The associated probability p_l for nonextensive statistical mechanics is also function of this product βE, however is a more elaborated expression and does not coincide with the corresponding Boltzmann factor [1–4]; for $q \to 1$ its limit is the above standard p_l. The same condition on Φ for our case, the entropy (11), gives a more involved expression in which the corresponding p_l results a function also of the product βE but p_l can not be written explicitly as a function of this product it is an implicit function. It has, however, as a limit the usual $p_l \sim e^{-\beta E}$. This and other aspects of β distributions and generalized entropies depending on p_l in the context of superstatistics will be discussed in a forthcoming work [17].

3. Newton's modified gravity

Verlinde [9] followed Bekenstein argument to deduce his famous entropy formula; a particle of mass m is dropped in the black hole just before the horizon. The increase of the mass of the black hole can be made infinitely small due to the infinite redshift. If one takes a gas of particles this would lead to problems with the second law of thermodynamics, Bekenstein argued that when a particle is one Compton wavelength from the horizon it should be considered to be part of the black hole and solved this problem. Therefore it increases the mass and horizon area by a small amount identified with one bit of information. Mimicking this reasoning, in his holographic considerations, Verlinde takes a flat nonrelativistic space and a particle that approaches it from the side at which space has emerged. He assumed then that the change of entropy associated with the information on the boundary is linear in the displacement Δx

$$\Delta S = 2\pi k \frac{mc}{\hbar} \Delta x \tag{13}$$

When a particle has an entropic reason to be on one side of the membrane and the membrane carries a temperature it will experience an effective force equal to

$$F \Delta x = T \Delta S \tag{14}$$

and this results in the same expression as the entropic force in standard thermodynamics. Then to have a nonzero force the temperature should not vanish, the force leads to an acceleration that is identified with Unruh's temperature for an observer in an accelerated frame

$$kT = \frac{1}{2\pi} \frac{\hbar a}{c} \tag{15}$$

Verlinde's picture is an attempt to model nonrelativistic gravity as a force originating from an entropy gradient, analogous to the known entropic forces on polymers immersed in a heath bath. A number of results have been obtained recently in connection with this entropic force [18–32]. Also the following considerations were taken into account by Verlinde [9,26,32]; the energy of the surface $\mathcal{S}$ is identified with the relativistic rest mass of the source $E = Mc^2$, and the energy of the surface $\mathcal{S}$ is equipartitioned among N bytes, $E = \frac{1}{2}NkT$, these bytes of information scale proportionally to the area of the surface $A = QN$, where Q is a fundamental constant, ΔS is one fundamental unit of entropy when $\Delta x = \eta\lambda_m$, with λ_m the Compton's wavelength, being N the number of bytes, $\Delta N = 1$ and hence $\Delta A = Q$.

We are interested in expressing the entropy (11,12) for the equiprobability case, i. e. $p_l = 1/\Omega$, it results in

$$S = k\Omega\left[1 - \frac{1}{\Omega^{1/\Omega}}\right] \tag{16}$$

To study the nonrelativistic gravity associated with the entropy (16), we first express it in terms of the Boltzmann entropy $S_B = k\ln\Omega$,

$$S = e^{S_B}\left[1 - e^{-S_B e^{-S_B}}\right] \tag{17}$$

where the entropies have been redefined and the factor $1/k$ has been suppressed. Verlinde's derivation of Newton's law of gravitation is based on the assumption of the entropy-area relationship and corresponds to the standard entropy $S_B = \frac{A}{4l_p^2}$ for black holes in Einstein gravity, where $A = 4\pi R^2$ represents the area of the horizon and $l_p^2 = G\hbar/c^3$ is the Planck length. We note that the entropy (17), for large S_B gives $S \sim S_B$. The expansion of (17) gives

$$S = S_B - \frac{S_B^2}{2!}e^{-S_B} + \frac{S_B^3}{3!}e^{-2S_B} - \frac{S_B^4}{4!}e^{-3S_B} + \cdots \tag{18}$$

With all these previous assumptions it can be shown [26,32] that it is possible to obtain a modified Newton's law from a certain modified entropy, namely

$$F = -\frac{GMm}{R^2}\left[1 + 4l_p^2\frac{\partial \mathbf{s}}{\partial A}\right]_{A=4\pi R^2} \tag{19}$$

where for $S_B = \frac{A}{4l_p^2}$,

$$S = \frac{A}{4l_p^2} + \mathbf{s} \tag{20}$$

For the entropy (17,18) assumed in this work, consequence of the superstatistics model for the $f_{p_l}(\beta)$ distribution (2), s can be read from (18) and

$$\begin{aligned} 4l_p^2\frac{\partial \mathbf{s}}{\partial A} = \frac{\partial \mathbf{s}}{\partial S_B} = &-\frac{A}{4l_p^2}e^{-A/4l_p^2}\left[1 - \frac{1}{2}\frac{A}{4l_p^2}\right] + \left(\frac{A}{4l_p^2}\right)^2 e^{-2A/4l_p^2}\left[\frac{1}{2} - \frac{1}{3}\frac{A}{4l_p^2}\right] \\ &- \frac{1}{6}\left(\frac{A}{4l_p^2}\right)^3 e^{-3A/4l_p^2} + \cdots \end{aligned} \tag{21}$$

as already noted from the general expression for the entropy (17,18) for large $S_B = \dfrac{A}{4l_p^2}$ the terms in (18,20) will not significantly contribute to modify the standard Newtonian force, corresponding to the first terms in (19).

As $A = 4\pi R^2$, $S_B = A/4l_p^2 = \pi R^2/l_p^2$. From (21) it can be seen that corrections to Newton's law of gravitation are of certain relevance for $R^2 \sim l_p^2$. For this case one can expand the exponential functions and substituting them in (19), the modified Newtonian force results in

$$F = -\frac{GMm}{R^2}\left[1 - \frac{\pi R^2}{l_p^2} + \frac{2\pi^2 R^4}{l_p^4} - \frac{5}{2}\frac{\pi^3 R^6}{l_p^6} + \ldots\right] \tag{22}$$

The nature of these terms is different to the one of those appearing in other formalisms with different assumptions. By example, quantum gravity models arising from a Wheeler-DeWitt equation for a black hole [33,34] or those considering loop quantum gravity [26,35,36]. There the terms that modify Newton's law are proportional to inverse powers of the radius R and are relevant for radius very close to the Planck's length.

In our case (22) the standard Newton's force will significantly change only for extremely small radius too. The correction terms grow with the power of the radius. Our assumptions, as those in [8,9] have nothing to do with quantizing gravity; the question we are exploring is of a very different nature, namely what kind of emergent gravity will we get if the spacetime is a nonequilibrium system with a long-term stationary state that possess a spatio-temporally fluctuating quantity (in our example β) and that after averaging a superstatistics arises providing the entropy (11,12,17,18).

4. Discussion and Outlook

The entropy (11,12,17,18) is a consequence of the Gamma distribution (2) and its associated Boltzmann factor (3). Unfortunately, as mentioned in the Introduction and shown in [5], for the log-normal and F-distributions it is not possible to get analytic expressions for the associated entropies. However, their approximated Boltzmann factor (4) is the same for all these distributions up to the first correction term and the next term is similar for these three cases. Consequently, one expects their approximated entropies to be analogous and by this means their associated modified Newtonian forces (22).

In [8] the Einstein equations were derived from the requirement that the Clausius relation $dS = \delta Q/T$ holds for all local accelerating horizons through each spacetime point, where dS is proportional to the area change in Planck length units and δQ and T are the energy flux across the horizon and the Unruh temperature seen by an accelerating observer inside the horizon. Then, in a nonequilibrium setting [12,13] the entropy balance relation $dS = \delta Q/T + d_i S$, with $d_i S$ a bulk viscosity entropy was considered in an attempt to extend the approach in [8] to the case of $f(R)$ theories of gravity.

Considering the spacetime as a nonequilibrium system with a long term stationary state that has β as a spatio-temporally fluctuating quantity after averaging the fluctuations we have shown a particular model that has a generalized entropy (11,16,17). The exact differential of this entropy is then given in terms of dS_B and S_B itself as

$$dS = dS_B e^{S_B} + dS_B e^{-S_B e^{-S_B}}\left[1 - S_B - e^{S_B}\right] \tag{23}$$

As it happens with the entropy (17,18) $dS \simeq dS_B$ for large S_B and the procedure in [8] to get the Einstein equations follows with $dS_B = \eta\delta A$, where η is a universal entropy density per unit horizon area. The entropy (17) and its differential (23) are now complex functions of A and δA and by considering the Clausius relation $dS = \delta Q/T$ it is not at all clear how to try to obtain the gravitation equations for this proposal. In [8] the heat was defined as the mean flux of the boost energy current of matter across the horizon. Taking the integral over a short segment of a thin pencil of horizon generators centered on the one that terminates at p and taking the Killing vector $\chi^a = -\lambda k^a$ and $T = \hbar/2\pi$ to order $\mathcal{O}(x^3)$, it was shown that

$$\frac{\delta Q}{T} = \frac{2\pi}{\hbar}\int T_{ab}k^a k^b(-\lambda)d\lambda d^2A \tag{24}$$

The entropy change $\delta S_B = \eta\delta A$ was determined by the area change of the horizon by means of the Raychaudhuri equation; $\theta = d(\ln d^2A)/d\lambda$, the expansion of the congruence of null geodesics generating the horizon was obtained in terms of the Ricci tensor $\theta = -\lambda R_{ab}k^a k^b + \mathcal{O}(\lambda^2)$ and by means of it

$$\delta S_B = \eta\int R_{ab}k^a k^b(-\lambda)d\lambda d^2A \tag{25}$$

In our model, even for the limiting case $S_B < 1$ in which $\delta S = \delta S_B(1 - S_B)$, to be able to relate (24) with this modified δS one would need to find S_B itself as certain integral over (25) and this would already result in a complicated expression where it is to be expected that the integrals on both sides of the Clausius relation will not cancel out. To define the appropriate assumptions to search for the most simple modified gravity associated with the entropy (17,23) is beyond the scope of this work.

The generalized statistical description of nonequilibrium complex systems [1,2] was assumed for the spacetime. These systems have a spatio-temporally inhomogeneous dynamics that can be effectively described by a superposition of several statistics, a "superstatistics". We have assumed a $f(\beta)$ distribution (2), depending on p_l by means of which one gets the generalized Boltzmann factor (3) and from it [5] the entropy (11). This, for the equiprobability case can be written in terms of S_B (17). Being S_B the standard entropy that has been related with the area of a black hole, we have obtained now a very different entropy-area relation (17,20,21) and this allowed us to compute the associated Newtonian force (22). As discussed above it is not at all clear how to find the generalized gravitation equations. This requires a detailed analysis that will be the subject of future work.

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