Entropy **2010**, *12*, 932-952; doi:10.3390/e12040932

OPEN ACCESS

entropy

ISSN 1099-4300

www.mdpi.com/journal/entropy

Article

Entropy in the Present and Early Universe: New Small Parameters and Dark Energy Problem

Alexander Shalyt-Margolin

National Centre for Particle and High Energy Physics, Bogdanovich Street 153, Minsk 220040, Belarus; E-Mail: a.shalyt@mail.ru; alexm@hep.by; Tel./Fax: +375 17 292 6075

Received: 22 March 2010; in revised form: 7 April 2010 / Accepted: 10 April 2010 / Published: 14 April 2010

Abstract: It is demonstrated that entropy and its density play a significant role in solving the problem of the vacuum energy density (cosmological constant) of the Universe and hence the dark energy problem. Taking this in mind, two most popular models for dark energy—Holographic Dark Energy Model and Agegraphic Dark Energy Model—are analysed. It is shown that the fundamental quantities in the first of these models may be expressed in terms of a new small dimensionless parameter that is naturally occurring in High Energy Gravitational Thermodynamics and Gravitational Holography (UV-limit). On this basis, the possibility of a new approach to the problem of Quantum Gravity is discussed. Besides, the results obtained on the uncertainty relation of the pair "cosmological constant–volume of space-time", where the cosmological constant is a dynamic quantity, are reconsidered and generalized up to the Generalized Uncertainty Relation.

Keywords: dark energy; deformed quantum theory; new small parameters

---

1. Introduction

The Dark Energy Problem is one of the key problems in modern theoretical physics [1–20]. The vacuum energy is still the major candidate to play a role of this energy. Provided the Dark Energy is actually the vacuum energy, the indicated problem is reduced to getting better insight into the essence of the vacuum energy. This problem has attracted the attention of researchers fairly recently with understanding that a cosmological constant determining the vacuum energy density is still nonzero, despite its smallness. As is known, the cosmological constant Λ has been first introduced in the works of

A. Einstein [21] who has used it as an antigravitational term to obtain solutions for the equations of the General Relativity (GR) in the stationary case. However, when A. Friedmann found the solutions for GR in case of expanding Universe [22] and E. Hubble derived an extension of the latter, A. Einstein refused from the cosmological constant considering its introduction to be erroneous [23]. But the situation was not so simple. In [24] it has been stated that any contribution into the vacuum energy acts exactly as the cosmological constant Λ and the Vacuum Energy Density is proportional to Λ. The principal problem of the cosmological constant resides in the fact that its experimental value is smaller by a factor of 10^{123} than that derived using a Quantum Field Theory (QFT) [25,26].

And the theories actively developed at the present time (e.g., superstring theory, loop quantum gravity, *etc.*) offer a modified quantum theory including, in particular, the fundamental length at Planck's scale. The estimates of Λ obtained on the basis of these theories may differ greatly from the initial ones derived from standard QFT.

In this paper, some of the properties of the Vacuum Energy Density are studied within the scope of a Quantum Field Theory with UV-cutoff (minimal length). Such a theory arises in the Early Universe in all the models without exception, since the fundamental length (probably on the order of Planck's length but not necessarily) is acknowledged to be of a crucial importance in this case. It is shown that for this case, the experimental and theoretical values are close and may be expressed in terms of a new small parameter introduced in physics at Planck's scales. Here some explanation is needed. The point is that a Quantum Field Theory with minimal length (QFTML) or, what is the same, UV-cutoff is always originating as a deformation of QFT. This deformation is understood as an extension of some theory with the use of one or several additional parameters in such a way that the initial theory shows itself in the limiting process [27]. One of such extensions generated by an additional small dimensionless parameter, in terms of which the Dark Energy Problem is formulated and successfully solved, is described in this paper. In so doing entropy of the Universe and its dynamics play a significant role. Additionally, within the scope of a dynamic approach to Λ, its behaviour associated with the Generalized Uncertainty Principle is studied for the pair "cosmological constant–volume of space-time". In what follows, there is no differentiation between the notions of the cosmological constant Λ and Vacuum Energy Density ρ_{vac}. Besides, it is demonstrated that a new small parameter occurs in High Energy Gravitational Thermodynamics and Gravitational Holography (UV-limit) as well. On this basis the possibility for a new approach to the problem of Quantum Gravity is discussed.

2. Vacuum Energy Density and Most Popular Modern Dark Energy Models

As noted in Introduction, the Vacuum Energy is a major candidate for the Dark Energy. At the same time, due to a factor of 10^{123} distinction between the experimental value ρ_{vac}^{exp} [1] and the value ρ_{vac}^{QFT} calculated using standard QFT interpretation [25] (*i.e.*, $\rho_{vac}^{QFT} \approx m_p^4$) of Dark Energy as a Vacuum Energy presents great difficulties.

$$\frac{\rho_{vac}^{exp}}{\rho^{QFT}} \approx 10^{-123} \quad (1)$$

Nevertheless, there are several methods enabling one to obviate the difficulties. We can name two most popular phenomenological models for the dark energy problem at the present time.

2.1. Holographic Dark Energy Models

The basic relation for this model is the "energy" inequality [28–30]

$$E_{\overline{\Lambda}} \leq E_{BH} \rightarrow l^3 \rho_{\overline{\Lambda}} \leq m_p^2 l \tag{2}$$

Here $\rho_{\overline{\Lambda}} = \overline{\Lambda}^4$ is vacuum energy density with the UV-cutoff $\overline{\Lambda}$ and l is the length scale (IR-cutoff) of the system. For the equality in (2) we have the **holographic energy density**

$$\rho_{\overline{\Lambda}} \sim \frac{m_p^2}{l^2} \sim \frac{1}{(l_{\rm p} l)^2} \tag{3}$$

Also, from (2) we can get the "entropic" inequality (entropy bound)

$$S_{\overline{\Lambda}} \leq (m_p^2 A)^{3/4} \tag{4}$$

where $A = 4\pi l^2$ is the area of this system in the spherically symmetric case.

The number of works devoted to the Holographic Dark Energy Models, beginning from the first publication [28], is ever growing [31–48] to relieve us of citing the whole list.

2.2. Agegraphic Dark Energy Models

Agegraphic Dark Energy Models became the subject of study only two years ago [49]. These relations were based on the result of Károlyházy for quantum fluctuations of time [50–52]

$$\delta t = \lambda t_{\rm p}^{2/3} t^{1/3} \tag{5}$$

Using the uncertainty relation of "energy-time" in the flat space

$$\Delta E \sim t^{-1} \tag{6}$$

we can obtain the **agegraphic energy density** [30,53],

$$\rho_{\boldsymbol{T}} \sim \frac{\Delta E}{(\delta t)^3} \sim \frac{m_{\rm p}^2}{\boldsymbol{T}^2} \tag{7}$$

where $\boldsymbol{T}$ is the Universe age.

The number of publications associated with models of this type is constantly increasing too [54–60]. This is caused by their relative simplicity and by a sufficiently good coincidence between the agegraphic energy density $\rho_{\boldsymbol{T}}$ and ρ_{vac}^{exp}.

3. Dark Energy Problem and Quantum Theory with UV-Cutoff

By Holographic Dark Energy Models (explicitly) and by Agegraphic Dark Energy Models (implicitly) it is suggested that QFT, where they are valid, is actually QFT with the UV-cutoff or with the fundamental length.

As it has been repeatedly demonstrated earlier, a Quantum Mechanics of the Early Universe (Planks Scale) is a Quantum Mechanics with the Fundamental Length (QMFL) [61]. The main approach to framing of QFT with UV-cutoff is that associated with the Generalized Uncertainty Principle (GUP) [62–70] and with the corresponding Heisenberg algebra deformation produced by this principle [71–74].

Besides, QMFL has been framed first using the deformed density matrix and then in the produced corresponding Heisenberg algebra deformation [75–84], the density matrix deformation $\rho(\alpha)$ in QMFL being a starting object called the density pro-matrix and the deformation parameter (additional parameter) $\alpha = l_{min}^2/x^2$, where x is the measuring scale and $l_{min} \sim l_p$. As indicated in this paper, the deformation parameter α varies within the limits $0 < \alpha \leq 1/4$. Moreover, $\lim\limits_{\alpha \to 0} \rho(\alpha) = \rho$, where ρ is the density matrix in the well-known Quantum Mechanics (QM), and the following condition must be fulfilled:

$$Sp[\rho(\alpha)] - Sp^2[\rho(\alpha)] = \alpha + a_0\alpha^2 + ... \tag{8}$$

The explicit form of the above-mentioned deformation gives an exponential ansatz:

$$\rho^*(\alpha) = exp(-\alpha) \sum_i \omega_i |i >< i| \tag{9}$$

where all $\omega_i > 0$ are independent of α and their sum is equal to 1. Here the superscript * in (9) is used only to emphasize that $\rho^*(\alpha)$ is a particular solution of (8).

In the corresponding deformed Quantum Theory (denoted as QFT^α) for average values we have

$$< B >_\alpha = exp(-\alpha) < B > \tag{10}$$

where $< B >$ is the average in well-known QFT [80,81] denoted as QFT^α. All the variables associated with the considered α-deformed quantum field theory are hereinafter marked with the superscript $^\alpha$.

Note that the deformation parameter α is absolutely naturally represented as a ratio between the squared UV- and IR-limits

$$\alpha = (\frac{UV}{IR})^2 \tag{11}$$

where UV-limit is fixed and IR-limit is varying.

As follows from the holographic principle [85–89], maximum entropy that can be stored within a bounded region $\Re$ in 3-D space must be proportional to the value $A(\Re)^{3/4}$, where $A(\Re)$ is the surface area of $\Re$. Of course, this is associated with the case when the region $\Re$ is not an inner part of a particular black hole. Provided a physical system contained in $\Re$ is not bounded by the condition of stability to the gravitational collapse, *i.e.*, this system is simply non-constrained gravitationally, then according to the conventional QFT $S_{\max}(\Re) \sim V(\Re)$, where $V(\Re)$ is the bulk of $\Re$. However in the Holographic Principle case, as it has been demonstrated originally by G. 't Hooft [85] and later by other authors (for example R. V. Buniy and S. D. H. Hsu [90]), we have

$$S_{\max}(\Re) \leq \frac{A(\Re)^{3/4}}{{l_p}^2} \tag{12}$$

In terms of the deformation parameter α, the principal values of the Vacuum Energy Problem may be simply and clearly defined. Let us begin with the Schwarzschild black holes, whose semi-classical entropy is given by

$$S = \pi R_{Sch}^2/l_p^2 = \pi R_{Sch}^2 m_p^2 = \pi \alpha_{R_{Sch}}^{-1} \tag{13}$$

with the assumption that in the formula for $\alpha\ R_{Sch} = x$ is the measuring scale and $l_p = 1/m_p$. Here R_{Sch} is the adequate Schwarzschild radius, and $\alpha_{R_{Sch}}$ is the value of α associated with this radius. Then, as it has been pointed out in [91], in case the Fischler–Susskind cosmic holographic conjecture [92] is valid, the entropy of the Universe is limited by its "surface" measured in Planck units [91]:

$$S \leq \frac{A}{4} m_p^2 \tag{14}$$

where the surface area $A = 4\pi R^2$ is defined in terms of the apparent (Hubble) horizon

$$R = \frac{1}{\sqrt{H^2 + k/a^2}} \tag{15}$$

with curvature k and scale a factors.

Again, interpreting R from (15) as a measuring scale, we directly obtain(14) in terms of α:

$$S \leq \pi \alpha_R^{-1} \tag{16}$$

where $\alpha_R = l_p^2/R^2$. Therefore, the average entropy density may be found as

$$\frac{S}{V} \leq \frac{\pi \alpha_R^{-1}}{V} \tag{17}$$

Using further the reasoning line of [91] based on the results of the holographic thermodynamics, we can relate the entropy and energy of a holographic system [93,94]. Similarly, in terms of the α parameter one can easily estimate the upper limit for the energy density of the Universe (denoted here by ρ_{hol}) [95]:

$$\rho_{hol} \leq \frac{3}{8\pi R^2} m_p^2 = \frac{3}{8\pi} \alpha_R m_p^4 \tag{18}$$

which is drastically differing from the one obtained with conventional QFT:

$$\rho^{QFT} \sim m_p^4 \tag{19}$$

Here by ρ^{QFT} we denote the energy vacuum density calculated from conventional QFT (without fundamental length) [25]. Obviously, as α_R for R determined by (15) is very small, actually approximating zero, ρ_{hol} is by several orders of magnitude smaller than the value expected in QFT ρ^{QFT}.

Since $m_p \sim 1/l_p$, the right-hand side of (18) is actually nothing else but the right-hand side of (3) in Holographic Dark Energy Models (Section 2.1). Thus, in Holographic Dark Energy Models the principal quantity, **holographic energy density** ρ_{Λ} (3), may be estimated in terms of the deformation parameter α.

In fact, the upper limit of the right-hand side of (18) is attainable as it has been demonstrated in [95] and indicated in [91]. The "overestimation" value of r for the energy density ρ^{QFT}, compared to ρ_{hol}, may be determined as

$$r = \frac{\rho^{QFT}}{\rho_{hol}} = \frac{8\pi}{3} \alpha_{\mathbf{R}}^{-\mathbf{1}} = \frac{8\pi}{3} \frac{R^2}{l_p^2} = \frac{8\pi}{3} \frac{S}{S_p} \tag{20}$$

where S_p is the entropy of the Plank mass and length for the Schwarzschild black hole. It is clear that, due to smallness of α_R, the value of α_R^{-1} is on the contrary too large. It may be easily calculated (e.g., see [91])

$$r = 5.44 \times 10^{122} \tag{21}$$

in a good agreement with the astrophysical data.

Naturally, on the assumption that the vacuum energy density ρ_{vac} is involved in ρ as a term

$$\rho = \rho_M + \rho_{vac} \tag{22}$$

where ρ_M is the average matter density, in case of ρ_{vac} we can arrive at the same upper limit (right-hand side of the formula (18)) as for ρ.

4. Some Comments on Dynamic Character of Cosmological Constant and GUP

Generally speaking, Λ is referred to as a constant just because it is such in the equations, where it occurs: Einstein equations [21]. But in the last few years the dominating point of view has been that Λ is actually a dynamic quantity, now weakly dependent on time [96–98]. It is assumed therewith that, despite the present-day smallness of Λ or even its equality to zero, nothing points to the fact that this situation was characteristics for the Early Universe as well. Some recent results [99–102] are rather important pointing to a potentially dynamic character of Λ. Specifically, of great interest is the Uncertainty Principle derived in these works for the pair of conjugate variables (Λ, V):

$$\Delta\Lambda\, \Delta V \sim \hbar \tag{23}$$

where Λ is the vacuum energy density (cosmological constant). It is a dynamic value fluctuating around zero; V is the space-time volume. Here the volume of space-time V results from the Einstein-Hilbert action [100]:

$$S_{EH} \supset \Lambda \int d^4x\sqrt{-g} = \Lambda V \tag{24}$$

In this case "the notion of conjugation is well-defined but approximate as implied by the expansion about the static Fubini–Study metric" (Section 6.1 of [99]). Unfortunately, in the proof per se (23), relying on the procedure with a non-linear and non-local Wheeler–de-Witt-like equation of the background-independent Matrix theory, some unconvincing arguments are used, making it insufficiently rigorous (Appendix 3 of [99]). But, without doubt, this proof has a significant result, though failing to clear up the situation.

Let us attempt to explain (23)(certainly at an heuristic level) using simpler and more natural terms involved with the other, more well-known, conjugate pair: "energy-time" (E, t). We use the notation of [99,100]. In this way the four-dimensional volume will be denoted, as previously, by V.

Just from the start, the Generalized Uncertainty Principle (GUP) is used. Then a change over to the Heisenberg Uncertainty Principle at low energies will be only natural. As is known, the Uncertainty Principle of Heisenberg at Planck's scales (energies) may be extended to the Generalized Uncertainty Principle. To illustrate, for the conjugate pair "momentum-coordinate" (p, x) this fact has been noted in many works [62,65,68,71,72]:

$$\triangle x \geq \frac{\hbar}{\triangle p} + \alpha' l_p^2 \frac{\triangle p}{\hbar} \tag{25}$$

In [77,83] it is demonstrated that the corresponding Generalized Uncertainty Relation for the pair "energy-time" may be easily obtained from

$$\Delta t \geq \frac{\hbar}{\Delta E} + \alpha' t_p^2 \frac{\Delta E}{\hbar} \tag{26}$$

where l_p and t_p represent Planck's length and time, respectively. Now we assume that in the space-time volume $\int d^4x\sqrt{-g} = V$ the temporal and spatial parts may be separated (factored out) in the explicit form:

$$V(t) \approx t\overline{V}(t) \tag{27}$$

where $\overline{V}$ is the spatial part V. For the expanding Universe such an assumption is quite natural. Then it is obvious that

$$\Delta V(t) = \Delta t\overline{V}(t) + t\Delta\overline{V}(t) + \Delta t\Delta\overline{V}(t) \tag{28}$$

Now we recall that for the inflation Universe the scaling factor is $a(t) \sim e^{Ht}$. Consequently, $\Delta\overline{V}(t) \sim \Delta t^3 f(H)$, where $f(H)$ is a particular function of Hubble's constant. From (26) it follows that

$$\Delta t \geq t_{min} \sim t_p \tag{29}$$

However, it is suggested that, even though Δt is satisfying (29), its value is sufficiently small in order that ΔV be contributed significantly by the terms containing Δt to the power higher than the first. In this case the main contribution on the right-hand side of (28) is made by the first term $\Delta t\overline{V}(t)$ only. Then, multiplying the left- and right-hand sides of (26) by $\overline{V}$, we have

$$\Delta V \geq \frac{\hbar\overline{V}}{\Delta E} + \alpha' t_p^2 \frac{\Delta E\overline{V}}{\hbar} = \frac{\hbar}{\Delta\Lambda} + \alpha' t_p^2 \overline{V}^2 \frac{\Delta\Lambda}{\hbar} \tag{30}$$

It is not surprising that a solution of the quadratic inequality (30) leads to a minimal volume of the space-time $V_{min} \sim V_p = l_p^3 t_p$ since (25) and (26) result in minimal length $l_{min} \sim l_p$ and minimal time $t_{min} \sim t_p$, respectively. (30) is of interest from the viewpoint of two limits: (1) IR-limit: $t \to \infty$, (2) UV-limit: $t \to t_{min}$.

In the case of IR-limit we have large volumes $\overline{V}$ and V at low $\Delta\Lambda$. Because of this, the main contribution on the right-hand side of (30) is made by the first term, as great $\overline{V}$ in the second term is damped by small t_p and $\Delta\Lambda$. Thus, we arrive at

$$\lim_{t\to\infty} \Delta V \approx \frac{\hbar}{\Delta\Lambda} \tag{31}$$

in accordance with (23) [99]. Here, similar to [99], Λ is a dynamic value fluctuating around zero.

And for case (2) $\Delta\Lambda$ becomes significant

$$\lim_{t\to t_{min}} \overline{V} = \overline{V}_{min} \sim \overline{V}_p = l_p^3; \lim_{t\to t_{min}} V = V_{min} \sim V_p = l_p^3 t_p \tag{32}$$

As a result, we have

$$\lim_{t\to t_{min}} \Delta V = \frac{\hbar}{\Delta\Lambda} + \alpha_\Lambda V_p^2 \frac{\Delta\Lambda}{\hbar} \tag{33}$$

where the parameter α_Λ absorbs all the above-mentioned proportionality coefficients.

For (33) $\Delta\Lambda \sim \Lambda_p \equiv \hbar/V_p = E_p/\overline{V}_p$.

It is easily seen that in this case $\Lambda \sim M_p^4$, in agreement with the value obtained using a naive (*i.e.*, without super-symmetry and the like) quantum field theory [25,26]. Despite the fact that Λ at Planck's scales (referred to as $\Lambda(UV)$) (33) is also a dynamic quantity, it is not directly related to well-known Λ (23),(31) (called $\Lambda(IR)$) because the latter, as opposed to the first one, is derived from Einstein's equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N \left(-\Lambda g_{\mu\nu} + T_{\mu\nu}\right) \tag{34}$$

However, Einstein's equations (34) are not valid at the Planck scales and hence $\Lambda(UV)$ may be considered as some high-energy generalization of the conventional cosmological constant, leading to $\Lambda(IR)$ in the low-energy limit.

In conclusion, it should be noted that the right-hand side of (25), (26) in fact is a series. Of course, a similar statement is true for (33) as well.

Then, we obtain a system of the Generalized Uncertainty Relations for the Early Universe (Planck's scales) in the symmetric form as follows:

$$\left\{ \begin{array}{lll} \Delta x & \geq & \frac{\hbar}{\Delta p} + \alpha' \left(\frac{\Delta p}{p_{pl}}\right) \frac{\hbar}{p_{pl}} + \ldots \\ & & \\ \Delta t & \geq & \frac{\hbar}{\Delta E} + \alpha' \left(\frac{\Delta E}{E_p}\right) \frac{\hbar}{E_p} + \ldots \\ & & \\ \Delta V & \geq & \frac{\hbar}{\Delta \Lambda} + \alpha_\Lambda \left(\frac{\Delta \Lambda}{\Lambda_p}\right) \frac{\hbar}{\Lambda_p} + \ldots \end{array} \right. \tag{35}$$

The latter of relations (35) may be important when finding the general form for $\Lambda(UV)$, low-energy limit $\Lambda(IR)$, and also may be a step in the process of deriving future quantum-gravity equations, the low-energy limit of which is represented by Einstein's equations (34).

It should be noted that a system of inequalities (35) may be complemented by the Generalized Uncertainty Relation in thermodynamics [77,83,103]. Let us consider the thermodynamic uncertainty relations between the inverse temperature and interior energy of a macroscopic ensemble

$$\Delta \frac{1}{T} \geq \frac{k}{\Delta U}, \tag{36}$$

where k is the Boltzmann constant. N. Bohr [104] and W. Heisenberg [105] have been the first to point out that such a kind of uncertainty principle should be involved in thermodynamics. The thermodynamic uncertainty relations (36) have been proven by many authors and in various ways [106–111]; their validity is unquestionable. Nevertheless, relation (36) has been proven in terms of the standard model of the infinite-capacity heat bath encompassing the ensemble. But it is obvious from the above inequalities that at very high energies the capacity of the heat bath can no longer be assumed infinite at the Planck scale. Indeed, the total energy of the "heat bath-ensemble" pair may be arbitrarily large but finite merely as the Universe was borne at the finite energy. Hence the quantity that can be interpreted as a temperature of the ensemble must have the upper limit and so does its main quadratic deviation. In other words, the quantity $\Delta(1/T)$ must be bounded from below. But in this case an additional term should be introduced into (36) [77,83,103]:

$$\Delta \frac{1}{T} \geq \frac{k}{\Delta U} + \eta \, \Delta U \tag{37}$$

where η is a coefficient. Dimension and symmetry reasons give

$$\eta \sim \frac{k}{E_p^2} \quad .$$

Similar to the previous cases, inequality (37) leads to the fundamental (inverse) temperature [77,83,103]:

$$T_{max} = \frac{\hbar}{\Delta t_{min} k} \sim \frac{\hbar}{t_p k}, \quad \beta_{min} = \frac{1}{kT_{max}} = \frac{\Delta t_{min}}{\hbar} \tag{38}$$

In the recently published work [112] the black hole horizon temperature has been measured with the use of the Gedanken experiment. In the process the Generalized Uncertainty Relations in thermodynamics (37) have been also derived. Expression (37) has been considered in the monograph [113] within the scope of the mathematical physics methods.

Besides, note that one of the first studies of the cosmological constant within the scope of the Heisenberg Uncertainty Principle has been presented in several works [114–116] demonstrating the inference: **"vacuum fluctuation of the energy density can lead to the observed cosmological constant"** [114]. In these works, however, no consideration has been given to GUP, whereas UV-cutoff has been derived artificially.

5. Gravitational Thermodynamics and Gravitational Holography at Low and High Energy

In the last decade several very interesting works have been published. We can primary name the works of T. Padmanbhan [115–126], where gravitation, at least for the spaces with horizon, is directly associated with thermodynamics, and the results obtained demonstrate a holographic character of gravitation. Of the greatest significance is a pioneer work written by T. Jacobson [93]. For black holes this association has been first revealed by Bekenstein and Hawking [127,128], who related the black-hole event horizon temperature to the surface gravitation. T. Padmanbhan, in particular in [125], has shown that this relation is not accidental and may be generalized to the spaces with horizon. As all the foregoing results have been obtained in a semi-classical approximation, *i.e.*, for sufficiently low energies, the problem arises: how these results are modified when going to higher energies. In the context of this paper, the problem may be stated as follows: since we have some infra-red (IR) cutoff L and ultraviolet (UV) cutoff l, we naturally have a problem how the above-mentioned results on Gravitational Thermodynamics are changed for

$$L \rightarrow l \tag{39}$$

According to Section 3 of this paper, they should become dependent on the deformation parameter α. After all, in the already mentioned Section 3 (11) α is indicated as nothing else but

$$\alpha = \frac{l^2}{L^2} \tag{40}$$

In fact, in several papers [129–135] it has been demonstrated that thermodynamics and statistical mechanics of black holes in the presence of GUP (*i.e.*, at high energies) should be modified. To illustrate, in [134] the Hawking temperature modification has been computed in the asymptotically flat space for this case in particular. It is easily seen that in this case the deformation parameter α arises naturally. Indeed, modification of the Hawking temperature is of the following form (formula (10) in [134]):

$$T_{GUP} = (\frac{d-3}{4\pi})\frac{\hbar r_+}{2\alpha'^2 l_p^2}[1-(1-\frac{4\alpha'^2 l_p^2}{r_+^2})^{1/2}] \tag{41}$$

where d is the space-time dimension, and r_+ is the uncertainty in the emitted particle position by the Hawking effect expressed as

$$\Delta x_i \approx r_+ \tag{42}$$

and being nothing else but a radius of the event horizon; α' is the dimensionless constant from GUP. But as we have $2\alpha' l_p = l_{min}$, in terms of α (41) may be written in a natural way as follows:

$$T_{GUP} = (\frac{d-3}{4\pi})\frac{\hbar\alpha_{r_+}^{-1}}{\alpha' l_p}[1-(1-\alpha_{r_+})^{1/2}] \tag{43}$$

where α_{r_+} - parameter α associated with the IR-cutoff r_+. In such a manner T_{GUP} is dependent on the constants including the fundamental ones and on the deformation parameter α only. The dependence of the black hole entropy on α may be derived in a similar way. For a semi-classical approximation of the Bekenstein-Hawking formula [127,128]

$$S = \frac{1}{4}\frac{A}{l_p^2} \tag{44}$$

where A is the surface area of the event horizon, provided the horizon event has radius r_+, then $A \sim r_+^2$ and (44) is clearly of the form

$$S = \sigma\alpha_{r_+}^{-1} \tag{45}$$

where σ is some dimensionless denumerable factor. The general formula for quantum corrections [133] is given as

$$S = \frac{A}{4l_p^2} - \frac{\pi\alpha'^2}{4}\ln\left(\frac{A}{4l_p^2}\right) + \sum_{n=1}^{\infty} c_n \left(\frac{A}{4l_p^2}\right)^{-n} + \text{const} \tag{46}$$

where the expansion coefficients $c_n \propto \alpha'^{2(n+1)}$ can always be computed to any desired order of accuracy [133], may also be written as a power series in $\alpha_{r_+}^{-1}$ (or Laurent series in α_{r_+})

$$S = \sigma\alpha_{r_+}^{-1} - \frac{\pi\alpha'^2}{4}\ln\left(\sigma\alpha_{r_+}^{-1}\right) + \sum_{n=1}^{\infty} c_n \left(\sigma\alpha_{r_+}^{-1}\right)^{-n} + \text{const} \tag{47}$$

Note that here no consideration is given to the restrictions on the IR-cutoff

$$L \leq L_{max} \tag{48}$$

and to those corresponding to the extended uncertainty principle (EUP) that leads to a minimal momentum [134]. This problem will be considered separately in further publications of the author.

A black hole is a specific example of the space with horizon. It is clear that for other horizon spaces [125] a similar relationship between their thermodynamics and the deformation parameter α should be exhibited.

Quite recently in a series of papers, and specifically in [117–123], it has been shown that Einstein equations may be derived from the surface term of the GR Lagrangian, in fact containing the same information as the bulk term.

And as the Einstein-Hilbert Lagrangian has the structure $L_{EH} \propto R \sim (\partial g)^2 + \partial^2 g$, in the conventional approach the surface term arising from $L_{surf} \propto \partial^2 g$ has to be cancelled to get Einstein equations from $L_{bulk} \propto (\partial g)^2$ [124]. But due to the relationship between L_{bulk} and L_{surf} [119–121,124], we have

$$\sqrt{-g}L_{suf} = -\partial_a\left(g_{ij}\frac{\partial\sqrt{-g}L_{bulk}}{\partial(\partial_a g_{ij})}\right) \tag{49}$$

In such a manner one can suggest a holographic character of gravity in that the bulk and surface terms of the gravitational action contain identical information. However, there is a significant difference between the first case, when variation of the metric g_{ab} in L_{bulk} leads to Einstein's equations, and the second case associated with derivation of the GR field equations from the action principle using only the surface term and virtual displacements of horizons [116], whereas the metric is not treated as a dynamic variable [124].

In the case under study, it is assumed from the beginning that we consider the spaces with horizon. It should be noted that in the Fischler–Susskind cosmic holographic conjecture it is implied that the Universe represents spherically symmetric space-time, on the one hand, and has a (Hubble) horizon (15), on the other hand. But proceeding from the results of [117–124], the entropy boundary is actually given by the surface of horizon measured in Planck's units of area [120]:

$$S = \frac{1}{4}\frac{A_R}{l_p^2} \tag{50}$$

where A_R is the horizon area corresponding to the Hubble horizon R (15).

To sum up, an assumption that space-time is spherically symmetric and has a horizon is the only natural assumption held in the Fischler–Susskind cosmic holographic conjecture to support its validity. Thus, the arguments in support of the Fischler–Susskind cosmic holographic conjecture are given on the basis of the results obtained lately on Gravitational Holography and Gravitational Thermodynamics.

It should be noted that Einstein's equations may be obtained from the proportionality of the entropy and horizon area together with the fundamental thermodynamic relation connecting heat, entropy, and temperature [93]. In fact [117–124], this approach has been extended and complemented by the demonstration of holographicity for the gravitational action (see also [125]). And in the case of the Einstein-Hilbert gravity, it is possible to interpret Einstein's equations as the thermodynamic identity [126]:

$$TdS = dE + PdV \tag{51}$$

The above-mentioned results in the last paragraph have been obtained at low energies, *i.e.*, in a semi-classical approximation. Because of this, the problem arises how these results are changed in the case of high energies? Or more precisely, how the results of [93,117–126] are generalized in the UV-cutoff? It is obvious that, as in this case all the thermodynamic characteristics become dependent on the deformation parameter α, the corresponding results should be modified (deformed) to meet the following requirements:

(a) to be clearly dependent on the deformation parameter α at high energies;

(b) to be duplicated, with high precision, at low energies due to the limiting transition $\alpha \to 0$.

(c) let us clear up what is meant by the adequate α-deformation of Einstein's equations (General Relativity) and by the Holographic Principle [85–89].

The problem may be more specific.

As, according to [93,125,126] and some other works, gravitation is greatly determined by thermodynamics and at high energies the latter is a deformation of the classical thermodynamics, it is interesting whether gravitation at high energies (or what is the same, quantum gravity or Planck scale)is determined by the corresponding deformed thermodynamics. The formulae (43) and (47) are elements of the high-energy α-deformation in thermodynamics, the general pattern of which still remains to be formed. Obviously, these formulae should be involved in the general pattern giving better insight into the quantum gravity, as they are applicable to black mini-holes (Planck's black holes) which may be a significant element of such a pattern. But what about other elements of this pattern? How can we generalize the results [93,125,126] when the IR-cutoff tends to the UV-cutoff (formula (39))? What are modifications of the thermodynamic identity (51) in a high-energy deformed thermodynamics and how is it applied in high-energy (quantum) gravity? What are the aspects of using the Generalized Uncertainty Relations in thermodynamics [77,83,103] (37),(37) in this respect? It is clear that these relations also form an element of high-energy thermodynamics.

By author's opinion, the methods developed to solve the problem mentioned in point (c) and elucidation of other indicted problems may form the basis for a new approach to solution of the quantum gravity problem. And one of the keys to the **quantum gravity** problem is a better insight into the **high-energy thermodynamics**.

6. QFT with UV-Cutoff for Different Approaches and Some Comments

(i) As shown by numerous authors (to start with [73]), the Quantum Mechanics with the fundamental length (UV-cutoff) generated by GUP is in line with the following deformation of Heisenberg algebra

$$[\vec{x},\vec{p}] = i\hbar(1+\beta^2\vec{p}^2+...) \tag{52}$$

and

$$\Delta x_{\min} \approx \hbar\sqrt{\beta} \sim l_p \tag{53}$$

In the recent works [136] it has been demonstrated that the Holographic Principle is an outcome of this approach, actually being integrated in the approach.

We can easily show that the deformation parameter β in (52),(53) may be expressed in terms of the deformation parameter α (see Section 3 of the text) that has been introduced in the approach associated with the density matrix deformation. Indeed, from (52),(53) it follows that $\beta \sim \mathbf{1/p^2}$, and for $x_{\min} \sim l_p$, β corresponding to $x_{\min}$ is nothing else but

$$\beta \sim 1/P_{pl}^2 \tag{54}$$

where P_{pl} is Planck's momentum: $P_{pl} = \hbar/l_p$.

In this way β is changing over the following interval:

$$\lambda/P_{pl}^2 \le \beta < \infty \tag{55}$$

where λ is a numerical factor and the second member in (52) is accurately reproduced in momentum representation (up to the numerical factor) by $\alpha = l_{min}^2/l^2 \sim l_p^2/l^2 = p^2/P_{pl}^2$

$$[\vec{x},\vec{p}] = i\hbar(1+\beta^2\vec{p}^2+...) = i\hbar(1+a_1\alpha+a_2\alpha^2+...) \tag{56}$$

As indicated in the previous Section (formula (45)), the parameter α has one more interesting feature:

$$\alpha_l^{-1} \sim l^2/l_p^2 \sim S_{BH} \tag{57}$$

Here α_l is the parameter α corresponding to l, S_{BH} is the black hole entropy with the characteristic linear size l (for example, in the spherically symmetric case $l = R$ - radius of the corresponding sphere with the surface area A), and

$$A = 4\pi l^2, S_{BH} = A/4l_p^2 = \pi\alpha_l^{-1} \tag{58}$$

This note is devoted to the demonstration of the fact that in case of the holographic principle validity in terms of the new deformation parameter α in QFT^{α}, considered above and introduced as early as 2002 [137–139], all the principal values associated with the Vacuum (Dark) Energy Problem may be defined simply and naturally. At the same time, there is no place for such a parameter in the conventional QFT, whereas in QFT with the fundamental length, specifically in QFT^{α}, it is quite natural [75,76,78,80,81,83].

(ii) It should be noted that smallness of α_R (Section 3) leads to a very great value of r in (20),(21). Besides, from (20) it follows that there exists some minimal entropy $S_{min} \sim S_p$, and this is possible only in QFT with the fundamental length.

(iii) This Section is related to Section 3 in [115] as well as to Sections 3 and 6 in [116]. The constant l_Λ introduced in these works is such that in the case under consideration $\Lambda \equiv l_\Lambda^{-2}$ is equivalent to R, *i.e.*, $\alpha_R \approx \alpha_{l_\Lambda}$ with $\alpha_{l_\Lambda} = l_p^2/l_\Lambda^2$. Then the expression on the right-hand side of (18) is the major term of the formula for ρ_{vac}, and its quantum corrections are nothing else but a series expansion in terms of α_{l_Λ} (or α_R)

$$\rho_{\text{vac}} \sim \frac{1}{l_p^4}\left(\frac{l_p}{l_\Lambda}\right)^2 + \frac{1}{l_p^4}\left(\frac{l_p}{l_\Lambda}\right)^4 + \cdots = \alpha_{l_\Lambda} m_p^4 + \ldots \tag{59}$$

In the first variant presented in [115] and [116] the right-hand side of (59) (formulae (12),(33)) in [115] and [116], respectively)reveals an enormous additional term $m_p^4 \sim \rho_{QFT}$ for renormalization. As indicated in the previous Section, it may be, however, ignored because the gravity is described by a pure surface term. And in the case under study, owing to the Holographic Principle, we may proceed directly to (59). Moreover, in QFT^{α} there is no need in renormalization as from the start we are concerned with the ultraviolet-finiteness.

Moreover, a series expansion of (59) in terms of α is a complete analogue of the expansion in terms of the same parameter, re-determining the measuring procedure in $QMFL^{\alpha}$ [76,78,80,83]:

$$Sp[\rho(\alpha)] - Sp^2[\rho(\alpha)] = \alpha + a_0'\alpha^2 + \ldots \tag{60}$$

As indicated in [84], the same expansion may be used to obtain quantum corrections to the semi-classical Bekenstein-Hawking formula (50) for the black hole entropy.

(iv) Besides, the Heisenberg algebra deformations are introduced due to the involvement of a minimal length in quantum mechanics. These deformations are stable in the sense of [140]. But this is not true for the unified algebra of Heisenberg and Poincare. This algebra does not carry the indicated immunity. It is suggested that the Lie algebra for the interface of the gravitational and quantum realms is in its stabilized form. Now it is clear that such a stability should be raised to the status of a physical principle. In a very interesting work of Ahluwalia-Khalilova [140] it has been demonstrated that the stabilized form of the Poincare–Heisenberg algebra [141,142] carries three additional parameters: "the length scale pertaining to the Planck/unification scale, the second length scale associated with cosmos, and a new dimensionless constant with the immediate implication that 'point particle' ceases to be a viable physical notion. It must be replaced by objects which carry a well-defined, representation space dependent, minimal spatiotemporal extent".

Thus, within the scope of a Quantum Field Theory with the UV-cutoff (fundamental length), closeness of the theoretical and experimental values for ρ_{vac} is adequately explained. In this case an important role is played by new parameters appearing in the corresponding Heisenberg Algebra deformation. Specifically, by the new small dimensionless parameter α, in terms of which one can adequately interpret both the smallness of ρ_{vac} and its modern experimental value. Besides, it is shown that the Generalized Uncertainty Principle (GUP) may be an instrument in studies of a dynamic character of the cosmological constant Λ.

7. Conclusions

In conclusion it should be noted that in a series of the author's works [75–84] a minimal α-deformation of QFT has been formed. By "minimal" it is meant that no space-time noncommutativity was required, *i.e.*, there was no requirement for noncommutative operators associated with different spatial coordinates

$$[X_i, X_j] \neq 0, i \neq j \tag{61}$$

However, all the well-known deformations of QFT associated with GUP (for example, [71–73]) contain (61) as an element of the corresponding deformed Heisenberg algebra. Because of this, it is necessary to extend (or modify) the above-mentioned minimal α-deformation of QFT (QFT^{α}) [75–84] to some new deformation $\widetilde{QFT}^{\alpha}$ compatible with GUP, as it has been noted in [143].

Besides, in this paper consideration has been given to QFT with a minimal length, *i.e.*, with the UV-cutoff. Consideration of QFT with a minimal momentum (or IR-cutoff) (48) necessitates an adequate extension of the α-deformation in QFT with the introduction of new parameters significant in the IR-limit. Proceeding from point (c) of Section 5, the problem may be stated as follows:

(c) Provided the α-deformation of GR describes the ultraviolet (quantum-gravity) limit of GR, it is interesting to examine the deformation type describing adequately the infrared limit of GR. It seems that some indications of a nature of such deformation may be found from the works devoted to the infrared modification of gravity [144,145].

8. Acknowledgement

This paper is devoted to the 65th anniversary of Vasili Strazhev.

References

1. Perlmutter, S.; Aldering, G.; Goldhaber, G.; *etc.* Measurements of Omega and Lambda from 42 high redshift supernovae. *Astrophys. J.* **1999**, *517*, 565–586.
2. Riess, A.G.; Filippenko, A.V.; Challis, P.; *etc.* Observational evidence from supernovae for an accelerating universe and a cosmological constant. *Astron. J.* **1998**, *116*, 1009–1038.
3. Riess A.G.; Kirshner, R.P.; Schmidt, B.P.; *etc.* BV RI light curves for 22 type Ia supernovae. *Astron. J.* **1999**, *117*, 707–724.

4. Sahni, V.; Starobinsky, A.A. The Case for a positive cosmological Lambda term. *Int. J. Mod. Phys. D* **2000**, *9*, 373–397.
5. Carroll, S.M. The Cosmological constant. *Living Rev. Rel.* **2001**, *4*, 1–50.
6. Padmanabhan, T. Cosmological constant: The Weight of the vacuum. *Phys. Rept* **2003**, *380*, 235–320.
7. Padmanabhan, T. Dark energy: the cosmological challenge of the millennium. *Current Sci.* **2005**, *88*,1057–1071.
8. Peebles, P.J.E.; Ratra, B. The cosmological constant and dark energy. *Rev. Mod. Phys.* **2003**, *75*, 559–606.
9. Ratra, B.; Peebles, J. Cosmological consequences of a rolling homogeneous scalar field. *Phys. Rev. D* **1988**, *37*, 3406–3422.
10. Caldwell, R.R.; Dave, R.; Steinhardt, P.J. Cosmological imprint of an energy component with general equation of state. *Phys. Rev. Lett.* **1998**, *80*, 1582–1585.
11. Armendariz-Picaon, C.; Damour,T.; Mukhanov, V. k - inflation. *Phys. Lett. B* **1999**, *458*, 209–218.
12. Garriga, J.; Mukhanov, V. Perturbations in k-inflation. *Phys. Lett. B* **1999**, *458*, 219–225.
13. Padmanabhan, T. Accelerated expansion of the universe driven by tachyonic matter. *Phys. Rev. D* **2002**, *66*, 021301.
14. Bagla, J.S.; Jassal, H.K.; Padmanabhan, T. Cosmology with tachyon field as dark energy. *Phys. Rev. D* **2003**, *67*, 063504.
15. Abramo, L.R.W.; Finelli, F. Cosmological dynamics of the tachyon with an inverse power-law potential. *Phys. Lett. B* **2003**, *575*, 165–171.
16. Aguirregabiria J.M.; Lazkoz, R. Tracking solutions in tachyon cosmology. *Phys. Rev. D* **2004**, *69*, 123502.
17. Guo, Z.K.; Zhang, Y.Z. Cosmological scaling solutions of the tachyon with multiple inverse square potentials. *JCAP* **2004**, *0408*, 010.
18. Copeland, E.J. *et al.* What is needed of a tachyon if it is to be the dark energy? *Phys. Rev. D* **2005**, *71*, 043003.
19. Sahni, V.; Shtanov, Y. Brane world models of dark energy. *JCAP* **2003**, *0311*, 014.
20. Elizalde, E.; Nojiri, S.; Odintsov, S.D. Late-time cosmology in (phantom) scalar-tensor theory: Dark energy and the cosmic speed-up. *Phys. Rev. D* **2004**, *70*, 043539.
21. Einstein, A. *Sitzungber. Preuss. Akad. Wiss.* **1917**, *1*, 142–152.
22. Friedmann, A. *Zs. Phys.* **1924**, *21*, 326–332.
23. Pais, A. *Subtle is the Lord... The Science and the Life of Albert Einstein*; Oxford Yniversity Press: New York, NY, USA, 1982.
24. Gliner, E.B. *ZHETF* **1965**, *49*, 542–549.
25. Zel'dovich, Y.B. The cosmological constant and the theory of elementary particles. *Sov. Phys. Uspehi.* **1968**, *11*, 381–393.
26. Weinberg, S. The cosmological constant problem. *Rev. Mod. Phys.* **1989**, *61*, 1–23.
27. Faddeev, L. Mathematical view on evolution of physics. *Priroda* **1989**, *5*, 11–18.
28. Cohen, A.; Kaplan, D.; Nelson, A. Effective field theory, black holes, and the cosmological constant. *Phys. Rev. Lett.* **1999**, *82*, 4971–4974.

29. Myung, Y. S. Holographic principle and dark energy. *Phys. Lett. B* **2005**, *610*, 18–22.
30. Myung, Y. S.; Min-Gyun Seo. Origin of holographic dark energy models. *Phys. Lett. B* **2009**, *617*, 435–439.
31. Huang, Q.G.; Li, M. The Holographic dark energy in a non-flat universe. *JCAP* **2004**, *0408*, 013.
32. Huang, Q.G.; Li, M. Anthropic principle favors the holographic dark energy. *JCAP* **2005**, *0503*, 001.
33. Huang, Q.G.; Gong, Y.G. Supernova constraints on a holographic dark energy model. *JCAP* **2004**, *0408*, 006.
34. Zhang, X.; Wu, F.Q. Constraints on holographic dark energy from Type Ia supernova observations. *Phys. Rev. D* **2005**, *72*, 043524.
35. Zhang, X. Statefinder diagnostic for holographic dark energy model. *Int. J. Mod. Phys. D* **2005**, *14*, 1597–1606.
36. Chang, Z.; Wu, F.Q.; Zhang, X. Constraints on holographic dark energy from X-Ray gas mass fraction of galaxy clusters. *Phys. Lett. B* **2006**, *633*, 14–18.
37. Wang, B.; Abdalla, E.; Su, R.K. Constraints on the dark energy from holography. *Phys. Lett. B* **2005**, *611*, 21–26.
38. Wang, B.;Lin, C.Y.; Abdalla, E. Constraints on the interacting holographic dark energy model. *Phys. Lett. B* **2006**, *637*, 357–361.
39. Zhang, X. Reconstructing holographic quintessence. *Phys. Lett. B* **2007**, *648*, 1–7.
40. Setare, M.R.; Zhang, J.; Zhang, X. Statefinder diagnosis in non-flat universe and the holographic model of dark energy. *JCAP* **2007**, *0703*, 007.
41. Zhang, J.; Zhang, X.; Liu, H. Statefinder diagnosis for the interacting model of holographic dark energy.*Phys. Lett. B* **2008**, *659*, 26–33.
42. Chen, B.; Li, M.; Wang, Y. Inflation with Holographic Dark Energy. *Nucl. Phys. B* **2007**, *774*, 256–267;
43. Zhang, J.; Zhang, X.; Liu, H. Holographic dark energy in a cyclic universe. *Eur. Phys. J. C* **2007**, *52*, 693–699.
44. Zhang, X.; Wu, F.Q. Constraints on Holographic Dark Energy from Latest Supernovae, Galaxy Clustering, and Cosmic Microwave Background Anisotropy Observations. *Phys. Rev. D* **2007**, *76*, 023502.
45. Feng, C.J. Holographic cosmological constant and dark energy. *Phys. Lett. B* **2008**, *663*, 367–371.
46. Ma, Y.Z.; Gong, Y., Features of holographic dark energy under the combined cosmological constraints. *Eur. Phys. J. C* **2009**, *60*, 303–315.
47. Li, M.; Lin, C.; Wang, Y. Some issues concerning Holographic Dark Energy. *JCAP* **2008**, *0805*, 023.
48. Li, M; Li, X.D.; Lin, C.; Wang, Y. Holographic gas as dark energy. *Commun. Theor. Phys.* **2009**, *51*, 181–186.
49. Cai, R.G. A dark energy model characterized by the age of the universe. *Phys. Lett. B* **2007**, *657*, 228–231.
50. Károlyházy, F. Gravitation and quantum mechanics of macroscopic objects. *Nuovo Cim. A* **1966**, *42*, 390–402.

51. Ng, Y.J.; Van Dam, H. Limit to space-time measurement. *Mod. Phys. Lett. A* **1994**, *9*, 335–340.
52. Sasakura, N. An uncertainty relation of space-time. *Prog. Theor. Phys.* **1999**, *102*, 169–179.
53. Maziashvili, M. Cosmological implications of Karolyhazy uncertainty relation. *Phys. Lett. B* **2007**, *652*, 165–168.
54. Wei, H.; Cai, R.G. Statefinder Diagnostic and w - w' analysis for the Agegraphic Dark Energy Models without and with interaction. *Phys. Lett. B* **2007**, *655*, 1–6.
55. Wei,H.; Cai, R.G. Cosmological constraints on new agegraphic dark energy. *Phys. Lett. B* **2008**, *663*, 1–6.
56. Neupane, I.P. Remarks on dynamical dark energy measured by the conformal age of the universe. *Phys. Rev. D* **2007**, *76*, 123006.
57. Maziashvili, M. Operational definition of (brane induced) space-time and constraints on the fundamental parameters. *Phys. Lett. B* **2008**, *666*, 364–370.
58. Zhang, J.; Zhang, X.; Liu, H. Agegraphic dark energy as a quintessence. *Eur. Phys. J. C* **2008**, *54*, 303–309.
59. Wu, J.P.; Ma, D.Z.; Ling, Y. Quintessence reconstruction of the new agegraphic dark energy model. *Phys. Lett. B* **2008**, *663*, 152–159.
60. Wei, H.; Cai, R.G. Interacting Agegraphic dark energy. *Eur. Phys. J. C* **2008**, *59*, 99–105.
61. Garay, L. Quantum gravity and minimum length. *Int. J. Mod. Phys. A* **1995**, *10*, 145–166.
62. Veneziano, G. A stringy nature needs just two constants *Europhys.Lett* **1986**, *2*, 199–211Universe as a nonuniform.
63. Amati, D.; Ciafaloni, M.; Veneziano, G. Can space-time be probed below the string size? *Phys. Lett. B* **1989**, **216**, 41–47.
64. Witten, E. Reflections on the fate of spacetime. *Phys. Today* **1996**, *49*, 24–28.
65. Scardigli, F. Generalized uncertainty principle in quantum gravity from micro–black hole Gedanken experiment. *Phys. Lett. B* **1999**, *452*, 39–44.
66. Adler, R. J.; Santiago, D. I. On gravity and the uncertainty principle. *Mod. Phys. Lett. A* **1999**, *14*, 1371–1378.
67. Bambi, C. A revision of the generalized uncertainty principle. *Class. Quant. Grav.* **2008**, *25*, 105003.
68. Ahluwalia, D.V. Wave particle duality at the Planck scale: Freezing of neutrino oscillations. *Phys. Lett.* **2000**, *A275*, 31–35.
69. Ahluwalia, D.V. Interface of gravitational and quantum realms. *Mod. Phys. Lett.* **2002**, *A17*, 1135–1146.
70. Maggiore, M. A generalized uncertainty principle in quantum gravity. *Phys. Lett.* **1993**, *B304*, 65–69.
71. Maggiore, M. Quantum groups, gravity and the generalized uncertainty principle. *Phys. Rev. D* **1994**, *49*, 5182–5187.
72. Maggiore, M. The Algebraic structure of the generalized uncertainty principle. *Phys. Lett. B* **1993**, *319*, 83–86.
73. Kempf, A.; Mangano, G.; Mann, R.B. Hilbert space representation of the minimal length uncertainty relation. *Phys. Rev. D* **1995**, *52*, 1108–1118.

74. Nouicer, K. Quantum-corrected black hole thermodynamics to all orders in the Planck length. *Phys. Lett. B* **2007**, *646*, 63–71.
75. Shalyt-Margolin, A.E.; Suarez, J.G. Quantum mechanics of the early universe and its limiting transition. *gr-qc/0302119*, pp. 16.
76. Shalyt-Margolin, A.E.; Suarez, J.G. Quantum mechanics at Planck's scale and density matrix. *Int. J. Mod. Phys. D* **2003**, *12*, 1265–1278.
77. Shalyt-Margolin, A.E.; Tregubovich, A.Ya. Deformed density matrix and generalized uncertainty relation in thermodynamics. *Mod. Phys. Lett. A* **2004**, *19*, 71–82.
78. Shalyt-Margolin, A.E. Nonunitary and unitary transitions in generalized quantum mechanics, new small parameter and information problem solving. *Mod. Phys. Lett. A* **2004**, *19*, 391–404.
79. Shalyt-Margolin, A.E. Pure states, mixed states and Hawking problem in generalized quantum mechanics. *Mod. Phys. Lett. A* **2004**, *19*, 2037–2045.
80. Shalyt-Margolin, A.E. The universe as a nonuniform lattice in finite volume hypercube. I. Fundamental definitions and particular features*Int. J. Mod. Phys D* **2004**, *13*, 853– 864.
81. Shalyt-Margolin, A.E. The universe as a nonuniform lattice in the finite-dimensional hypercube. II. Simple cases of symmetry breakdown and restoration. *Int. J. Mod. Phys. A* **2005**, *20*, 4951–4964.
82. Shalyt-Margolin, A.E.; Strazhev, V.I. The density matrix deformation in quantum and statistical mechanics in early universe. In Proceedings of the Sixth International Symposium "Frontiers of Fundamental and Computational Physics", 26-29 September 2004 , Udine, Italy; Sidharth, B.G., *et al.*, Eds.; Springer: New York, NY, USA, 2006; pp.131–134.
83. Shalyt-Margolin, A.E. The density matrix deformation in physics of the early universe and some of its implications. In *Quantum Cosmology Research Trends*, Horizons in World Physics. 246; Reimer, A., Ed.; Nova Science Publishers, Inc.: Hauppauge, NY, USA, 2005; pp. 49–91.
84. Shalyt-Margolin, A.E. Deformed density matrix and quantum entropy of the black hole. *Entropy* **2006**, *8*, 31–43.
85. Hooft, G. 'T. Dimensional reduction in quantum gravity. Essay dedicated to Abdus Salam *gr-qc/9310026*, p. 15.
86. Hooft, G. 'T. The Holographic Principle, *hep-th/0003004*, pp. 15;
87. Susskind, L. The World as a hologram. *J. Math. Phys.* **1995**, *36*, 6377–6396.
88. Bousso, R. The Holographic principle. *Rev. Mod. Phys.* **2002**, *74*, 825–874.
89. Bousso, R.A Covariant entropy conjecture. *JHEP* **1999**, *07*, 004.
90. Buniy, R.V.; Hsu, S.D.H. Entanglement entropy, black holes and holography. *Phys. Lett. B* **2007**, *644*, 72–76.
91. Balazs, C.; Szapudi, I. Naturalness of the vacuum energy in holographic theories. *hep-th/0603133*, p. 4.
92. Fischler, W.; Susskind, L. Holography and cosmology. *hep-th/9806039*, p. 7.
93. Jacobson, T. Thermodynamics of space-time: The Einstein equation of state. *Phys. Rev. Lett.* **1995**, *75*, 1260–1263.
94. Cai, R.-G.; Kim, S.P. First law of thermodynamics and Friedmann equations of Friedmann-Robertson-Walker universe. *JHEP* **2005**, *02*, 050.

95. Shalyt-Margolin, A.E.; Strazhev, V.I. Dark energy and deformed quantum theory in physics of the early universe. In *Non-Eucleden Geometry in Modern Physics*. In Proceedings of the 5th International Conference of Bolyai-Gauss-Lobachevsky, 2007; Kurochkin, Yu., Minsk, Red'kov, V., Eds.; pp. 173–178.
96. Mukohyama, S.; Randall, L. A Dynamical approach to the cosmological constant. *Phys. Rev. Lett.* **2004**, *92*, 211302.
97. Cai, R.G.; Hu, B.; Zhang, Y. Holography, UV/IR relation, causal entropy bound and dark energy. *Commun. Theor. Phys.* **2009**, *51*, 954–960.
98. Shapiro, I.L.; Sola, J. Can the cosmological "constant" run? - It may run. *arXiv:0808.0315*, p. 35.
99. Jejjala, V.; Kavic, M.; Minic, D. Time and M-theory. *Int. J. Mod. Phys. A* **2007**, *22*, 3317–3405.
100. Jejjala, V.; Kavic, M.; Minic, D. Fine structure of dark energy and new physics. *Adv. High Energy Phys.* **2007**, *2007*, 21586.
101. Jejjala, V.; Minic, D. Why there is something so close to nothing: Towards a fundamental theory of the cosmological constant. *Int. J. Mod. Phys. A* **2007**, *22*, 1797-1818.
102. Jejjala, V.; Minic, D.; Tze, C-H. Toward a background independent quantum theory of gravity. *Int. J. Mod. Phys. D* **2004**, *13*, 2307–2314.
103. Shalyt-Margolin, A.E.; Tregubovich, A.Ya. Generalized uncertainty relation in thermodynamics. *gr-qc/0307018*, p. 7.
104. Bohr, N. *Faraday Lectures*; Chemical Society: London, UK, 1932; pp. 349-384, 376-377.
105. Heisenberg, W. Der Teil und Das Ganze ch 9 R.Piper, Munchen, 1969.
106. Lindhard, J. Complementarity between energy and temperature. In *The Lesson of Quantum Theory*; de Boer, J., Dal, E., Ulfbeck, O., Eds.; North-Holland Publishing: Amsterdam, The Netherlands, 1986.
107. Lavenda, B. *Statistical Physics: a Probabilistic Approach*; J.Wiley and Sons: New York, NY, USA, 1991.
108. Mandelbrot,B. An outline of a purely a phenomenological of statistical thermodynamics: I. Canonical Ensembles. *IRE Trans. Inform. Theory* **1956**, *IT-2*, 190–198.
109. Rosenfeld L. In *Ergodic theories*; Caldrirola, P., Eds.; Academic Press: New York, NY, USA, 1961.
110. Schlogl, F. Thermodynamic uncertainty relation. *J. Phys. Chem. Solids* **1988**, *49*, 679–687.
111. Uffink, J.; Lith-van Dis, J. Thermodynamic uncertainty relation. *Found. Phys.***1999**, *29*, 655–679.
112. Farmany, A. Probing the Schwarzschild horizon temperature. *Acta Phys. Pol. B* **2009**, *40*, 1569–1574.
113. Carroll, R. Fluctuations, information, gravity and the quantum potential. *Fundam. Theor. Phys.*, **2006**, *148*, 454.
114. Padmanabhan, T. Vacuum fluctuations of energy density can lead to the observed cosmological constant. *Class. Quant. Grav.* **2005**, *22*, L107-L110.
115. Padmanabhan, T. Darker side of the universe ... and the crying need for some bright ideas! In Proceedings of the 29th International Cosmic Ray Conference, Pune, India, 2005; pp. 47–62.

116. Padmanabhan, T. Dark Energy: Mystery of the Millennium. *Paris 2005, Albert Einstein's century*, AIP Conference Proceedings 861; American Institute of Physics: New York, NY, USA, 2006; pp. 858–866.
117. Padmanabhan, T. A New perspective on gravity and the dynamics of spacetime. *Int. Jorn. Mod. Phys.* **2005**, *D14*, 2263–2270.
118. Padmanabhan, T. The Holography of gravity encoded in a relation between entropy, horizon area and action for gravity. *Gen. Rel. Grav.* **2002**, *34*, 2029–2035.
119. Padmanabhan, T. Holographic gravity and the surface term in the Einstein-Hilbert Action. *Braz. J. Phys.* **2005**, *35*, 362–372.
120. Padmanabhan, T. Gravity: A New holographic perspective. *Int. J. Mod. Phys. D* **2006**, *15*, 1659–1676.
121. Mukhopadhyay, A.; Padmanabhan, T. Holography of gravitational action functionals. *Phys. Rev. D* **2006**, *74*, 124023.
122. Padmanabhan, T. Dark energy and gravity. *Gen. Rel. Grav.* **2008**, *40*, 529–564.
123. Padmanabhan, T.; Paranjape, A. Entropy of null surfaces and dynamics of spacetime. *Phys. Rev. D* **2007**, *75*, 064004.
124. Padmanabhan, T. Gravity as an emergent phenomenon: A conceptual description. International Workshop and at on Theoretical High Energy Physics (IWTHEP 2007), AIP Conference Proceedings 939; American Institute of Physics: New York, NY, USA, 2007; pp. 114–123.
125. Padmanabhan, T. Gravity and the thermodynamics of horizons. *Phys. Rept* **2005**, *406*, 49–125.
126. Paranjape, A.; Sarkar, S.; Padmanabhan, T. Thermodynamic route to field equations in Lancos-Lovelock gravity. *Phys. Rev. D* **2006**, *74*, 104015.
127. Bekenstein, J.D. Black holes and entropy. *Phys. Rev. D* **1973**, *7*, 2333–2345.
128. Hawking, S. Black holes and thermodynamics. *Phys. Rev. D* **1976**, *13*,191–204.
129. Adler, R.J.; Chen, P.; Santiago, D.I. The generalized uncertainty principle and black hole remnants. *Gen. Rel. Grav.* **2001**, *13*, 2101-2108.
130. Custodio, P.S.; Horvath, J.E. The Generalized uncertainty principle, entropy bounds and black hole (non)evaporation in a thermal bath. *Class. Quant. Grav.* **2003**, *20*, L197-L203.
131. Cavaglia, M.; Das, S. How classical are TeV scale black holes? *Class. Quant. Grav.* **2004**, *21*, 4511–4523.
132. Bolen, B.; Cavaglia, M. (Anti-)de Sitter black hole thermodynamics and the generalized uncertainty principle. *Gen. Rel. Grav.* **2005**, *37*, 1255–1263.
133. Medved, A.J.M.; Vagenas, E.C. When conceptual worlds collide: The GUP and the BH entropy. *Phys. Rev. D* **2004**, *70*, 124021.
134. Park, M.-I. The Generalized Uncertainty Principle in (A)dS Space and the Modification of Hawking Temperature from the Minimal Length. *Phys. Lett. B* **2008**, *659*, 698–702.
135. Kim, W.; Son, E.J.; Yoon, M. Thermodynamics of a black hole based on a generalized uncertainty principle. *JHEP* **2008**, *08*, 035.
136. Kim, Y.W.; Lee, H.W.; Myung, Y.S. Entropy bound of local quantum field theory with generalized uncertainty principle. *Phys. Lett. B* **2009**, *673*, 293-296.

137. Shalyt-Margolin, A.E. Fundamental length, deformed density matrix and new view on the black hole information paradox. *gr-qc/0207074*, p. 12.
138. Shalyt-Margolin, A.E.; Suarez, J.G. Density matrix and dynamical aspects of quantum mechanics with fundamental length. *gr-qc/0211083*, p. 14.
139. Shalyt-Margolin, A.E.; Tregubovich, A.Ya. Generalized uncertainty relations, fundamental length and density matrix. *gr-qc/0207068*, p. 11.
140. Ahluwalia-Khalilova, D.V. Minimal spatio-temporal extent of events, neutrinos, and the cosmological constant problem. *Int. J. Mod. Phys. D* **2005**, *14*, 2151–2166.
141. Vilela Mendes, R. Geometry, stochastic calculus and quantum fields in a noncommutative space-time. *J. Math. Phys* **2000**, *41*, 156–186.
142. Chryssomalakos, C.; Okon, E. Generalized quantum relativistic kinematics: A stability point of view. *Int. J. Mod. Phys. D* **2004**, *13*, 2003–2034.
143. Shalyt-Margolin, A.E. Entropy in the present and early universe. *Symmetry* **2007**, *18*, 299–320.
144. Patil, S.P. Degravitation, inflation and the cosmological constant as an afterglow. *JCAP* **2009**, *0901*, 017.
145. Park, Mu-in. The black hole and cosmological solutions in IR modified Horava Gravity. *JHEP* **2009**, *0909*, 123.