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The Connection Between Entropy and the Absorption Spectra of Schwarzschild Black Holes for Light and Massless Scalar Fields

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Abstract: We present heuristic arguments suggesting that if EM waves with wavelengths somewhat larger than the Schwarzschild radius of a black hole were fully absorbed by it, the second law of thermodynamics would be violated, under the Bekenstein interpretation of the area of a black hole as a measure of its entropy. Thus, entropy considerations make the well known fact that large wavelengths are only marginally absorbed by black holes, a natural consequence of thermodynamics. We also study numerically the ingoing radial propagation of a scalar field wave in a Schwarzschild metric, relaxing the standard assumption which leads to the eikonal equation, that the wave has zero spatial extent. We find that if these waves have wavelengths larger than the Schwarzschild radius, they are very substantially reflected, fully to numerical accuracy. Interestingly, this critical wavelength approximately coincides with the one derived from entropy considerations of the EM field, and is consistent with well known limit results of scattering in the Schwarzschild metric. The propagation speed is also calculated and seen to differ from the value c , for wavelengths larger than R_s , in the vicinity of R_s . As in all classical wave phenomena, whenever the wavelength is larger or comparable to the physical size of elements in the system, in this case changes in the metric, the zero extent 'particle' description fails, and the wave nature becomes apparent.

Keywords: Physics of black holes; Classical black holes; Quantum aspects of black holes; evaporation; thermodynamics.

1. Motivation

The presence of a black hole divides the universe into two causally distinct regions. Essentially, an event horizon hides a singularity from the external universe with an inside region which is causally disconnected from the outside one. The simplest example of a black hole is the one given by the Schwarzschild metric. These Schwarzschild black holes have a “spherical” event horizon. These holes are described by a single parameter, their total collapsed mass M located at a single point, in the “centre” of the event horizon.

The problem of accounting for the apparent entropy decrease for the universe when a body is swallowed by a black hole, hence making its entropy disappear from the outside region, has been given an answer [1] by associating an entropy to the area of the black hole horizon. This last quantity remains accessible to measurements performed by external observers through its dependence on the black hole mass. The extensive work on black hole thermodynamics of the 60s and 70s notably by Bekenstein and Hawking (cf. [2] and references therein) has led to the establishment of the so called laws of black hole thermodynamics, where black holes appear as classical thermodynamical objects, having entropy S_{BH} and temperature T_H given by:

$$\frac{S_{BH}}{k_B} = \frac{A_{SH}}{4A_P} \quad (1)$$

and

$$T_H = \frac{hc}{8\pi^2 k_B R_s}. \quad (2)$$

In the above equations $A_{SH} = 4\pi R_s^2$ is the area of the Schwarzschild event horizon, $A_P = \hbar G/c^3$ is the Planck area and $R_s = (2GM)/c^2$, the Schwarzschild radius of a black hole of mass M . It can be seen, for example, that the merger of two black holes of equal mass will result in a net increase in entropy, and hence an event one should expect could happen, in terms of the second law of thermodynamics.

We now point to a particular process which would appear to violate the second law. Suppose a black hole of mass M is absorbing an amount of classical black body radiation having an energy E_{EM} , temperature T_{EM} and entropy S_{EM} . When far from the black hole, this entropy is given by

$$\frac{S_{EM}}{k_B} = \frac{4}{3} \frac{E_{EM}}{k_B T_{EM}}. \quad (3)$$

The total entropy before the radiation is swallowed by the black hole will therefore be $S_{BH} + S_{EM}$. After swallowing the radiation, the black hole will experience an increase in mass $\Delta M = E_{EM}/c^2$, and hence an increase in entropy given by equation (1) as:

$$\frac{\Delta S_{BH}}{k_B} = \frac{8\pi^2 G}{hc} M \Delta M, \quad (4)$$

in this case,

$$\frac{\Delta S_{BH}}{k_B} = \frac{8\pi^2 G}{hc^3} M E_{EM}. \quad (5)$$

We can now write the quotient of this increase to the original black body radiation entropy, which was lost to the universe on it being swallowed, as:

$$\frac{\Delta S_{BH}}{S_{EM}} = \frac{6\pi^2 G}{hc^3} k_B T_{EM} M. \quad (6)$$

Now using equation (2) we can write,

$$\frac{\Delta S_{BH}}{S_{EM}} = \frac{3\pi}{8} \frac{T_{EM}}{T_H}. \quad (7)$$

The right hand side of the above equation becomes < 1 for $T_{EM} < 8T_H/3\pi$. We see that the process of a black hole swallowing Planck radiation of a temperature somewhat lower than T_H results in an overall decrease in the entropy of the universe. Seen in this way, this process would violate the second law of thermodynamics. It would appear to follow that at least part of the impinging radiation should not be absorbed. The following considerations point to which part of this radiation one might expect not to be absorbed.

We now consider a black hole of mass M swallowing a single photon of wavelength λ_γ and carrying an energy $E_\gamma = hc/\lambda_\gamma$. The increase in the black hole mass will now be $\Delta M = h/c\lambda_\gamma$, to which there corresponds an increase in the area of the Schwarzschild horizon of:

$$\Delta A = 8\pi R_S \frac{hG}{c^3 \lambda_\gamma}, \quad (8)$$

which we can write as:

$$\frac{\Delta A}{A_P} = 16\pi^2 \frac{R_S}{\lambda_\gamma}. \quad (9)$$

We see that the increase in the black hole area becomes arbitrarily small, in particular less than the Planck area, for photons having wavelengths somewhat larger than the Schwarzschild radius of the black hole swallowing them. This last might seem uncomfortable if one adopts the point of view that the smallest dimension of area which should appear in any physical theory or process is the Planck area, e.g. a loop quantum gravity approach. The case of a “single photon” is of course an idealisation which strictly should be treated in the quantum regime. For a black hole in the quantum regime itself, the results of [3] already imply no violation of the second law of thermodynamics, it is the macroscopic limit what we will consider here.

The preceding two thought experiments lead to the conclusion that either equation (1) is not valid and arbitrarily small area increases are allowed for black holes, or radiation colder than T_H (or photons with wavelengths longer than R_S) can not be fully swallowed by black holes, if one wants to keep the second law of thermodynamics. We see that entropy considerations dictate the outcome of physical processes around black holes. The detailed physical mechanism involved has to be explored in the scope of scattering problems in the Schwarzschild space–time, as it has been done in the past by several

authors [4–7], however entropy constrains offer qualitative indications of the results to be expected. The results of [4] already point to an upper limit for the wavelength of an EM wave above which absorption by a black hole strongly decreases, of order R_S . We note that past studies of the interaction between waves and black holes which identify upper critical wavelengths, typically fall within the scope of WKB approximations, where monochromatic waves are assumed to remain as such. As will be seen in the following, the particular nature of the problem makes the above assumption invalid. Indeed, as already noted by [4], the error on the transmission coefficients under such approaches is of order M/λ , somewhat worrisome as the critical wavelengths are typically of the order M .

The suggestion that EM wavelengths longer than the Schwarzschild radius cannot be absorbed by a black hole is interesting, since going back to equation (9) implies that the smallest increase in the area of a black hole which can result from the absorption of a photon, will be of order $16\pi^2 A_p$. Bearing in mind that whereas the quanta of action \hbar can be inferred from experiments due to its direct effect on observables of the electro-magnetic field, the quanta of area cannot, and has only been estimated to lie close to A_p on dimensional grounds. The above results might hint at the quanta of area A_q , being of the order of $16\pi^2 A_p$, with the quanta of length resulting an order of magnitude larger than the canonical Planck length.

We now turn to the propagation of EM radiation in the vicinity of a black hole, to see if any mechanism to prevent radiation with wavelengths larger than R_S from being swallowed might naturally arise. The full EM problem, even with some simplifications, leads to complicated, coupled differential equations. As a first approach we simplify to an equation which is valid for the problem of the interaction of a scalar field and a black hole.

2. Scalar waves in the Schwarzschild space–time

We start this section with the problem of a propagating EM wave, moving towards a Schwarzschild black hole. On first impression one would naively jump to the conclusion that as photons can be treated as massless particles, and hence, any form of EM radiation, composed of photons, should simply follow null geodesics into the black hole. However, such a treatment is derived under the assumption of vanishingly small wavelengths. See for example [8, 9], where the geometrical optics approximation for EM waves is derived by requiring that the metric can be treated as locally flat over the spatial variations in the studied wave. The eikonal equation, trajectories described by $ds = 0$ are, strictly speaking, approximations to the propagation of EM radiation valid only when the wavelengths can be treated as zero. In general, whenever the dimensions of a system are much larger than the wavelength of radiation moving around in it, light can accurately be treated as point particles having zero extent. However, whenever elements appear with dimensions comparable or smaller than the wavelengths of the radiation present, the wave nature of light is immediately apparent, and EM radiation must be treated explicitly as a wave. In the case of radiation having wavelengths comparable or larger than the Schwarzschild radius of a black hole, it is the variations in the metric which become comparable to the wavelength of the EM waves. We must therefore treat the problem outside of the eikonal approximation, and study the full physics of it.

As it will be shown below, the problem becomes highly complex near $R \sim 1$, with even simple wave pulses developing complex structures in frequency space and exhibiting a large range of propagation speeds. Thus, a tortoise coordinate system where the characteristics of the equations we are trying to

solve correspond to waves with propagation speeds equal to that of light, will result cumbersome and impractical. The coordinate singularity at the event horizon in the Schwarzschild space–time will not be of concern as we are only interested in a careful treatment of the reflection of long wavelength waves which occurs before they reach the event horizon. Examples of studies treating wave reflection of black holes in the Schwarzschild metric can be found in [7, 10, 11]. The well known absorption of short wavelength waves will not be treated in this article.

The electromagnetic potential one–form \mathbf{A} is related to the Faraday 2–form \mathbf{F} by [12] $\mathbf{F} := \mathbf{dA}$, where \mathbf{d} represents the exterior derivative. In the absence of charge–currents, one pair of Maxwell’s equations are given by $\delta F = \delta \mathbf{dA} = 0$, where δ represents the codifferential operator. With this, the electromagnetic potential one–form satisfies the relation $\Delta \mathbf{A} - \mathbf{d} \delta \mathbf{A} = 0$, where $\Delta = (\mathbf{d} + \delta)^2$ represents Laplace–de Rham’s operator. By imposing the Lorenz gauge given by $\delta \mathbf{A} = 0$ it then follows that the electromagnetic potential one–form satisfies a wave–like equation given by [13] $\Delta \mathbf{A} = 0$. In components, this equation yields:

$$\begin{aligned}
 A^{\alpha;\beta}{}_{;\beta} = & \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\beta} \left\{ \sqrt{-g} \frac{\partial A^\alpha}{\partial x_\beta} \right\} \\
 & + 2\Gamma^\alpha{}_{\lambda\beta} \frac{\partial A^\lambda}{\partial x_\beta} + \frac{A^\theta \Gamma^\alpha{}_{\theta\rho}}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} (\sqrt{-g} g^{\mu\rho}) \\
 & + g^{\beta\rho} A^\theta \frac{\partial \Gamma^\alpha{}_{\theta\rho}}{\partial x^\beta} + g^{\beta\rho} \Gamma^\alpha{}_{\lambda\beta} \Gamma^\lambda{}_{\theta\rho} A^\theta = 0.
 \end{aligned}
 \tag{10}$$

Limit and approximate solutions of this problem exist in the literature [7]. In what follows, we include a simplified presentation leading to a full numerical integration. If we retain only the first term of the above sum, for a particular component Ψ of A^μ this results in [see for example 10]:

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\beta} \left\{ \sqrt{-g} \frac{\partial \Psi}{\partial x_\beta} \right\} = 0.
 \tag{11}$$

We note that this equation rigorously represents the propagation of a massless scalar field Ψ in a curved space–time, which is given by $\Delta \Psi = 0$ [7]. The solution of equation (11) will shed some light as to the qualitative behaviour of the full EM solution, e.g. [10] studies the reflection of scalar waves from black holes under a spectral decomposition analysis, as a first order qualitative model for EM and gravitational waves. We shall now concentrate on solving equation (11), rigorously valid for a scalar field in vacuum.

Taking a $(-, +, +, +)$ signature the Schwarzschild metric becomes

$$ds^2 = \left(-1 + \frac{2M}{r} \right) dt^2 + \left(1 - \frac{2M}{r} \right)^{-1} dr^2 + r^2 d\Omega^2,
 \tag{12}$$

where

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2.
 \tag{13}$$

resulting in:

$$g_{\mu\nu} = \text{diag} \left[\left(-1 + \frac{2M}{r} \right), \left(1 - \frac{2M}{r} \right)^{-1}, r^2, r^2 \sin^2\theta \right],
 \tag{14}$$

$$\sqrt{-g} = r^2 \sin\theta \tag{15}$$

and,

$$g^{\mu\nu} = \text{diag} \left[\left(-1 + \frac{2M}{r} \right)^{-1}, \left(1 - \frac{2M}{r} \right), r^{-2}, \frac{1}{r^2 \sin^2\theta} \right] \tag{16}$$

We will retain the approximation that the metric is not modified by the presence of the scalar field, vanishing field strength, but otherwise introduce no approximations on the derivatives of Ψ . We will not introduce a vanishing wave dimension. For the Schwarzschild metric, equation (11) reduces to:

$$\left\{ \left(-1 + \frac{2M}{r} \right)^{-1} \frac{\partial^2}{\partial t^2} + \left[\frac{2}{r} \left(1 - \frac{2M}{r} \right) \frac{\partial}{\partial r} + \frac{2M}{r^2} \frac{\partial}{\partial r} + \left(1 - \frac{2M}{r} \right) \frac{\partial^2}{\partial r^2} \right] \right\} \Psi = 0, \tag{17}$$

where we have dismissed all non-radial spatial derivatives, as we are interested only on a purely radially propagating scalar wave. Further algebra reduces the above equation to:

$$\frac{1}{c^2} \frac{\partial^2 \Psi}{\partial t^2} = \frac{(r - R_S)^2}{r^2} \left(\frac{\partial^2 \Psi}{\partial r^2} + \frac{(2r - R_S)}{r(r - R_S)} \frac{\partial \Psi}{\partial r} \right). \tag{18}$$

It is clear that for $r \gg R_S$ the above equation reduces to the classical spherical wave propagation equation, as should be expected, since the Schwarzschild metric reduces to Minkowski space-time for $r \gg R_S$.

Defining dimensionless quantities for the problem, $T := ct/R_S$ and $R := r/R_S$, equation (18) becomes:

$$\frac{\partial^2 \Psi}{\partial T^2} = \left(\frac{R - 1}{R} \right)^2 \frac{\partial^2 \Psi}{\partial R^2} + \frac{(2R - 1)(R - 1)}{R^3} \frac{\partial \Psi}{\partial R}. \tag{19}$$

If we now propose a solution of the form $\Psi(R, T) = F(T)G(R)$, we can separate equation (19) to obtain:

$$\frac{d^2 F}{dT^2} = -W^2 F \tag{20}$$

and

$$\frac{(R - 1)^2}{R^2} \left(\frac{d^2 G}{dR^2} + \frac{(2R - 1)}{R(R - 1)} \frac{dG}{dR} \right) = -W^2 G \tag{21}$$

Where $W = \omega R_S/c$ and the separation constant was chosen < 0 to guarantee periodic wave propagation. The solution to equation (20) is trivial, while equation (21) requires somewhat more treatment.

We propose an approximate solution of the form $G(R) = U(R)\exp(iKR)$, which is then introduced into (21), the imaginary component of which yields:

$$2R \frac{dU}{dR} + \left(\frac{2R - 1}{R - 1} \right) U = 0, \tag{22}$$

and so,

$$U = \frac{C}{R^{1/2}(R-1)^{1/2}}. \quad (23)$$

Again, it is clear that for $R \gg 1$ the geometric dilution factor of $1/R$ on the potential, and hence $1/R^2$ on the energy, for spherical wave propagation is recovered. The real part of equation (21) after introducing the trial solution gives the dispersion relation for the problem:

$$K^2 = \frac{1 + 4W^2R^4}{4R^2(R-1)^2}. \quad (24)$$

We can see that the standard $K = W$ dispersion relation is recovered for $R \gg 1, W \gg 1$. At this point we can calculate the propagation velocity as W/K ,

$$V = \frac{2WR(R-1)}{(1 + 4W^2R^4)^{1/2}}. \quad (25)$$

We obtain the surprising result that the radial propagation of the scalar wave towards a black hole does not always proceed at speed c (recall that $c = 1$ in the above units), but actually slows down on approaching R_S , at a rate that depends on the frequency of the wave, the effect becoming increasingly strong as W decreases. Part of the above effect, that at $W \gg 1$ is simply a consequence of the Schwarzschild coordinate system, in which an inertial observer calculates infinite travel times for waves travelling towards a black hole, however, a chromatic effect is introduced by having considered a fuller physics than the standard zero wavelength approximation. Again, in the limit $W \gg 1, R \gg 1$ we recover $V = 1$.

We have obtained a dispersion relation which shows clear deviations from the standard expression for the standard $ds = 0$ solution corresponding to $W \sim 1$ and smaller. We notice that the above analysis offers only a local approximation, a first correction valid for non-zero, but small wavelengths, for which K can be considered approximately constant; a fuller solution to the problem requires a numerical treatment of equation (19). The dependence of K with R exemplifies the limitations of any spectral WKB approach to the problem.

3. Numerical calculations

We note that the extensive literature on scattering of waves from black holes treats the problem either from the point of view of perturbations to the metric (non-vanishing field strength for the wave), limit behaviour, or using a Fourier decomposition analysis [e.g. 2, 5–7, 10, 11, 15, 16, 19–21, 24–26], or the recent article by Dolan [14] and references therein. That the full complexity of the problem makes a spectral decomposition approach only an approximation, specially for $R \sim 1$, is already suggested by the inconsistencies reached after the proposed $G(R)$ solution to equation (21). In equation (24), K is seen to be dependent on R , contradicting the original assumption. We shall perform a direct integration of the resulting equations with the aim of following explicitly the full behaviour of the problem, in the vanishing field strength limit, a test field interacting with the black hole.

In order to obtain numerical solutions to equation (19) we have used an explicit finite-difference leapfrog scheme (central approximation in both time and space) which affords a local truncation error (LTE) of order $k^2 + h^2$, where k is the fixed time step and h is the fixed position step. All of the results

presented in this section have been obtained using $k = h = 0.001$, so that the LTE remains at all times quite small. Note that in the numerical scheme, the right hand side of equation(19) is evaluated, and used to obtain the evolution of the field through the left hand side of equation(19). In this way, there is no singularity at $R = 1$, only a zero, which is in any case explicitly avoided by choosing a discretisation which avoids having a grid point at $R = 1$.

It has also been checked that these choices of k and h satisfy the well known Courant-Friedrichs-Lewy condition for the stability of an explicit finite-difference scheme for a hyperbolic partial differential equation, such as equation (19), [18]. Reducing the size of k and h by a factor of 10 yields results with only minimal differences from those shown, and then, only at the peaks, proving numerical convergence of the scheme.

Figure 1. The figure shows four time steps in the propagation of a scalar pulse of characteristic length R towards a black hole. We see substantial deformation of the pulse on approaching $R = 1$, accompanied by a propagation speed which tends to zero. A video of this simulation can be found at <http://www.mendoza.org/sergio/bouncing-bh> and at <http://www.youtube.com/watch?v=jBoHQv2BrF0>

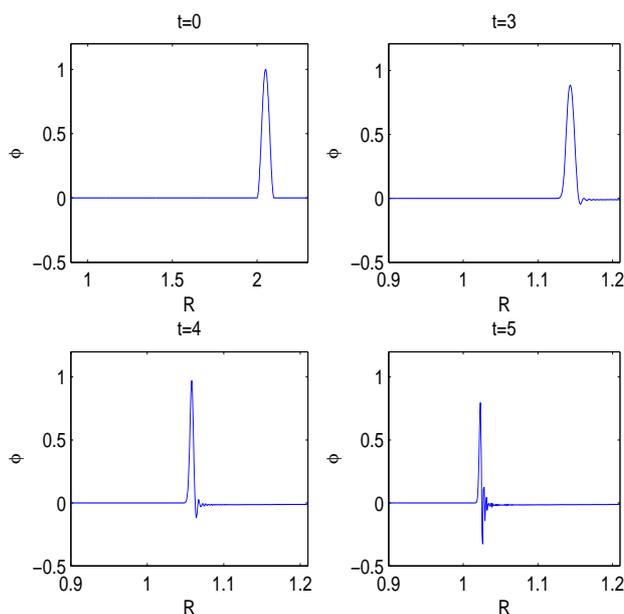
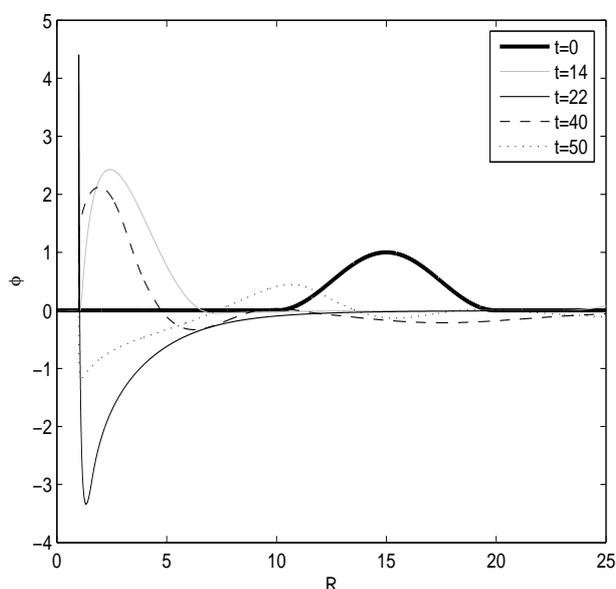


Figure 1 shows results for a pulse of wavelength $0.1R$, a regime where the standard small wavelength approximation would be expected to hold. In the above, as in all that follows, the term wavelength refers to the typical extent of the pulse when far from $R = 1$. We see that although propagation towards $R = 1$ proceeds with deformations of the original pulse, the pulse approaches $R = 1$ with a speed given by equation (25) (see figure 4), which appears indistinguishable from the $ds = 0$ solution of $c = (R - 1)/R$. The spatial extent of the pulse is progressively reduced, in consistency with standard gravitational blueshift. If played in reverse, we would see the gravitational redshift of a wave emitted from an $R \rightarrow 1$, tending to infinity. The pulse essentially stalls and would reach $R = 1$ in an infinite

time, given the Schwarzschild description of the problem from the point of view of a distant observer.

Figure 2. The figure shows four time steps in the propagation of a scalar pulse of characteristic length $10R$ towards a black hole. We see substantial deformation of the pulse as it approaches $R = 1$, followed by the emission of a wavetrain of wavelength comparable to the original wave $10R$, after a finite time. A video of the simulation can be obtained from <http://www.mendoza.org/sergio/bouncing-bh>, <http://www.youtube.com/watch?v=u52CqjVerlQ> and <http://www.youtube.com/watch?v=g1AruhdDzGA>



On the other hand, figure 2 shows the propagation of a pulse having an initial extent of $10R$, starting at $R = 10$. This pulse approaches $R = 1$, is deformed substantially, and produces a reflected wave train, with a residual amplitude in the vicinity of $R = 1$ which slowly decays. We see a reflected pulse appearing in a finite coordinate time. Notice that a wave solution ceases to be valid for large pulses, as the original pulse shape is completely lost. This shows that any description of the scattering problem in terms of a WKB treatment is at best a first approximation as the evolution near $R \sim 1$ invalidates the assumptions of spectral decomposition analysis, e.g. as noted in [22], and treated in the improved analytical method given there. A single component in frequency space gives rise to complex spectra as $R \rightarrow 1$ [21, 23].

In terms of the thought experiments of the opening section, we see that a strictly standard mechanism naturally arises, such that scalar waves larger than the Schwarzschild radius are essentially prevented from entering the black hole. It is interesting to see that the critical wavelength, for the scalar field, appears approximately at the scale identified by the heuristic entropy considerations of Section 1, when considering EM waves. We have identified the critical wavelength to lie somewhat below $1R$. By comparing reflected scalar waves of various initial wavelengths, we conclude that the characteristic wavelength of the reflected wavetrain is of the order of the initial wavelength, as happens in the case of

gravitational radiation [6].

Away from $R = 1$ we observe that, as expected, the physics is very similar to spherical propagation, as the limiting form of equation (19) as $R \rightarrow \infty$ is precisely the equation for a spherically symmetric scalar wave. As $R \rightarrow 1^+$, however, the propagation velocity decreases noticeably and the wave amplitude increases, giving the impression that the waveform is being smeared against $R = 1$. This is explained by the fact that the propagation term is proportional to $(R - 1)^2$ while the spherical spreading term is proportional to $(R - 1)$, so that the influence of the propagation term becomes smaller at a more rapid rate than that of the spreading term.

Figure 3. The figure shows four time steps in the propagation of a scalar pulse initially with an extent of $1R$ towards a black hole. We see substantial deformation of the pulse as it approaches $R = 1$, followed by the emission of a wavetrain of wavelength comparable to the original wave, after a finite time.

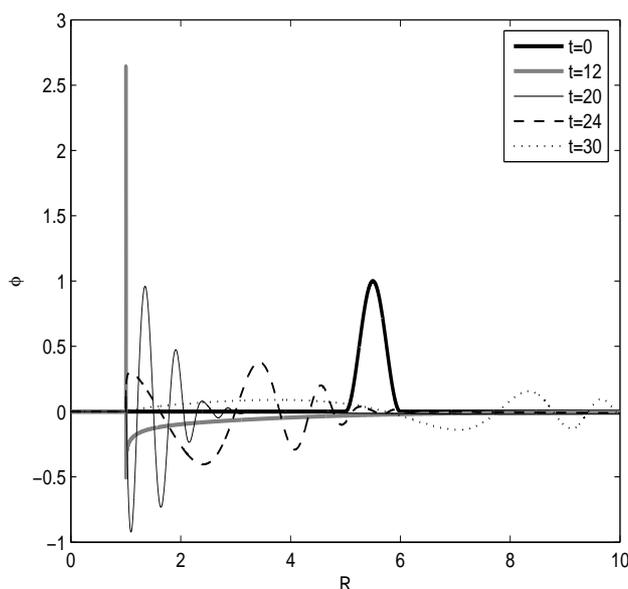


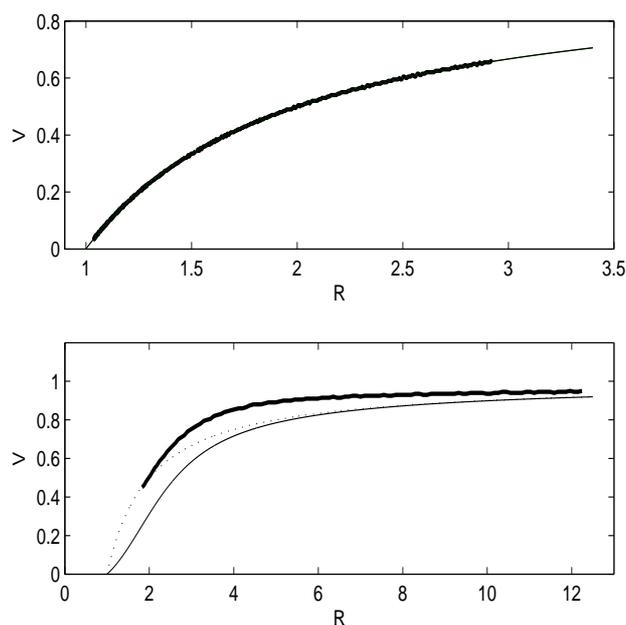
Figure 3 is analogous to figure 2, but for a pulse having an initial extent of $1R$, close to the critical value, we see an essentially equivalent behaviour to that of the $10R$ pulse. The critical value for the strong qualitative change in regime, from that of figure 1 to that of figures 2 and 3, lying slightly below this point.

The preceding results are not unexpected, if one considers existing studies of gravitational backscattering of light, where the fact that light rays can travel along non-null geodesics is well known. We note important precedents in the work of [19] who showed that in the limit as $\lambda \rightarrow \infty$, electromagnetic and gravitational waves will be reflected off Kerr black holes. Also, [20] demonstrated analytically, under the assumption that a strictly wave solution should always apply, that the reflection coefficient for EM waves tends to 1 for wavelengths of the order of R_S . A comprehensive study of scattering from black holes can be found in [7]. However a full numerical solution of scattering of a massless scalar test field

is not found in the literature, neither are the entropy arguments relating limiting absorption wavelengths and the second law of thermodynamics, as presented here.

More recent numerical studies include [21, 23–25] and references therein. It has been previously found that for the case of backscattering by Schwarzschild black holes, although mostly treated in terms of perturbations on the metric, the corresponding space–time works as a nonuniform medium with a varying refraction index for electromagnetic waves. The magnitude of the back-scattered wave depends on the frequency spectrum of the radiation: it becomes negligible in the short wave limit and can be significant in the long wave regime [17, 21, 24]. For scalar waves, the full numerical treatment presented here suggests that this process saturates and leads to zero absorption of light for wavelengths larger than R_s .

Figure 4. Propagation speeds V of the maxima of the pulses shown in figures (1) and (2), upper and lower panels, respectively, thick lines, compared to the corresponding solutions of equation(25), thin line. The dotted curves show the speed along a null geodesic, $c = (R - 1)/R$.



Finally, we have calculated numerically the pulse propagation speeds. We have evaluated propagation speeds as the time derivatives of the position of the maximum of the pulse, they are hence phase velocities in the case where the pulse retains its shape and behaves as a wave, and group velocities when significant distortions in the shape of the pulse appear. Figure 4 shows a comparison of the actual pulse propagation speeds, thick lines, and the approximation of equation(25), thin lines, as a function of radius, for the pulses shown in figures (1) and (2). The pulse which started with a wavelength of $0.1R$ is shown in the upper panel, and is seen to propagate exactly at the speed predicted by equation (25) for that wavelength. This was to be expected, as over the extent of the pulse no significant variations in the metric occur, until

only very close to $R = 1$. The wavelength of the pulse changes, as seen in figure (1), but given the large initial value of $W = 100$, and the rapid approach to the asymptote of equation (25) for large W , this does not introduce significant variations.

The lower panel of figure (3) shows the propagation speed of a pulse starting with a wavelength $10R$. This time, the difference with the speed predicted by equation (25) is much more obvious, as significant distortions in the metric, over the extent of the pulse, are apparent inwards of around 1 wavelength, $R = 10$. In all cases, the actual propagation speed is seen to deviate upwards of the prediction of equation (25), inwards of a certain critical radius. As in the previous case, the asymptote towards $V = 1$ is evident, both for the numerical wave and the solutions of equation (25). The function $c = (R - 1)/R$, the propagation corresponding to the standard vanishing wavelength limit of $ds = 0$, is also shown, seen completely superimposed onto the other two curves for the short pulse, and appearing just below the numerical speed in the $10R$ case. This last merely a result of the pulse deformation, as the maximum moves within the pulse towards the incoming edge of it, turning the numerical speed into a group velocity of the problem. The above highlighting the dispersive nature of the problem and the intrinsic limitations of fully analytic approaches. Results for the full electromagnetic vector field can be expected to be qualitatively similar to the more limited scalar wave problem treated here, e.g. [10, 14].

To check the validity of the numerical scheme, we have performed two tests. The first one regarding the sensibility of the problem to the complications that may arise at $R = 1$, we have repeated all simulations after shifting the numerical grid by a fraction of the radial interval in such a way that the first grid point to the right of $R = 1$ changes slightly. Results are completely insensitive to these shifts, showing that the results are not sensitive to the coordinate singularity that appears at $R = 1$ in the Schwarzschild space-time. The reflection we observe occurs very close to $R = 1$, still outside the event horizon. The second test has been an explicit calculation of the total energy E of the scalar field, given by [8]

$$E = 4\pi \int T^{00} r^2 dr, \quad (26)$$

where T^{00} is the time-time component of the stress-energy tensor for a scalar field given by [27]

$$T^{00} = g^{0\alpha} g^{0\beta} \Psi_{,\alpha} \Psi_{,\beta} - g^{\mu\nu} \Psi_{,\alpha} \Psi_{,\alpha} / 2 = (1/2) [(1 - 1/r)^{-1} \Psi_{,t}^2 + \Psi_{,r}^2]. \quad (27)$$

The net result of this calculation is that the total energy is conserved in all simulations to better than one percent accuracy, over 100 units of T , 10^5 simulation time steps. Thus, the total initial energy in the large pulse simulations remains the same when compared with the energy after observing the wave far away from $R = 1$ after it has “bounced” completely. We have found full reflection of pulses larger than the critical wavelength, to our numerical accuracy of 99%. This however, does not constitute a rigorous proof of full reflection for the problem being treated, indeed, going back to the heuristic arguments of section 1 on the EM field, the apparent inconsistency vanishes not only in the case of full reflection of large wavelengths, but also for any number of scenarios, provided large wavelengths are very substantially reflected to possibly slightly varying degrees. Our numerical experiments allow us to identify very substantial reflections of over 95% of the incoming energy in large pulses. This is not seen to any degree in the case of small pulses.

These results suggest the possibility of direct detection of black holes through the study of radiation

being reflected off them, as already mentioned by [4, 17], in connection with fractional backscattering in general. For stellar black holes, the gravitational radii would be in the kilometre range, substantial EM backscattering would hence be in this wavelength range, which is totally blocked by the atmosphere, hence requiring presently non-existent orbital or moon based observatories for their detection. The possibility of much smaller black holes appearing in particle accelerators has been suggested, a case where substantial backscattering, would be expected to occur in detectable ranges.

4. Conclusions

We have provided heuristic arguments suggesting that if a macroscopic black hole is allowed to swallow classical Planck radiation colder than itself, the Bekenstein interpretation of the horizon area of a black hole as a measure of its entropy is not consistent with the second law of thermodynamics. In the macroscopic regime, entropy considerations suggest a cut off limit for the light absorption spectrum of black holes, or very substantial reflection of waves above a critical wavelength $\sim R$. It is interesting to note that this critical wavelength is consistent with previously identified critical wavelengths for the interaction of waves and black holes, calculated in absence of any entropy considerations.

The standard $ds = 0$ treatment of an eikonal approximation necessarily fails when variations in the metric appear over a scale comparable to the wavelengths present. In the particular case of scalar waves, this variations imply a non-achromatic effect for their interaction with black holes, in particular, propagation velocities which fall below c , even in the wave's proper frame, as the frequency is decreased.

Scalar waves having wavelengths longer than the Schwarzschild radius of a black hole are very substantially reflected, or perhaps even fully bounce off. It is interesting that the critical values appear precisely at the wavelength scale identified through entropy considerations on the interaction of Planck radiation and black holes.

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