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Digital Impedance Bridge for Four-Terminal-Pair AC Resistor Calibration up to 20 kHz

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Abstract: For this study, a substitution principle-based impedance bridge has been developed to calibrate AC resistors in a four-terminal-pair (4TP) configuration. The calibration is performed in the full complex plane for resistances ranging from 100 m Ω to 400 Ω and frequencies of between 50 Hz and 20 kHz. The automated bridge is based on four resistors associated with two high-impedance stages. The balancing of the bridge is achieved by means of PXI modules. The bridge is automatically balanced via a simplex top-down algorithm. The new bridge is primarily used for the measurement chain of AC standard resistors defined in a 4TP configuration at LNE, which are used for routine customer calibrations. The traceability of LNE's standard resistors when defined in a 4TP configuration is ensured by a measurement chain from a 1 k Ω reference resistor using the new bridge. The reference resistor was calibrated previously via comparison with a calculable resistor up to 20 kHz. The bridge was validated via comparison with calibration results obtained in 1983 and 2009. For a resistor of 1 Ω at 1 kHz, the uncertainty of the series resistance variation and the phase shift are less than 6 $\mu\Omega/\Omega$ ($k = 1$) and 6 μrad ($k = 1$), respectively.

Keywords: impedance metrology; calibration; impedance bridge; AC resistors; uncertainty; Wheatstone bridge; PXI modules



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1. Introduction

The traceability of electrical resistors to the International System of Units (SI) is a crucial step before they can be used [1]. In electrical metrology, the calibration of AC resistors for frequencies up to 1 MHz is performed via several methods: an impedance comparison bridge [2–7], measurement by the current/voltage method [8], the self-balancing bridge method or impedance analyzers [9,10], and the network analyzer method [11].

Impedance (of a resistor, a capacitor, or an inductor) is mainly defined according to its four configurations: a two-wire/single terminal (2T), a four-wire/single terminal (4T), a two-terminal pair (2TP), or a four-terminal pair (4TP) [12]. For impedance calibration at the highest level of accuracy, the 2TP and 4TP configurations are the most commonly used forms, especially due to the ability to add a potential reference terminal to eliminate leakage resistances with the reference potential (ground) [12–18]. The 4TP configuration greatly reduces the influence of contact impedances (typically less than a few tens of micro-ohms) on the impedance measurement; thus, it is mainly used for low impedance values (typically less than 400 Ω). However, the 2TP configuration is usually used for high impedance values (typically above 400 Ω) while neglecting the influence of contact impedances [16,17].

At LNE, the traceability of AC resistance measurements up to 10 MHz is ensured by employing methods based on the principle of impedance comparison bridges for frequencies between 50 Hz and 20 kHz [18–21]. For frequencies up to 20 kHz, three methods, representing three levels of uncertainty, are used: a comparison bridge, based on a “two-stage” autotransformer (uncertainty level 1) [20], a Wheatstone bridge based on four resistors (uncertainty level 2), and a comparison bridge based on a sampling technique (uncertainty level 3) [21]. In terms of uncertainty level 2, a new automated Wheatstone bridge has been

implemented for the calibration of AC resistors defined in a 2TP configuration (between 400 Ω and 2 M Ω) [18].

In this paper, an evolution of LNE's impedance bridge (Wheatstone bridge) is used to calibrate AC resistors defined in a 4TP configuration for resistors between 100 m Ω and 400 Ω . This paper is a supplement to a previous paper [18]. However, it differs from the calibration bench that was previously presented, in terms of:

- The range of resistors to be calibrated. In the first paper [18], the resistors range from 400 Ω to 2 M Ω . In the present paper, the calibration system is developed for low resistance values of between 100 m Ω and 400 Ω .
- The type of resistors. In the first paper [18], the resistors are defined in a 2TP configuration. In this paper, the measured resistors are defined in a 4TP configuration, which requires triaxial connections to limit the internal leakage (internal guard).
- Although the power source and voltage injection system are the same as in the first paper [18], the calibration system presented in this paper has been improved to measure low resistance values. In the new calibration bench, two developed high-impedance stages and standard capacitors have been used to ensure bridge balance for resistors defined in a 4TP configuration. Consequently, the calibration equations to define the frequency variation of the calibrated resistors are different from those in the first paper [18].

We will be focusing on the calibration of AC resistance standards that are defined and measured in a 4TP configuration. A detailed uncertainty budget is also presented in this paper.

2. Description of the Impedance Bridge

The calibration bridge principle proposed for resistors defined in a 4TP configuration is similar to that presented in [18] for resistors defined in a 2TP configuration. Figure 1 shows the Wheatstone bridge schematic developed for resistors defined in a 4TP configuration up to 20 kHz. The calibrated resistor values range from 100 m Ω to 400 Ω . The bridge is based on the substitution principle between the impedance to be calibrated, Z_X , and the standard impedance, Z_E . It is composed of four impedances: Z_E (or Z_X), Z_1 (or Z'_1), and two impedances, Z_2 and Z_3 , for adjusting the bridge ratio. The bridge is balanced by keeping the vertices of the sensing diagonal (nodes P and Q) at ground potential. This balance is achieved by injecting a voltage in-phase and quadrature with the input voltage through two injection systems and by adjusting a capacitor C_1 (or C_2), placed in parallel with impedance Z_2 (or Z_3). Null detectors D_1 and D_2 are placed at nodes P and Q to determine the balance of the bridge. This grounding of nodes P and Q allows us:

- To ensure a 4TP configuration for impedances Z_E (or Z_X) and Z_1 (or Z'_1) and a 2TP configuration for impedances Z_2 and Z_3 ;
- To eliminate the influence of parasitic impedances between the P and Q nodes and the ground;
- To eliminate the influence of the common mode on the main null detector, D_2 .

The bridge is supplied by a commercial dual precision arbitrary waveform generator, which has been designed for low-frequency metrological applications (DualDAC™) [22]. Its two voltage sources are independently controlled in terms of amplitude and phase in a frequency range of between 0.1 Hz and 20 kHz. A preliminary study was carried out on how the power source should be selected [23]. At each DualDAC™ voltage output, a power amplifier is used to feed the Wheatstone bridge with the necessary current.

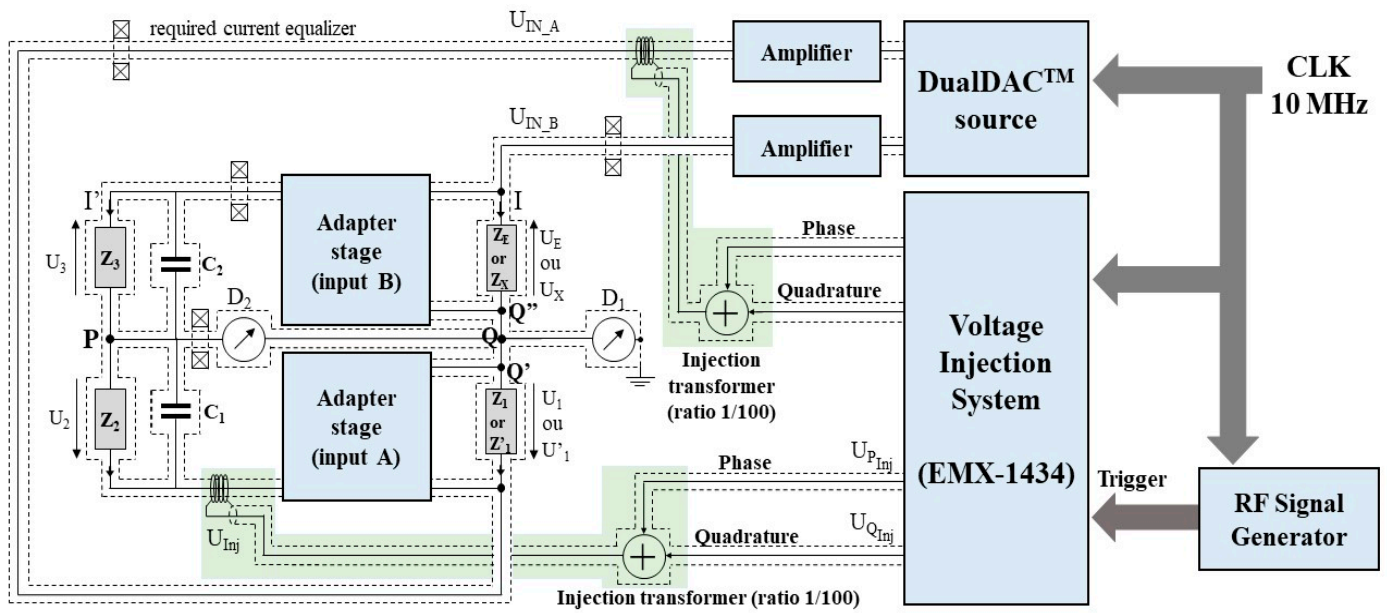


Figure 1. Schematic of the Wheatstone bridge for AC resistors defined in a four-terminal pair up to 20 kHz.

The bridge is balanced using the injection system, based on the PXI EMX-1434 modules [24] and by adjusting one of the standard capacitors, C_1 or C_2 . The use of PXI EMX-1434 modules as a voltage injection system is demonstrated in a previous study [25]. The results of a comparison with other PXI modules showed that the EMX-1434 module is characterized by the ability to provide finely adjustable voltages in phase and quadrature (a few μV in amplitude and a few millidegrees in phase) with low signal distortion, a low DC residual offset, low temperature drift, high linearity, and a low signal-to-noise ratio. Two voltage injection systems are used: the first is employed to ensure the metrological configuration of the resistors, the balance of which is measured using null detector D_1 . The second system is used to ensure the main balance of the bridge; its balance is measured using null detector D_2 . Each voltage injection system has two generators of the EMX-1434 module (in quadrature and in phase). An injection transformer is used to perform the vector sum of the supply voltage vector and the two injection voltages in phase and quadrature [26]. Capacitors C_1 and C_2 are used to reduce the level of the in-phase voltage that is injected by the EMX-1434 modules, which is necessary to ensure the bridge balance (showing a minimum on the detector D_2).

A 10 MHz signal with an external trigger signal is used to synchronize the DualDAC™ source and the EMX-1434 module. A calibrated phasemeter is also used to correct any phase errors before starting to balance the bridge.

A single digital lock-in amplifier with two inputs (N_A and N_B) is used for null detectors D_1 and D_2 . For null detector D_1 , the lock-in amplifier input mode is configured to input N_A . However, the configuration of the lock-in amplifier input mode to N_A-N_B allows the lock-in amplifier to be positioned on D_2 .

Current equalizers are used to minimize the influence of electromagnetic noise and to ensure a coaxial configuration [27,28]. For the same purpose, mutual induction coupling is minimized by twisting the power cables and the cables connected to the adapter stages. Short cables are used between nodes P or Q and the lock-in amplifier to minimize the leakage impedance to the ground.

Considering impedances Z_2 and Z_3 , which are defined in 2TP configurations and are used to adjust the bridge ratio, high-impedance differential stages (adapter stages A and B) are used between the two bridge sections. These stages eliminate the influence of voltage losses between nodes $Q Q'$ and $Q Q''$. For the same purpose, the connection lengths

between these nodes have been minimized. The adapter stages are designed with triaxial input connectors to limit the internal leakage (internal guard) [21].

The high-impedance stages used are based on an instrumentation amplifier (differential input). These stages represent the main difference with the bridge used for resistors defined in a 2TP configuration [18] and ensure the 4TP configuration. The schematic diagram is shown in Figure 2a. The amplifiers used (OP27) were selected to provide broadband stability up to a few megahertz. Two guard amplifiers of LT1028 (A5 and A6), combined with a triaxial input circuit, not only eliminated the influence of cable losses but also limited their effect on amplifier stability. The settings for phase errors are made by fine adjustment of the capacitance, C_{h+} , C_{h-} , C_{b+} , and C_{b-} , of each adapter stage. The high-impedance stages consist of the following electronic modules:

- Two adapter stages A and B, representing the main module of the high-impedance stages.
- A safety module to protect and warn against voltage limits or saturation levels for the input module amplifiers (3 V).
- Power supply module to supply power to the high-impedance stages (± 12 V).

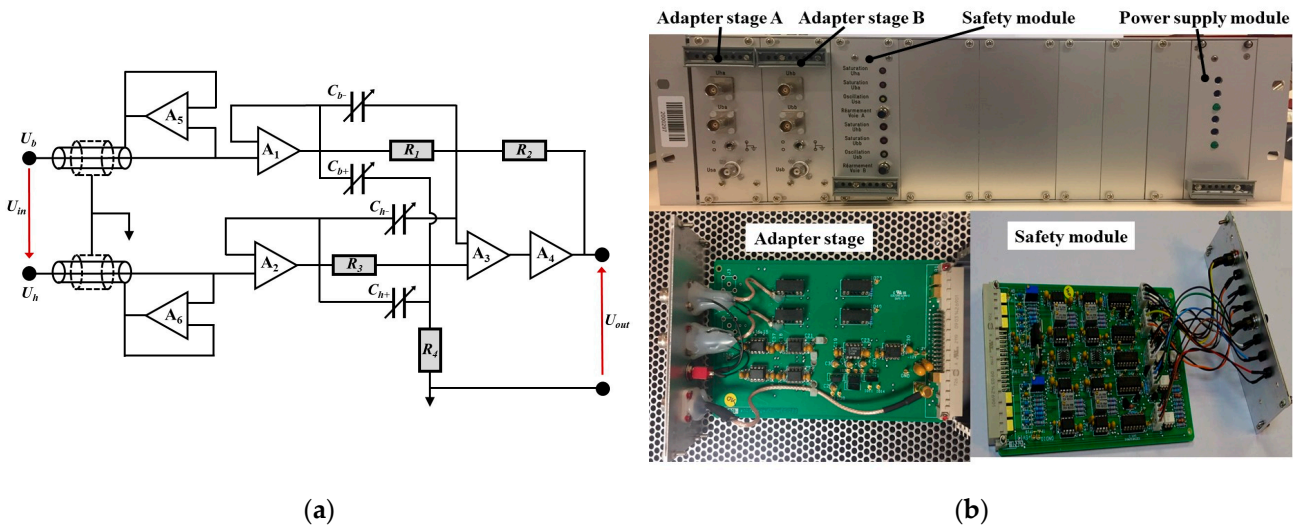


Figure 2. High-impedance stages. (a) Schematic diagram; (b) implementation.

The implementation of high-impedance stages is shown in Figure 2b. The differential inputs of the high-impedance stages are calibrated and adjusted before use. The purpose of this calibration is to verify the negligible effect of the gain and phase shift (the phase shift of the U_{out} output with respect to the U_b input) of the high-impedance stages on the voltages supplying impedances Z_2 and Z_3 (see Figure 1). The calibration results are not used to determine the characteristics of the AC resistors (using the substitution method), as described in Section 3. The gain and phase shift errors are measured after a period of thermal stabilization of the input stages (at least 24 h). Three configurations are verified:

- Follower configuration: voltage is applied to the high input U_h and grounded low input U_b .
- Inverter configuration: voltage is applied to the low input U_b and grounded to the high input U_h .
- Differential configuration: the same voltage is applied to the low input U_b and high input U_h .

The maximum absolute gain and phase shift corrections are less than $(40 \pm 3) \mu\text{V/V}$ and $(26 \pm 7) \mu\text{rad}$, respectively, up to 20 kHz. The use of the substitution principle, the stability of the corrections, and the associated low uncertainties of the two matched adapter stages allow us to disregard their influence on the calibration of the AC resistor up to 20 kHz; this part is detailed in Section 3.

The input impedance Z_p of each adapter stage (A and B) is modeled by a capacitor C_p and a resistor R_p in parallel. These components are measured using a HIOKI IM3536 RLC bridge at each frequency, and the values considered are the average of 100 acquisitions. Table 1 shows the measured values of the capacitors C_p and the resistors R_p . The measured input impedances of each adapter stage (A and B) are sufficiently large to ensure a 4TP configuration when calibrating AC resistors up to 20 kHz.

Table 1. The measured values, with uncertainties ($k = 2$), of the capacitors C_p and the resistors R_p of each adapter stage.

Frequency (kHz)	Adapter Stage A		Adapter Stage A	
	Capacitance C_p (fF)	Resistance R_p (GΩ)	Capacitance C_p (fF)	Resistance R_p (GΩ)
0.1	95.1 ± 0.5	3011.4 ± 0.9	95.1 ± 0.5	2589.2 ± 0.9
0.2	95.0 ± 0.4	2055.2 ± 0.9	95.0 ± 0.4	2375.3 ± 0.9
0.5	95.1 ± 0.4	1971.3 ± 0.9	95.0 ± 0.4	2347.6 ± 0.9
1	95.2 ± 0.2	922.9 ± 0.9	95.2 ± 0.2	753.1 ± 0.9
2	95.1 ± 0.4	235.3 ± 0.9	95.1 ± 0.4	217.6 ± 0.9
5	95.0 ± 0.8	43.1 ± 0.9	95.0 ± 0.8	41.9 ± 0.9
10	95.1 ± 1.6	11.5 ± 0.9	95.0 ± 1.6	11.5 ± 0.9
20	95.2 ± 1.6	3.1 ± 0.9	95.2 ± 1.6	3.1 ± 0.9

The measurement system is automated, performing all the tasks necessary to compare AC resistors defined in a 4TP configuration, automatically balancing the bridge, and controlling the maximum electrical current according to the nominal current values of the compared resistors. The main steps of the measuring procedure are:

1. Construction of the impedance bridge with impedances Z_1 , Z_2 , Z_3 , and Z_E ;
2. Generation of the supply voltages U_{IN_A} and U_{IN_B} by the DualDAC™ source, based on the bridge ratio and the maximum input voltage of the adapter stages (3 V);
3. Correction of phase errors between the DualDAC™ source and the EMX-1434 module;
4. Adjustment by successive iterations of balances on detectors D_1 and D_2 , using the voltage injection system and the capacitor C_1 (or C_2);
5. Repetition of steps 2 to 4 for each measurement frequency;
6. Substitution of Z_1 and Z_E by Z'_1 and Z_X , respectively (an electrical circuit with impedances Z'_1 , Z_2 , Z_3 and Z_X);
7. Repetition of steps 2 to 5.

3. AC Resistor Calibration

The impedance bridge is based on the substitution principle between the impedance to be calibrated, Z_X , and the standard impedance, Z_E . The calibration equations are detailed for the standard resistor Z_E . The bridge then consists of impedances Z_E and Z_1 and two impedances, Z_2 and Z_3 , for the bridge ratio. Applying Ohm's law to the balance of the bridge, the complex voltages across impedances Z_1 , Z_2 , Z_3 , and Z_E are given by:

$$\begin{cases} U_1 = -Z_1 I \\ U_E = Z_E I \\ U_2 = -(Z_2 // Z_{C1}) I' \\ U_3 = (Z_3 // Z_{C2}) I' \end{cases} \quad (1)$$

where Z_{C1} and Z_{C2} are the impedances of capacitors C_1 and C_2 at the measurement frequency f . The voltages U_E , U_1 , U_2 , and U_3 are the voltages across impedances Z_E , Z_1 , $(Z_2 // Z_{C1})$, and $(Z_3 // Z_{C2})$, respectively. The currents I and I' are the electrical currents in each bridge arm.

The relations between the bridge voltages are ascertained by applying Kirchhoff's voltage law:

$$\begin{cases} U_1 = U_{IN_A} \\ U_E = U_{IN_B} \\ G_A \cdot U_1 = U_2 + U_{Inj} \\ G_B \cdot U_E = U_3 \end{cases} \quad (2)$$

where G_A and G_B are the complex gains of the high-impedance stages of inputs A and B , respectively. The voltages U_{IN_A} and U_{IN_B} are the bridge supply voltages. The complex voltage U_{Inj} at the output of the injection transformer is the voltage that is injected to ensure bridge balance.

The complex transfer coefficient k_{Inj} of the voltage injection system is defined by the ratio between the injected voltage U_{Inj} and the voltage U_1 that is supplied to impedance Z_2 (in the bridge balance):

$$k_{Inj} = \frac{U_{Inj}}{U_1} = \frac{U_{P_Inj} + j U_{Q_Inj}}{N_T U_{IN_A}} = \frac{1}{G_A} (k_{P_Inj} + j k_{Q_Inj}) \quad (3)$$

where N_T is the gain ratio of the injection transformer ($=100$). U_{P_Inj} and U_{Q_Inj} are the injected voltages in phase and quadrature, respectively, to ensure bridge balance. k_{P_Inj} and k_{Q_Inj} are the transfer coefficients in phase and in quadrature, respectively, of the voltage injection system.

Using Equations (1)–(3), we obtain:

$$\begin{cases} \frac{Z_1}{Z_E} = -\frac{U_1}{U_E} = -\frac{U_{IN_A}}{U_{IN_B}} \\ \frac{(Z_2 // Z_{C_1})}{(Z_3 // Z_{C_2})} = -\frac{U_2}{U_3} = -\frac{G_A}{G_B} \frac{U_{IN_A}}{U_{IN_B}} - k_{Inj} \end{cases} \quad (4)$$

Thus:

$$\frac{Z_1}{Z_E} \frac{G_A}{G_B} = \frac{(Z_2 // Z_{C_1})}{(Z_3 // Z_{C_2})} + k_{Inj} \quad (5)$$

By developing Equation (5) to the first order:

$$\frac{Z_E}{Z_1} \frac{\left(1 + \frac{Z_3}{Z_{C_2}}\right)}{\left(1 + \frac{Z_2}{Z_{C_1}}\right)} \left(1 + k_{Inj} \frac{(Z_3 // Z_{C_2})}{(Z_2 // Z_{C_1})}\right) = \frac{Z_3}{Z_2} \frac{G_A}{G_B} \quad (6)$$

From Equation (6), considering the substitution of Z_1 and Z_E by Z'_1 and Z_X , respectively, and the gain stability of adapter stages A and B , we obtain:

$$\frac{Z_X}{Z'_1} \frac{\left(1 + \frac{Z_3}{Z_{C'_2}}\right)}{\left(1 + \frac{Z_2}{Z_{C'_1}}\right)} \left(1 + k'_{Inj} \frac{(Z_3 // Z_{C'_2})}{(Z_2 // Z_{C'_1})}\right) = \frac{Z_3}{Z_2} \frac{G_A}{G_B} \quad (7)$$

where $k'_{Inj} = k'_{P_Inj} + j k'_{Q_Inj}$ is the complex transfer coefficient of the voltage injection system with impedances Z'_1 and Z_X . $Z_{C'_1}$ and $Z_{C'_2}$ are, respectively, the impedances of the capacitors C'_1 and C'_2 , adjusted to obtain the balance of the bridge (for a circuit with impedances Z'_1 and Z_X).

The substitution method avoids errors caused by the gains (G_A and G_B) of the adapter stages. The errors of the adapter stages, which are below the maximum values of $40 \mu\text{V/V}$ for the gain and $26 \mu\text{rad}$ for the phase shift up to 20 kHz , allow this hypothesis to be

validated. These corrections were verified and validated before the adapter stages were used. By combining Equations (6) and (7), we obtain:

$$\frac{Z_E}{Z_1} \left(1 + \frac{Z_3}{Z_{C_2}}\right) \left(1 + k_{Inj} \frac{(Z_3 // Z_{C_2})}{(Z_2 // Z_{C_1})}\right) = \frac{Z_X}{Z'_1} \left(1 + \frac{Z_3}{Z_{C'_2}}\right) \left(1 + k'_{Inj} \frac{(Z_3 // Z_{C'_2})}{(Z_2 // Z_{C'_1})}\right) \quad (8)$$

The variation with the frequency of the series resistance of the impedance to be calibrated, Z_X , is given by:

$$\Delta s_X = \Delta s_E - \Delta s_1 + \Delta s'_1 - \frac{R_{DC2}}{R_{DC3}} (k_{P_{inj}} - k'_{P_{inj}}) + \omega^2 (R_{DC2} t_2 (C_1 - C'_1) + R_{DC3} t_3 (C_2 - C'_2)) \quad (9)$$

where Δs_E , Δs_1 , and $\Delta s'_1$ are the relative variations with the frequency of impedances Z_E , Z_1 , and Z'_1 , respectively. R_{DC2} and R_{DC3} are the DC resistances of impedances Z_2 and Z_3 , respectively. t_2 and t_3 are the time constants, respectively, of impedances Z_2 and Z_3 .

Similarly, the phase shift with the frequency of the impedance to be calibrated, Z_X , is given by:

$$\omega t_X = \omega (t_E - t_1 + t'_1) - \frac{R_{DC2}}{R_{DC3}} (k_{Q_{inj}} - k'_{Q_{inj}}) + \omega (R_{DC2} (C_1 - C'_1) + R_{DC3} (C_2 - C'_2)) \quad (10)$$

where t_E , t_1 , and t'_1 are the time constants of impedances Z_E , Z_1 , and Z'_1 , respectively.

The parameters Δs_X and ωt_X of the impedance to be calibrated, Z_X , are determined as a function of the two complex transfer coefficients and the parameters of the normalized impedances, Z_E , Z_1 , Z'_1 , Z_2 , Z_3 , Z_{C1} , Z_{C2} , Z'_{C1} , and Z'_{C1} . Impedances Z_E , Z_1 , Z'_1 , Z_2 , and Z_3 were previously calibrated using the bridge formed by a measurement chain from a 1 k Ω reference resistor. The measurement chain is presented in Section 5.

4. Uncertainty Budget

The uncertainty budget is similar to that for the resistances defined for a two-terminal pair and detailed in [18]. The additional uncertainty components being considered were consistent with the observations made during the calibration process:

- Influence of adapter stages (BL1): This error relates to the calibration and adjustment of the differential inputs of the adapter stages. The errors on variation Δs are corrected. The associated uncertainty ($k = 1$) can be increased by a component equal to 1 $\mu\text{V/V}$ up to 20 kHz. For phase shift measurements, considering a rectangular distribution law, the estimated uncertainty ($k = 1$) is less than 4.1 μrad up to 20 kHz.
- Influence of parasitic impedances (BL2): The leakage impedances to the ground are eliminated by the Wagner arm. The input impedance of the adapters is likely to interfere with the measurements. This component is given according to the resistor values:
 - For the relative variation, Δs : The values that were estimated for impedance Z_p during the calibration of the adapter stages were used to apply a correction term. The uncertainty ($k = 1$) associated with the internal resistance value R_p of the impedance Z_p of the adapter stages can be considered to be less than 1 G Ω . The correction term $\beta_{\Delta s_Z}$, related to the influence of the parasitic impedance Z_p on the variation Δs , is equal to:

$$\beta_{\Delta s_Z} = -\frac{R_X}{R'_1} (R'_1 - R_X) \left[\frac{1}{R_p} + \frac{R'_1 - R_X}{2} (2\pi f_s C_p)^2 \right] \quad (11)$$

where f_s is the frequency (in Hz), and C_p is the internal capacitance of the impedance Z_p of the adapter stages.

The uncertainty component *BL2* is then given by:

$$BL2 (k = 1) = \frac{R_X}{R'_1} |R'_1 - R_X| \cdot \sqrt{\frac{u^2(R_p)}{R_p^4} + \left[(R'_1 + R_X) (2\pi f_s)^2 C_p \right]^2 u^2(C_p)} \quad (12)$$

This component can be increased by considering $R_p = 3 \text{ G}\Omega$ and $C_p = 95 \text{ fF}$. We obtain:

$$BL5 (k = 1) \approx \frac{R_X}{R'_1} |R'_1 - R_X| \cdot \left[1.1 \times 10^{-10} + 9.5 \times 10^{-29} \cdot (R'_1 + R_X) (2\pi f_s)^2 \right] \quad (13)$$

- For the phase shift ωt , the uncertainty ($k = 1$) associated with the capacitance value, C_p , is less than 1 fF; thus, the applied correction term is equal to $(R_X - R'_1) \cdot C_p \cdot 2\pi f_s$. The uncertainty component *BL2* is given by:

$$BL2 (k = 1) = [|R_X - R'_1| \cdot 2\pi f_s \cdot u(C_p)] \quad (14)$$

- Mutual induction effect (*BL3*): This component corresponds to the error due to mutual parasitic coupling between a current circuit and a voltage circuit. It is equal to:

$$\beta_M = j M \cdot 2\pi f_s \cdot \frac{R_X}{R'_1} \cdot \left(\frac{1}{R'_1} - \frac{1}{R_X} \right) \quad (15)$$

where β_M is the correction term associated with the mutual inductance effect and M is the mutual inductance (in H).

This uncertainty component mainly affects the quadrature component (the parasitic time constant). The parasitic mutual inductance is estimated to be less than 0.2 nH. Therefore, its effect on the relative variation Δs is negligible. For the phase shift, the correction term is equal to $j\beta_M$. Considering a rectangular distribution law, the uncertainty associated with this component on the phase shift is given by:

$$BL3 (k = 1) = \left[1.3 \times 10^{-9} \cdot f_s \cdot \frac{R_X}{R'_1} \cdot \left(\frac{1}{R'_1} - \frac{1}{R_X} \right) \right] \quad (16)$$

Considering the major uncertainty components, Table 2 shows the example of Wheatstone bridge uncertainty budgets for the resistances SC1 (1 Ω) and SC0.1 (100 m Ω).

Table 2. Uncertainty components ($k = 1$) for the LNE’s impedance bridge.

Resistance Defined in 4TP Configuration	Frequency (kHz)	Estimated Uncertainty ($k = 1$) of the Different Uncertainty Components of the Wheatstone Bridge Due to:											Estimated Uncertainty ($k = 2$)	
		Impedances Z_E, Z_1 et Z'_1		Complex Transfer Coefficients k_{inj} and k'_{inj}		Influence of Temperature on the Stability of the Bridge (UB1)		Accuracy Error Associated with the Adapter Stages (BL1)		Influence of Parasitic Impedances (BL2)		Mutual Induction Effect (BL3)		
		Δs ($\mu\Omega/\Omega$)	ωt (μrad)	Δs ($\mu\Omega/\Omega$)	ωt (μrad)	Δs ($\mu\Omega/\Omega$)	ωt (μrad)	Δs ($\mu\Omega/\Omega$)	ωt (μrad)	Δs ($\mu\Omega/\Omega$)	ωt (μrad)	ωt (μrad)	Δs ($\mu\Omega/\Omega$)	ωt (μrad)
SC1 (1 Ω)	0.1	3.6	3.6	0.6	0.6	2.4	2.4	1.0	1.0	0.6	0.1	0.6	9.1	9.2
	1	5.0	5.1	0.6	0.6	2.4	2.4	1.0	1.0	0.6	1.4	0.6	11.5	11.9
	5	11.6	11.6	0.6	0.6	2.4	2.4	1.0	1.4	0.6	7.2	3.0	23.9	28.6
	10	19.9	20.0	0.6	0.6	2.4	2.4	1.0	2.2	0.6	14.4	6.0	40.2	51.2
	20	34.9	19.2	0.8	0.8	2.4	2.4	1.0	4.1	0.6	28.8	12.0	70.1	73.9
SC0.1 (100 m Ω)	0.1	5.5	6.0	0.4	0.4	2.8	2.8	1.0	1.0	0.0	0.0	3.2	12.5	14.8
	1	7.2	13.0	0.3	0.3	2.8	2.8	1.0	1.0	0.0	0.0	3.2	15.6	27.4
	5	15.2	46.6	0.3	0.3	2.8	2.8	1.0	1.4	0.0	0.1	15.8	31.1	98.5
	10	25.6	88.7	0.3	0.3	2.8	2.8	1.0	2.2	0.0	0.2	31.6	51.5	188.5
	20	45.3	170.5	0.5	0.5	2.8	2.8	1.0	4.1	0.0	0.3	63.2	90.8	363.9

5. AC Resistance Standards Measurement Chain

The first step of the measurement chain is based on the calibration of a 1 kΩ resistor (R1M) via comparison with a calculable Haddad-type resistor up to 20 kHz [19]. This resistor is then used as the reference element for the measurement chain. Figure 3 shows the measurement chain for AC reference resistors defined in a 4TP configuration at LNE.

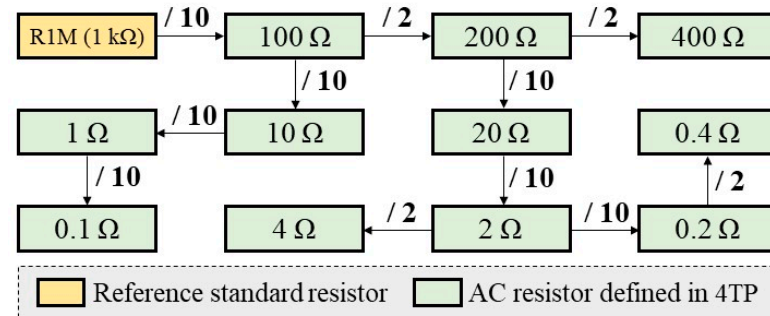


Figure 3. Measurement chain for AC resistors defined in a four-terminal pair at LNE to a 1 kΩ standard resistor (R1M). The chain is made by sequentially connecting pairs of standards for the following bridge ratios: 2 and 10.

Several LNE standard resistors defined in a 4TP configuration have been measured. The relative variation Δs and the phase shift ωt were characterized for each impedance at the following frequencies: 100 Hz, 1 kHz, 5 kHz, 10 kHz, and 20 kHz. The calibration current values for the measured resistors were between 3 mA and 1.5 A. The measurements made with the new Wheatstone bridge were compared with calibrations made in 1983 and 2009 [20,29]; these previous calibrations at LNE were made with a manual Wheatstone bridge (first generation) [20]. Tables 3 and 4 show the example calibration of the resistors SC0.1 (100 mΩ) and SC0.2 (200 mΩ), respectively. Normalized error ratios of relative variation Δs and the phase shift ωt were calculated for all calibrated resistors. This is a statistical test to compare the proficiency of measurement results, taking into account their uncertainties [30]. This normalized error ratio E_n is calculated in comparison with previous calibrations, using the following equation:

$$E_n = \left| \frac{x_m - x_{ref}}{\sqrt{U_m^2 + U_{ref}^2}} \right| \quad (17)$$

where x_m and x_{ref} are the values of the relative variation Δs (or the phase shift ωt), measured with the presented impedance bridge and obtained in one of the previous calibrations, respectively. U_m and U_{ref} are the expanded uncertainties associated with x_m and x_{ref} , respectively.

Table 3. Values and uncertainties ($k = 2$) of the variation Δs and the phase shift ωt of the resistor SC0.1 (100 mΩ).

Frequency (kHz)	Parameters	Year of Calibration		
		1983	2009	2022
0.1	Variation Δs ($\mu\Omega/\Omega$)	0 ± 11	0 ± 11	0 ± 13
	Phase shift ωt (mrad)	0.23 ± 0.01	0.23 ± 0.01	0.23 ± 0.01
1	Variation Δs ($\mu\Omega/\Omega$)	26 ± 15	28 ± 15	32 ± 16
	Phase shift ωt (mrad)	2.32 ± 0.03	2.32 ± 0.03	2.32 ± 0.03
5	Variation Δs ($\mu\Omega/\Omega$)	260 ± 33	260 ± 33	261 ± 31
	Phase shift ωt (mrad)	11.3 ± 0.1	11.3 ± 0.1	11.3 ± 0.1

Table 3. Cont.

Frequency (kHz)	Parameters	Year of Calibration		
		1983	2009	2022
10	Variation Δs ($\mu\Omega/\Omega$)	470 ± 56	459 ± 56	461 ± 52
	Phase shift ωt (mrad)	22.3 ± 0.3	22.3 ± 0.3	22.3 ± 0.2
20	Variation Δs ($\mu\Omega/\Omega$)	840 ± 100	813 ± 100	812 ± 91
	Phase shift ωt (mrad)	44.3 ± 0.5	44.3 ± 0.5	44.3 ± 0.4

Table 4. Values and uncertainties ($k = 2$) of the variation Δs and the phase shift ωt of the resistor SC0.2 (200 m Ω).

Frequency (kHz)	Parameters	Year of Calibration		
		1983	2009	2022
0.1	Variation Δs ($\mu\Omega/\Omega$)	0 ± 10	0 ± 10	0 ± 10
	Phase shift ωt (mrad)	0.19 ± 0.01	0.19 ± 0.01	0.18 ± 0.01
1	Variation Δs ($\mu\Omega/\Omega$)	25 ± 13	24 ± 13	38 ± 13
	Phase shift ωt (mrad)	1.86 ± 0.03	1.90 ± 0.03	1.89 ± 0.03
5	Variation Δs ($\mu\Omega/\Omega$)	125 ± 28	101 ± 28	131 ± 28
	Phase shift ωt (mrad)	9.1 ± 0.1	9.2 ± 0.1	9.1 ± 0.1
10	Variation Δs ($\mu\Omega/\Omega$)	225 ± 46	205 ± 46	218 ± 46
	Phase shift ωt (mrad)	18.2 ± 0.2	18.1 ± 0.2	18.1 ± 0.2
20	Variation Δs ($\mu\Omega/\Omega$)	400 ± 83	404 ± 83	417 ± 83
	Phase shift ωt (mrad)	36.2 ± 0.4	36.3 ± 0.4	36.3 ± 0.4

Figures 4 and 5 show the absolute values of the normalized error ratios of the relative variation Δs and the phase shift ωt for the automated Wheatstone bridge presented in this paper, compared to the calibration results for the years 1983 and 2009, respectively. The ratios were calculated for all the resistors in the measurement chain (one resistor for each value) between 100 m Ω and 400 Ω and for frequencies up to 20 kHz. The ratio values obtained were less than 1. The results obtained validate the bridge presented in this paper.

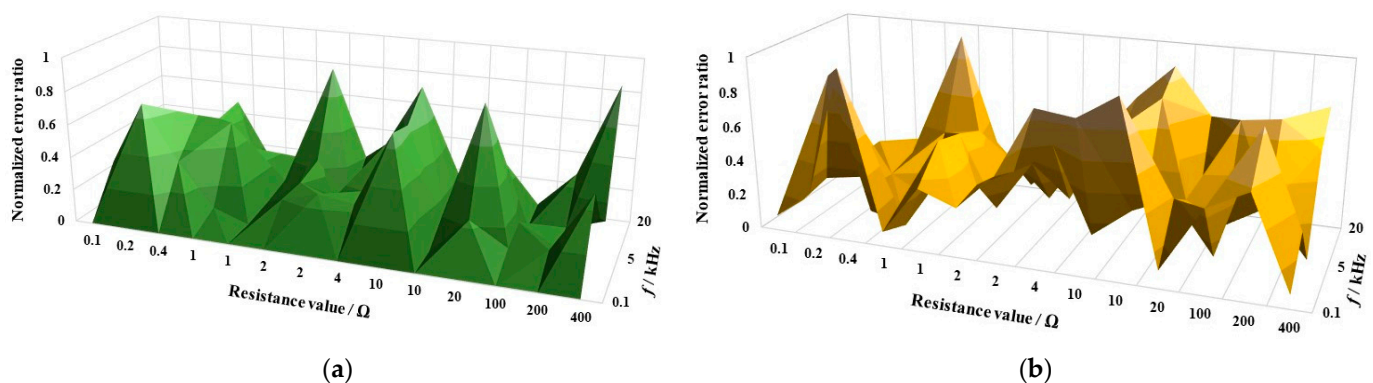


Figure 4. Normalized error ratio of measurements with the presented digital impedance bridge, compared with the calibration results in the year 1983 for values between 100 m Ω and 400 Ω and up to 20 kHz. (a) Values for the variation of the series resistance Δs . (b) for the phase ωt .

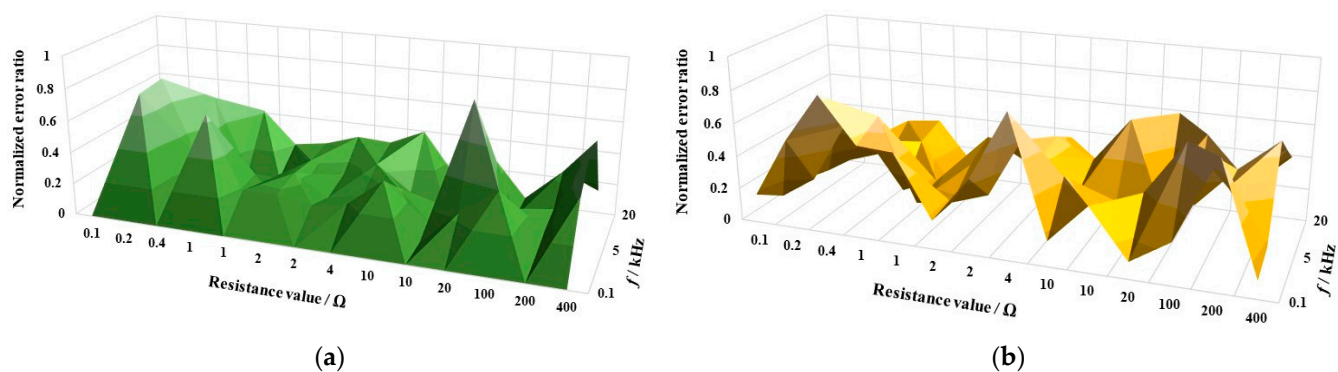


Figure 5. Normalized error ratio of measurements with the presented digital impedance bridge compared with the calibration results of the year 2009 for values between 100 mΩ and 400 Ω and up to 20 kHz. (a) Values for the variation of the series resistance Δs . (b) Values for the phase ωt .

6. Conclusions

In this paper, we present a new automated digital Wheatstone bridge for the calibration of AC resistors, defined in a 4TP configuration, from 100 mΩ to 400 Ω and from 20 Hz to 20 kHz. This bridge is mainly used at LNE to carry out the measurement chain process of standard resistors defined in a 4TP configuration with an AC current of up to 20 kHz. The measurement chain is ensured by a single reference resistor, R1M (1 kΩ), characterized via comparison with a calculable resistance of 1 kΩ and with the two-stage autotransformer bridge of LNE. The presented bridge is an improvement of the first-generation solution (a manual bridge). This bridge is based on a DualDAC™ commercial precision signal source, comprising a voltage injection system based on PXI modules and impedance adapter stages. At the end of this calibration, two main parameters are determined for each resistor as a function of frequency: the series resistance variation Δs and the phase shift ωt . An uncertainty budget has been developed, taking into account the main sources of uncertainty of the bridge. The most important uncertainty components of the uncertainty budget are the bridge resistor values, the influence of temperature on the stability of the bridge, and the mutual induction effect. For all measured resistors, the normalized error ratios calculated with reference to the calibrations made in 1983 and 2009 are less than 1, which validates the bridge presented in this paper.

However, as with the resistors defined in a 2TP configuration, using a calculable resistor as a reference for the measurement chain, with expanded uncertainties of less than 3 μΩ/Ω on the variation Δs and 3 μrad on the phase shift ωt up to 20 kHz, will reduce the combined uncertainties during the calibration of resistors defined in a 4TP configuration with the proposed bridge.

Author Contributions: M.O.: Design of the reference measurement set-ups, development of the generation systems, carrying out performance tests with separate and combined influence factors, and elaboration of the test procedures. R.V.: Validation of the measurement set-ups and carrying out performance tests with separate and combined influence factors. M.A.: Validation of the measurement set-ups and carrying out performance tests with separate and combined influence factors. All authors have read and agreed to the published version of the manuscript.

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