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Review

A Review on Fractional-Order Modelling and Control of Robotic Manipulators

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Abstract: Robot manipulators are widely used in many fields and play a vital role in the assembly, maintenance, and servicing of future complex in-orbit infrastructures. They are also helpful in areas where it is undesirable for humans to go, for instance, during undersea exploration, in radioactive surroundings, and other hazardous places. Robotic manipulators are highly coupled and non-linear multivariable mechanical systems designed to perform one of these specific tasks. Further, the time-varying constraints and uncertainties of robotic manipulators will adversely affect the characteristics and response of these systems. Therefore, these systems require effective modelling and robust controllers to handle such complexities, which is challenging for control engineers. To solve this problem, many researchers have used the fractional-order concept in the modelling and control of robotic manipulators; yet it remains a challenge. This review paper presents comprehensive and significant research on state-of-the-art fractional-order modelling and control strategies for robotic manipulators. It also aims to provide a control engineering community for better understanding and up-to-date knowledge of fractional-order modelling, control trends, and future directions. The main table summarises around 95 works closely related to the mentioned issue. Key areas focused on include modelling, fractional-order modelling type, model order, fractional-order control, controller parameters, comparison controllers, tuning techniques, objective function, fractional-order definitions and approximation techniques, simulation tools and validation type. Trends for existing research have been broadly studied and depicted graphically. Further, future perspective and research gaps have also been discussed comprehensively.

Keywords: approximation approaches; fractional calculus; fractional-order control; fractional-order model; industrial manipulators; optimization techniques; robotic manipulators

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1. Introduction

Robotic manipulators are electronically controlled mechanisms consisting of multiple segments that perform tasks by interacting with their environment. They can perform repetitive tasks at speeds and accuracies far exceeding human operators [1]. They can move or handle objects automatically depending upon the given number of DOF. The DOF of industrial robotic manipulators can range from two to ten, or more. As they are capable of automating, many automated applications have recently been seen. The most common include spot welding, assembly, handling, painting, and palletizing [2]. Technological advancements have greatly improved robotic manipulators' accuracy and precision, thus allowing them to automate new applications such as automated 3D printing. Robotic manipulator automation makes manufacturing processes more efficient, reliable, and productive. As a result, considerable attention has been given to modelling the robotic manipulators and designing practical controllers that are easy to implement and provide optimal controlled performance [3–5].

Recently, the fractional-order concept has attracted increasing attention in control research. Fractional-order modelling and control, using fractional-order derivatives/integrals, has been recognized as an alternative strategy to solve many robust control problems effectively [6,7]. This is also true in the case of robotic manipulators. In the last few years, extensive research has been performed on robotic manipulators using fractional-order concepts. Thus, this study thoroughly reviews the application of fractional calculus in modelling and controlling robotic manipulators. Therefore, a comprehensive literature review on fractional-order modelling and control techniques for various robotic manipulators is presented. This study is structured as follows:

- Different conventional and fractional-order modelling strategies for lower and higher DOF robotic manipulators are included in the review.
- A review of developed fractional-order controllers for various robotic manipulators evolved from PID, sliding mode, fuzzy, backstepping, active disturbance rejection control, and impedance control is presented.
- Fractional-order derivative definitions and approximation techniques are also presented.
- Trends for existing research and future developments in this area have been broadly presented and depicted in a graphical layout.

The paper's remaining sections are organized as follows: the preliminaries of fractional calculus, including the derivative definitions, are presented in Section 2. Section 3 summarizes the collected literature review and the graphical trend analysis. Section 4 offers the detailed dynamic modelling of robotic manipulators. The broad overview of fractional-order control strategies developed for various robotic manipulators is presented in Section 5. Finally, the paper concludes in Section 6.

2. Preliminaries of Fractional Calculus

The fractional-order differintegral operator $\mathcal{D}_t^\alpha$ for an order α of a given function $f(t)$ is defined as,

$$\mathcal{D}_t^\alpha f(t) = \begin{cases} \frac{d^\alpha}{dt^\alpha} f(t), & \alpha > 0, \\ f(t), & \alpha = 0, \\ \int_0^t f(\tau) d\tau, & \alpha < 0. \end{cases} \quad (1)$$

The three most frequently used definitions of fractional-order derivative $\mathcal{D}_t^\alpha$ for $\alpha > 0$ are Grünwald–Letnikov, Riemann–Liouville, and Caputo, as given in orange, blue, and grey coloured boxes of Figure 1, respectively. In the definitions, $\Gamma(\cdot)$ is Euler's Gamma function. On the other hand, among the various approximation techniques available in the literature, Oustaloup's technique is the most widely used frequency domain approximation method. The formula for computing the Oustaloup and refined Oustaloup approximations in red and green coloured boxes is in Figure 1. These approximation techniques are valid for estimating the Nth order approximation of order within the lower and higher frequencies of ω_l and ω_h, respectively.

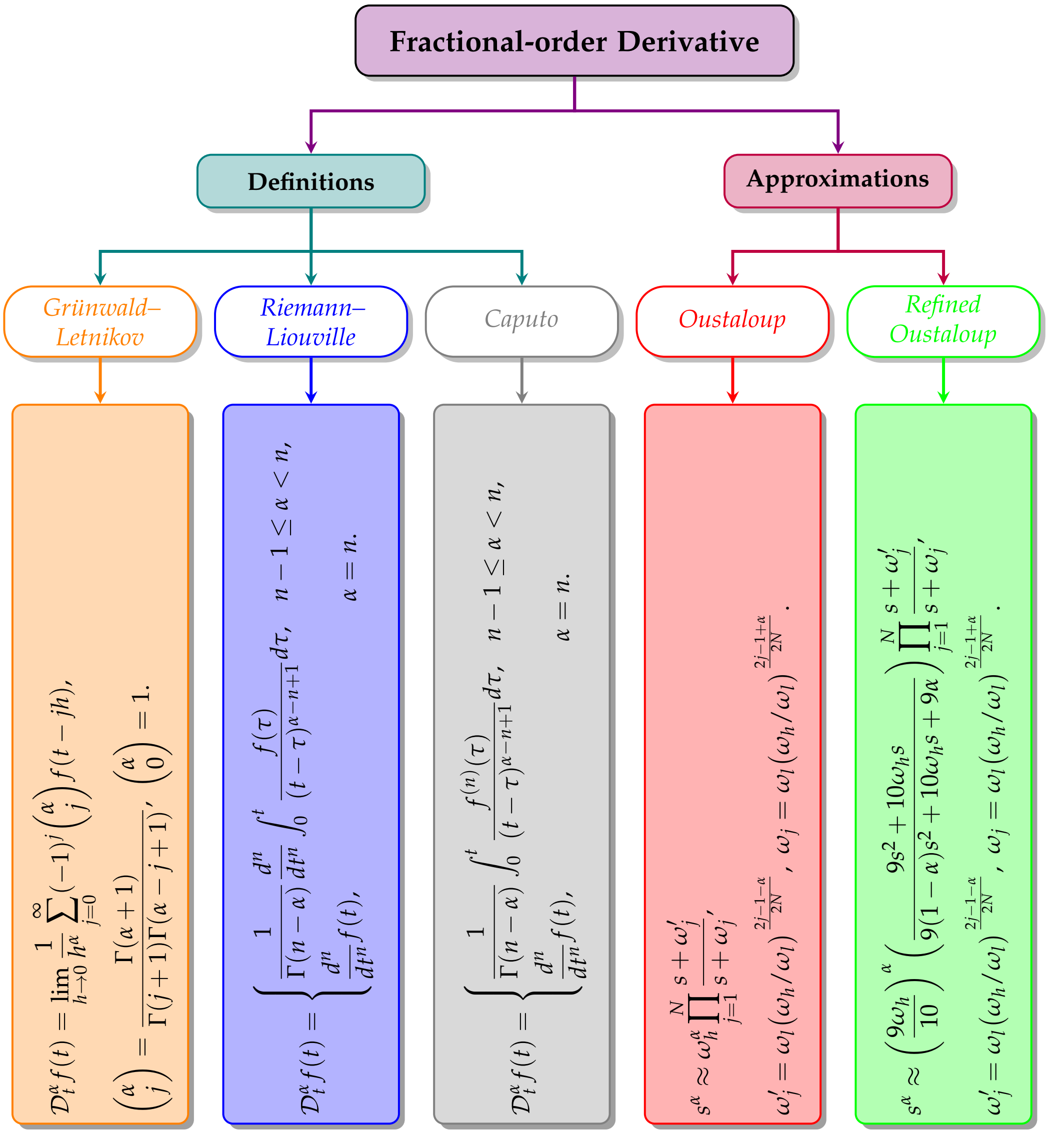


Figure 1. Definitions and approximation techniques of fractional-order derivative.

3. Survey With Trend Analysis

From the collected literature review in Table 1, a graphical trend analysis is made in this section. From the table, the summary of the manipulators' trend is given in Figure 2. As shown in the figure, research has been conducted on various manipulators of DOF ranging from 1 to 7. However, most of the research on developing either fractional-order models or controllers has been conducted on 1, 2, and 3 DOF manipulators, with 2 DOF being the highest, around 60% (see Figure 2a). Moreover, as shown in Figure 2b, about 66% of research has been conducted on robotic manipulators without any payload, and only 34% work with a load. Further, it can be observed from Figure 2c that the research on developing either fractional-order models or controllers has been performed primarily on two-link, rigid planar, and single-link manipulators. It is also worth highlighting that research has been conducted on some industrial manipulators, including PUMA 560, SCARA, Polaris -I, Stewart platform, Staubli RX-60, Robotino-XT, Mitsubishi RV-4FL, KUKA youBot, Fanuc, ETS-MARSE, EFFORT-ERC20C-C10, Delta robot, differential-drive mobile robot [8] and University of Maryland manipulators.

Table 1. Summary of works focussed on fractional-order modelling and controlling of robotic manipulators.

Ref.	Manipulator Details				Modelling Details			Controller Details						Tool	S/P
	Type	DOF	Payload	FOM	Method	Order	FOC	Controller	CP	Tuning Technique	Comparison Controllers	OF	Approximation		
[9]	2R robotic manipulator	2	✗	✗	Mathematical modelling	2	✓	Fractional-order D controller	2	Trial and error	PI and PD controllers	Transient response characteristics	Padé approximation	—	S
[10]	Redundant manipulator	—	✗	✗	Closed-Loop Pseudoinverse	2	✓	Pseudoinverse Algorithm	5	—	—	Tracking error	Grünwald–Letnikov's method	—	S
[11]	Single-link flexible manipulator	1	✓	✗	Mathematical modelling	2	✓	Fractional-order PD controller	3	Trial and error	PD controller	Stability	Digital IIR filter approximation	M	P
[12]	Robotic manipulator	2	✓	✗	Mathematical modelling	2	✓	Fractional fuzzy adaptive sliding mode controller	5	Trial and error	—	Tracking error	CRONE approximations	M	S
[13]	Rotational joints robotic manipulator	2	✓	✗	Mathematical modelling	2	✓	Fractional-order PD-PI controller	5	Trial and error	PD-PI controller	Transient response characteristics	—	—	S
[14]	Two-link robotic manipulator	2	✗	✗	Lagrangian formulation	2	✓	Adaptive fractional-order PID controller	5	Genetic Algorithm	PID controller	ISE	CRONE approximations	—	S
[15]	Polar robotic manipulator	2	✓	✗	State space model	4	✓	Fuzzy Fractional-order PD surface sliding mode controller	8	Genetic Algorithm	Classical PD surface sliding mode controller	RMSE	Caputo derivative	—	S
[16]	Two-link flexible joint manipulator	2	✗	✗	Lagrangian formulation	8	✓	Fractional order fuzzy sliding mode controller	6	Genetic Algorithm	Sliding mode controller, PD surface sliding mode controller, Sliding surfaces through fractional PD controller	IAE, ITAE, ISV	Caputo derivative	—	S
[17]	Two-link planar rigid robotic manipulator	2	✗	✗	Mathematical modelling	2	✓	Fractional-order PID controller	5	Particle Swarm Optimization	Fuzzy and PID controllers	RMSE, MAE, MMFAE	Riemann–Liouville method	—	S
[18]	Mechanical manipulator	2	✗	✗	Mathematical modelling	3	✓	Fractional variable structure control and sliding mode control	6	Trial and error	Integer variable structure control and sliding mode control	Switching activity	Taylor series expansion	—	P
[19]	Two-link planar rigid robotic manipulator	2	✗	✗	Mathematical modelling	2	✓	Fractional-order PID controller	5	Genetic Algorithm, Particle Swarm Optimization	—	RMSE, MAE, MMFAE	—	M	S
[20]	Manipulator robot (Fanuc)	6	✓	✗	Robust disturbance observer	1	✓	Fractional-order PI controller	3	Decentralized tuning	PI controller	Gain Margins	Refined Oustaloup Filter	M	P

Table 1. *Cont.*

Ref.	Manipulator Details				Modelling Details			Controller Details						Tool	S/P
	Type	DOF	Payload	FOM	Method	Order	FOC	Controller	CP	Tuning Technique	Comparison Controllers	OF	Approximation		
[21]	University of Maryland (UMD) manipulator	3	✓	✗	Mathematical modelling	2	✓	Fractional-order PID controller	5	Pattern search optimization	PID controller	MSE	—	—	S
[22]	Flexible link manipulator	2	✓	✗	Euler-Bernoulli method	2	✓	Fractional-order sliding mode controller	6	Particle Swarm Optimization	Sliding mode controller	ISE	Riemann–Liouville method	—	S
[23]	Angular manipulator	3	✗	✗	Lagrange model	2	✓	Fractional-order PID controller	5	Trial and error	—	—	Riemann–Liouville method	M, L	P
[24]	Robotic manipulator	6	✓	✗	Mathematical modelling	6	✓	Fractional-order PD controller	3	Bode tuning	PD controller	Linear and angular velocities	Grünwald–Letnikov method	M	S
[25]	Single-link flexible manipulator	1	✗	✓	Non-commensurate fractional-order model	0.71, 0.92	✓	Fractional order sliding mode controller	4	QR decomposition method	Sliding mode controller	Tracking error	Caputo derivative	M	P
[4]	Two-link planar rigid robotic manipulator	2	✗	✗	Mathematical modelling	2	✓	Fractional-order fuzzy PID controller	6	Cuckoo Search Algorithm	Fuzzy PID, fractional-order PID and PID controllers	IAE, IACCO	Oustaloup's approximation	M	S
[26]	Hydraulic manipulator	2	✓	✗	Mathematical modelling	2	✓	Fractional-order nonsingular terminal sliding mode controller	16	Trial and error	Integer-order nonsingular terminal sliding mode controller	RMSE	Refined Oustaloup filter	M	P
[27]	Single-link flexible manipulator	1	✗	✓	Non-commensurate fractional-order model	0.71, 0.92	✓	Observer-based fractional-order sliding mode controller	8	Stability criterion	Sliding mode controller	Tracking error	Caputo derivative	—	P
[5]	Two-link planar rigid robotic manipulator	2	✓	✗	Mathematical modelling	2	✓	Two-degree of freedom fractional-order PID controller	8	Cuckoo Search Algorithm	Two-degree of freedom PID controller	Weighted sum of ITAE and IACCO	Oustaloup's approximation	M	S
[28]	Two-link robotic manipulator	2	✗	✗	Mathematical modelling	2	✓	Adaptive fractional-order nonsingular fast terminal sliding mode controller	13	Trial and error	Nonsingular terminal, Second-order sliding mode controllers	Error, Reaching time, Chattering effect	Riemann–Liouville method	—	S
[29]	Two-link robotic manipulator	2	✗	✗	Mathematical modelling	2	✓	Fractional-order PID controller	5	Particle swarm optimization, Genetic algorithm and Estimation of distribution algorithm	—	RMSE	Riemann–Liouville method	M	S

Table 1. *Cont.*

Ref.	Manipulator Details				Modelling Details			Controller Details						Tool	S/P
	Type	DOF	Payload	FOM	Method	Order	FOC	Controller	CP	Tuning Technique	Comparison Controllers	OF	Approximation		
[30]	Robotic manipulator (PUMA 560)	2	✗	✗	Mathematical modelling	2	✓	Fractional-order fuzzy PID controller	5	Genetic Algorithm	PID, fractional-order PID and fuzzy PID controllers	ISE	—	M	S
[31]	Two-link planar rigid robotic manipulator (SCARA)	2	✓	✗	Mathematical modelling	2	✓	Two-layered fractional-order fuzzy logic controller	10	Cuckoo Search Algorithm	Two-layered, single-layred fuzzy logic, PID controllers	IAE	Oustaloup's approximation	M	S
[32]	Rotary manipulator	2	✗	✗	Mathematical modelling	2	✓	Fractional-order adaptive backstepping controller	7	Trial and error	Adaptive backstepping controllers	Tracking performance	Caputo derivative	M	P
[33]	Robotic manipulator	4	✗	✓	Pseudoinverse algorithm	0.5, 0.6, 0.8, 0.9, 0.99	✗	—	—	—	—	Tracking accuracy	Grünwald–Letnikov method	M	S
[34]	Inchworm/ Caterpillar robotic manipulator	1	✗	✗	Euler–Lagrange method	2	✓	Neural network-based fraction integral terminal sliding mode controller	5	Trial and error	Sliding mode controller, Integral terminal sliding mode controller, Fraction integral terminal sliding mode controller	Tracking error	—	M	S
[35]	Single-link direct joint driven robotic manipulator	1	✗	✗	Mathematical modelling	2	✓	Sliding mode based fractional-order PD type iterative learning control	5	Trial and error	Sliding mode based fractional-order D type iterative learning control, Higher-order iterative learning control	Tracking error	CRONE approximations	M	S
[36]	Robotic manipulator	2	✓	✗	Mathematical modelling	2	✓	Time delay estimation-based fractional-order nonsingular terminal sliding mode controller	9	Trial and error	Time delay estimation-based, continuous nonsingular terminal, Time delay estimation-based integer-order nonsingular terminal sliding mode controllers	Tracking error	Riemann–Liouville method	M	P
[37]	Inchworm/ Caterpillar robotic manipulator	1	✗	✗	Euler–Lagrange formalism	2	✓	Adaptive fractional-order PID sliding mode controller	5	Bat optimization algorithm	PID, fractional-order PID, sliding mode controller	Weighted sum of IAE and ISV	Oustaloup's recursive approximation	M	S

Table 1. *Cont.*

Ref.	Manipulator Details				Modelling Details			Controller Details						Tool	S/P
	Type	DOF	Payload	FOM	Method	Order	FOC	Controller	CP	Tuning Technique	Comparison Controllers	OF	Approximation		
[38]	Five-bar-linkage robotic manipulator	-	✗	✗	Mathematical modelling	2	✓	Fractional-order PID controller	5	Modified Particle Swarm Optimization	Fractional-order PID controller tuned using standard, constriction factor approach, random inertia weight-based particle swarm optimization algorithms	IAE, ISE, ITSE	Oustaloup's approximation	M	P
[39]	Two-link robotic manipulator	2	✓	✗	Mathematical modelling	2	✓	Interval type-2 fractional-order fuzzy PID controller	6	Artificial Bee Colony-Genetic Algorithm	Interval type-2 fuzzy PID, Type-1 fractional-order fuzzy PID, Type-1 fuzzy PID, PID	ITAE	Oustaloup's approximation	M	S
[40]	Single-link flexible manipulator	1	✗	✗	Mathematical modelling	2	✓	Fractional-order phase-lead compensator	4	Nyquist criterion	PID controller	Gain Margin	Grünwald–Letnikov method	—	P
[41]	Three and five links redundant manipulators	3, 5	✗	✓	Moore-Penrose pseudoinverse	—	✗	—	—	—	—	—	Grünwald–Letnikov method	M	S
[42]	Robotic manipulator	2	✓	✗	State space model	4	✓	Fractional-order global sliding mode controller	10	Trial and error	Sliding mode controller	Tracking error	Riemann–Liouville method	—	S
[43]	Robotic manipulator	2	✗	✗	Mathematical modelling	2	✓	Fractional-order fuzzy pre-compensated fractional-order PID controller	9	Hybrid artificial bee colony-genetic algorithm	Fuzzy pre-compensated PID, fuzzy PID and PID controllers	ITAE	Oustaloup's recursive approximation	M	S
[44]	Two-link planar rigid robotic manipulator	2	✓	✗	Mathematical modelling	2	✓	Non-linear adaptive fractional-order fuzzy PID controller	7	Backtracking search algorithm	Non-linear adaptive fuzzy PID controller	ITAE, ITACO	Grünwald–Letnikov method	L	S
[45]	Two-link robotic manipulator	2	✗	✓	Fractional adaptive neural network	—	✓	Fractional-order PID controller	5	Trial and error	—	Tracking error	Caputo derivative	—	S
[46]	Two-link rigid planar manipulator	2	✗	✗	Mathematical modelling	2	✓	Fractional-order PID controller	5	Genetic Algorithm	PID controller	Weighted sum of IAE and ISCCO	Short memory principle	L	P
[47]	Rotary flexible joint manipulator	1	✗	✗	Mathematical modelling	2	✓	Fractional-order integral controller	2	Gain margins	Integral controller	Tracking accuracy	Oustaloup's approximation	M	P
[48]	Electrically driven three-link rigid robotic manipulator	3	✗	✗	Mathematical modelling	3	✓	Fractional-order fuzzy PD+I controller	4	Cuckoo Search Algorithm	PID, Fractional-order PID, Integer-order fuzzy PD+I	IAE	Grünwald–Letnikov method	M	S

Table 1. *Cont.*

Ref.	Manipulator Details				Modelling Details			Controller Details						Tool	S/P
	Type	DOF	Payload	FOM	Method	Order	FOC	Controller	CP	Tuning Technique	Comparison Controllers	OF	Approximation		
[49]	Robotic manipulator (SCARA)	2	✗	✗	Linear model	2	✓	Fractional-order model reference adaptive controller	3	Trial and error	Model reference adaptive controller	Delay time	Oustaloup's approximation	—	S
[50]	Robotic manipulator (PUMA 560)	3	✗	✗	Mathematical modelling	2	✓	Fractional-order nonsingular fast terminal sliding mode control based fault tolerant control	7	Trial and error	Adaptive fractional-order nonsingular fast terminal sliding mode controller, Nonsingular fast terminal sliding mode control based active fault tolerant control	Convergence speed	Riemann–Liouville method	—	S
[51]	Two-link planar electrically-driven rigid robotic manipulator	2	✗	✗	Mathematical modelling	2	✓	Fractional-order self organizing fuzzy controller	6	Cuckoo Search Algorithm	Fractional-order fuzzy PID	IAE	Grünwald–Letnikov method	M	S
[52]	Serial link manipulator	2	✗	✗	Mathematical modelling	2	✓	Fractional-order PID and auxiliary controllers	5	Trial and error	Torque approach controller	Tracking error	CRONE approximations	M	S
[53]	Redundant manipulator (SCARA)	5	✗	✗	Mathematical modelling	2	✓	Fuzzy fractional-order PID controller	6	Artificial Bee Colony Algorithm	PID and fuzzy PID controllers	ITAE	—	M	S
[54]	Three-link robotic manipulator (Staubli RX-60)	6	✗	✗	Mathematical modelling	3	✓	Fractional-order PID controller	5	Cuckoo Search Algorithm	PID controller	IAE, ITAE, ISE and IACCO	—	M	S
[55]	Robotic manipulator	6	✗	✗	Kinematic modelling	2	✓	Fractional order nonsingular fast terminal sliding mode control	13	Trial and error	—	Tracking error	Riemann–Liouville method	—	S
[56]	Three-link planar rigid robotic manipulator	3	✗	✗	Euler–Lagrange formalism	3	✓	Fractional-order PID controller	5	Evaporation Rate-Based Water Cycle Algorithm	PID controller	Weighted sum of IAE and IACCO	Grünwald–Letnikov method	M	S
[57]	Two-link planar rigid robotic manipulator	2	✗	✗	Euler–Lagrange formalism	2	✓	Fractional-order fuzzy sliding mode PD/PID controller	8	Cuckoo Search Algorithm	Integer-order fuzzy sliding mode PD/PID controller	Weighted sum of IAE and chatter	Grünwald–Letnikov method	M	S
[58]	Two-link planar rigid robotic manipulator	2	✗	✗	Lagrangian-Euler formulation	2	✓	Fractional-order fuzzy sliding mode controller with proportional derivative surface	6	Genetic Algorithm	Integer-order fuzzy SMC with proportional derivative surface	Weighted sum of IAE and chatter	Grünwald–Letnikov method	M	S
[59]	Parallel robotic manipulators (Delta Robot)	3	✓	✗	Inverse kinematic model	3	✓	Fractional-order PID controller	5	FMINCON (Gradient descent algorithm)	PID controller	RMSE	—	M	P

Table 1. *Cont.*

Ref.	Manipulator Details				Modelling Details			Controller Details						Tool	S/P
	Type	DOF	Payload	FOM	Method	Order	FOC	Controller	CP	Tuning Technique	Comparison Controllers	OF	Approximation		
[60]	Robotic manipulator (SCARA)	2	✗	✓	Euler–Lagrange and Hamilton formalisms	1.14	✓	Fractional-order PI/PD controller	3	Particle Swarm Optimization	PI/PD controller	ITAE	Grünwald–Letnikov method	M	S
[61]	Serial robotic manipulator	6	✗	✗	Mathematical modelling	2	✓	Fractional-order adaptive nonsingular terminal siding mode controller	8	Trial and error	—	Tracking error	Riemann–Liouville method	M	S
[3]	Cable-driven manipulator (Polaris-I)	2	✓	✗	Mathematical modelling	2	✓	Time delay control scheme-based adaptive fractional-order nonsingular terminal sliding mode controller	15	Trial and error	Time delay estimation-based adaptive, continuous fractional-order nonsingular terminal sliding mode controller	RMSE	Riemann–Liouville method	M	P
[62]	Robotic manipulator	2	✗	✗	Euler–Lagrange formalism	2	✓	Fuzzy fractional-order PID controller	3	Heuristic Tuning	Sliding mode control, Super twisting sliding mode control, Fuzzy PID	ITAE, ISE	Grünwald–Letnikov method	C++	P
[63]	Rigid planar robotic manipulator	2	✓	✗	Mathematical modelling	2	✓	Collaborative fractional order PID and fractional order fuzzy logic controller	9	Cuckoo Search Algorithm	PID, Fractional-order PID, Fractional-order fuzzy PID	ITAE	Oustaloup's recursive approximation	M	S
[64]	Two-link robotic manipulator	2	✗	✗	Mathematical modelling	2	✓	Two-degree-of-freedom fractional-order fuzzy PI-D	16	Multi-objective non-dominated sorting genetic algorithm-II	Two-degree-of-freedom fractional-order PI-D	IAE	Grünwald–Letnikov method	M	S
[65]	Three-link planar rigid robotic manipulator	3	✓	✗	Euler–Lagrange formalism	3	✓	Self-regulated fractional-order fuzzy PID controller	6	Backtracking Search Algorithm	Self-regulated integer-order fuzzy PID controller	IAE, IACCO	Grünwald–Letnikov method	L	S
[66]	Single-link flexible manipulator	1	✓	✗	Lagrangian formulation	2	✓	Sliding fractional order controller	6	Trial and error	PD controller	Tracking error	—	—	S
[67]	Two-link robotic manipulator	2	✗	✗	Mathematical modelling	2	✓	Fractional-order fuzzy PID controller	6	Particle Swarm Optimization	Fractional-order PID controller	IAE, IACCO	Oustaloup's approximation	M	S
[68]	Single-link flexible manipulator	1	✓	✗	State space model	4	✓	Fractional-order sliding mode controller	10	Trial and error	PID, Sliding mode controller	RMSE, MAE	CRONE approximations	M	S
[69]	Cable-driven manipulator (Polaris-I)	2	✓	✗	Mathematical modelling	2	✓	Fractional-order nonsingular terminal sliding mode controller	12	Closed-loop control tuning	Time delay estimation-based and continuous fractional-order nonsingular terminal sliding mode controller	RMSE	Refined Oustaloup filter	M	P

Table 1. *Cont.*

Ref.	Manipulator Details				Modelling Details			Controller Details						Tool	S/P
	Type	DOF	Payload	FOM	Method	Order	FOC	Controller	CP	Tuning Technique	Comparison Controllers	OF	Approximation		
[70]	Serial Flexible Link Robotic Manipulator, Serial Flexible Joint Robotic Manipulator	2	✗	✓	Fractional transfer function model	0.3, 0.9	✓	Fractional-order PID controller	5	Trial and error	PID controller	Transient response characteristics	Oustaloup's approximation	M	P
[71]	Robotic manipulator	2	✗	✗	Kinematic modelling	2	✓	Fractional-order PID controller	5	Particle Swarm Optimization	PID controller	Error	—	—	S
[72]	Two-link flexible robotic manipulator	3	✗	✓	Euler–Lagrange formulation	0.98	✓	Fractional-order adaptive sliding mode controller	13	Trial and error	Adaptive sliding mode controller	Tracking error	—	M	S
[73]	Exoskeleton Robot (ETS-MARSE)	7	✗	✗	Mathematical modelling	2	✓	Adaptive neural network fast fractional integral terminal sliding mode control	6	Trial and error	Fast fractional integral terminal sliding mode controller	Tracking error	Grünwald–Letnikov method	M	P
[74]	Robotic manipulator	2	✓	✗	Mathematical modelling	2	✓	Adaptive fractional high-order terminal sliding mode controller	10	Trial and error	H∞-Adaptive control, intelligent PD, intelligent PID, Adaptive third-order sliding mode controller	Convergence speed and precision	Oustaloup method	M	S
[75]	Robotic manipulator (PUMA 560)	6	✓	✓	Euler–Lagrange formalism	12	✓	Fractional-order PI, PD controllers	9	Cuckoo Search Algorithm	PI, PD controllers	RMSE	Caputo–Fabrizio derivative, Atangana–Baleanu integral	—	P
[76]	3-RRR planar parallel robots	3	✗	✗	Inverse kinematics using Cayley–Menger determinants and bilateration	2	✓	Fractional-order PID controller	5	Bat optimization algorithm	PID controller	Weighted function	—	M	P
[77]	Muscle-actuated manipulator	2	✗	✓	Fractional order describing functions	2	✗	—	—	—	—	—	Grünwald–Letnikov method	—	P
[78]	Rigid robotic manipulator	2	✗	✗	Mathematical modelling	2	✓	Deep convolutional neural network based Fractional-order terminal sliding-mode controller	15	FMINCON (Gradient descent algorithm)	Nonsingular and conventional fractional-order terminal sliding-mode controllers	Fractional-order loss function	Caputo derivative	—	S

Table 1. *Cont.*

Ref.	Manipulator Details				Modelling Details			Controller Details						Tool	S/P
	Type	DOF	Payload	FOM	Method	Order	FOC	Controller	CP	Tuning Technique	Comparison Controllers	OF	Approximation		
[79]	Robotic manipulator	2	✗	✗	Mathematical modelling	2	✓	Fractional-order fuzzy PD and I controller	8	Multi-objective non-dominated sorting genetic algorithm-II, dragonfly algorithm, multi-verse optimization, ant lion optimizer algorithms	PID, fuzzy PID controllers	IAE	Grünwald–Letnikov method	M	P
[80]	Robotic manipulator (SCARA)	2	✗	✗	Mathematical modelling	2	✓	Fractional-order PID and Fractional-order pre-filter	5, 4	Genetic Algorithm, Trial and error	—	Gain Margins	CRONE approximations	M	S
[81]	Two-link robotic manipulator	2	✗	✗	Mathematical modelling	2	✓	Time delay estimation-based adaptive fractional-order nonsingular terminal sliding mode controller	12	Trial and error	Nonsingular fast terminal sliding mode controller, Second order nonsingular fast terminal sliding mode controller	Tracking error	Riemann–Liouville method	M	S
[82]	Parallel robotic manipulator	6	✓	✗	Kinematic modelling	3	✓	Fractional-order active disturbance rejection controller	16	Trial and error	Active disturbance rejection controller	Tracking accuracy	—	M	P
[83]	Single-link robotic manipulator	1	✗	✓	Euler–Lagrange formulation	0.5	✗	Feedback controller	8	Pole placement method	PID, LQR controllers	Tracking accuracy	Oustaloup’s approximation	M	P
[83]	Serial-link flexible robotic manipulator, Serial flexible joint robotic manipulator	2	✗	✓	Fractional value selection algorithm	0.3, 0.9	✓	Fractional-order PID controller	5	Trial and error	PID controller	Tracking accuracy	Oustaloup’s approximation	M	P
[84]	Rotary flexible joint manipulator	1	✗	✗	Mathematical modelling	2	✓	State-feedback-based fractional-order integral controller	2	Trial and error	Pure state-feedback control scheme and the modified state-feedback-based fractional-order integral controllers	Tracking error	CRONE, Oustaloup’s approximations	M	S
[85]	Robotic manipulator (PUMA 560)	3	✓	✗	State space model	2	✓	Fractional-order adaptive backstepping controller	6	Trial and error	PID and Computed torque controllers	Tracking error and convergence speed	Caputo method	M	S

Table 1. *Cont.*

Ref.	Manipulator Details				Modelling Details			Controller Details						Tool	S/P
	Type	DOF	Payload	FOM	Method	Order	FOC	Controller	CP	Tuning Technique	Comparison Controllers	OF	Approximation		
[86]	Two-link robotic manipulator	2	✗	✗	Mathematical modelling	2	✓	Time delay estimation-based adaptive fractional-order nonsingular terminal sliding mode controller	10	Trial and error	—	Tracking performance and speed	Oustaloup's recursive approximation	—	S
[83]	Single Rigid Link Robotic Manipulator, Serial Link Robotic Manipulator	2	✗	✗	Mathematical modelling	2	✓	Adaptive fractional-order controller	5	Trial and error	Integer-order and adaptive controllers	Transient response characteristics	Oustaloup's approximation	M	P
[2]	Cooperative manipulator (Mitsubishi RV-4FL)	6	✓	✗	Kinematic modelling	3	✓	Coupled fractional-order sliding mode control	5	Fuzzy tuning	PI, Sliding mode controllers, fractional-order sliding mode controller	IAE, ISE, STD	Oustaloup's approximation	M	P
[87]	Single flexible link robotic manipulator, Serial flexible joint robotic manipulator	1,2	✗	✓	Euler–Lagrange formulation	0.5	No	Feedback controller	8	Pole placement method	PID, LQR controllers	Tracking accuracy	Oustaloup's approximation	M	P
[88]	Single flexible link robotic manipulator, Serial flexible joint robotic manipulator	1,2	✗	✗	Euler–Lagrange formulation	2	✓	Fractional-order PID controller	5	Trial and error	PID controller	Transient response characteristics	Oustaloup's approximation	M	S
[89]	Stewart Platform	6	✗	✗	Lagrange-Euler approach	3	✓	Fractional order fuzzy PID controller	8	Particle Swarm Optimization	PID, fractional-order PID and fuzzy PID controllers	MAE, RMSE	Oustaloup's approximation	M	P
[90]	Robotic manipulator (PUMA 560)	3	✗	✗	Mathematical modelling	2	✓	Fractional-order backstepping fast terminal sliding mode controller	15	Trial and error	PID, Computed torque controller, Nonsingular fast terminal sliding mode controller	Position tracking error	Oustaloup's approximation	M	S
[91]	Robotic manipulator (EFFORT-ERC20C-C10)	6	✓	✗	Mathematical modelling	2	Yes	Fractional-order impedance control	3	Frequency design method	Impedance control	ITSE	Impulse response method	—	P

Table 1. *Cont.*

Ref.	Manipulator Details				Modelling Details			Controller Details						Tool	S/P
	Type	DOF	Payload	FOM	Method	Order	FOC	Controller	CP	Tuning Technique	Comparison Controllers	OF	Approximation		
[1]	Three-link omnidirectional mobile robot manipulator (KUKA youBot)	5	✗	✗	Lagrangian dynamics equation	3	✓	Adaptive fractional-order nonsingular terminal sliding mode controller	9	Trial and error	Fractional-order terminal sliding mode controller, Nonsingular terminal sliding mode controller	Tracking speed and accuracy	Riemann–Liouville method	M	P
[92]	Two-link Rigid Robotic Manipulator	2	✗	✗	Mathematical modelling	2	✓	Fractional-order fuzzy PID controller	6	Most valuable player algorithm	Integer-order fuzzy PID, One block fractional/Integer order fuzzy PID, Two block Fractional/Integer order fuzzy PID controllers	ITSE	Grünwald–Letnikov method	M	S
[93]	Robotic manipulator	2	✓	✗	Euler–Lagrange method	2	Yes	Fractional-order PID controller	5	Gradient-based optimization	PID controller	ISE	—	M	S
[94]	Single-segment soft continuum manipulator (Robotino-XT)	—	✓	✓	Fractional-order Bouc–Wen hysteresis model	16	—	—	—	—	—	Absolute pose error	Grünwald–Letnikov method	—	P
[95]	Two-link robotic manipulator	2	✓	✗	Mathematical modelling	—	✓	Fractional-order fuzzy PID controller	8	Hybrid grey wolf optimizer and artificial bee colony algorithm	PID	Tracking error	—	M	P
[96]	Robotic manipulator	—	✗	✓	Fractional-order Euler–Lagrange formulation	—	—	—	—	—	—	—	—	—	P
[97]	Stewart Platform	6	✓	✗	Kinematic modelling	2	✓	Fractional-order KDHD impedance control	2	Transient response-based tuning	KD controller	Error	Grünwald–Letnikov method	M	S
[98]	3-PUU parallel robotic manipulator	3	✗	✗	Kinematic modelling	2	✓	PDD1/2 controller	2	Transient response-based tuning	PD controller	Error	Grünwald–Letnikov method	M	S
[99]	Flexible link manipulator	2	✗	✗	Euler–Lagrange formulation	2	✓	Fractional-order phase-lag compensator	3	Optimization process	2DOF PID controller	Tracking error	Grünwald–Letnikov method	M	P

Table 1. *Cont.*

Ref.	Manipulator Details				Modelling Details			Controller Details						Tool	S/P
	Type	DOF	Payload	FOM	Method	Order	FOC	Controller	CP	Tuning Technique	Comparison Controllers	OF	Approximation		
[100]	Single-link flexible manipulator	2	✓	✗	Euler–Bernoull formulation	2	✓	Fractional-order PD	2	Bode Specifications	PD controller	Bode Margins	Grünwald–Letnikov method	M	P
[101]	KUKA LWR IV	7	✓	✓	Inverse Kinematics Model	3.04	✓	Impedance control	4	Genetic Algorithm	—	MSE, MAD	—	—	P
[102]	Single-link flexible manipulator	2	✓	✗	Pseudo-clamped approach	2	✓	Fractional-order PID	2	Bode Specifications	PID controller	Tracking error	Frequency response-based technique	M	P

The notations used in the table header are as follows: DOF—degree of freedom; FOM—fractional-order model; FOC—fractional-order control; CP—controller parameters; OF—objective function; M—MATLAB; L—LabVIEW; S/P—simulation/practical.

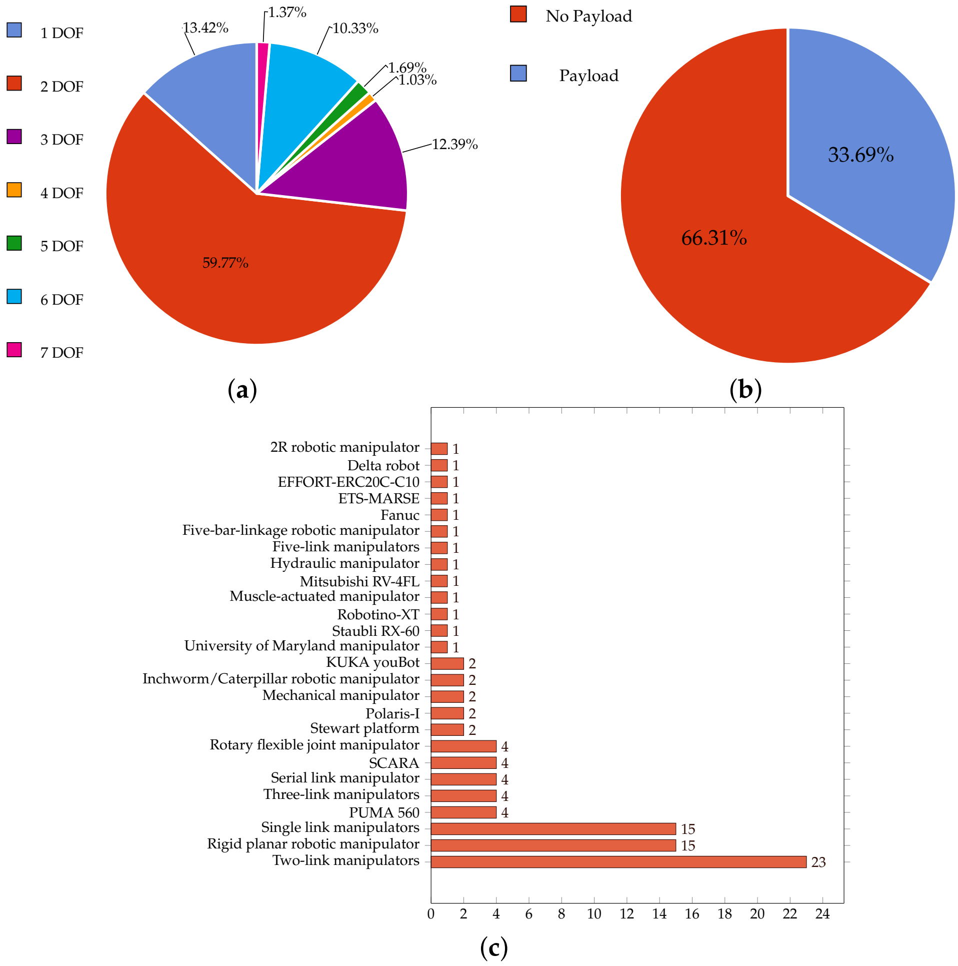


Figure 2. Summary of manipulator details from Table 1. (**a**) Manipulators' DOF trend; (**b**) Payload trend; (**c**) Manipulator's type.

Figure 3 gives a summary of the modelling approach and techniques used for robotic manipulators. As shown in Figure 3a, approximately 85% of modelling approaches used in the literature are conventional/integer-order type only. The remaining 15% of works have developed a fractional-order model of orders 0.3, 0.5, 0.6, 0.71, 0.8, 0.9, 0.92, 0.99, 1.14 and 3.04. Figure 3b shows that Euler–Lagrange relations have often been used to develop the manipulator's dynamic model in the conventional model category. In the fractional-order model category, various approaches, including adaptive neural network, describing functions, value selection algorithm, the Bouc–Wen hysteresis model, and the Euler–Lagrange formulation, have been used to develop commensurate and non-commensurate fractional-order models of manipulators. The following section will give a more detailed review of these modelling stargates.

Similarly, Figure 4 shows the summary of controllers, optimization, and approximation techniques used during the manipulators' control design. As shown in Figure 4a, the most widely developed fractional-order controllers use PID, sliding mode, and fuzzy. This is because PID is often used in the industry due to the advantages of simplicity and easy tuning and implementation. At the same time, the sliding mode offers the benefits of computational simplicity, less sensitivity to parameter uncertainties, being highly robust to disturbances, and fast dynamic response. On the other hand, fuzzy achieves better servo and regulatory response. However, sliding mode and fuzzy requires more controller parameters to be tuned. Researchers have used various optimization algorithms for tuning, as shown in Figure 4b. The figures show that about 70% have used genetic algorithms, cuckoo search, and particle swarm optimization. This is because these are the most popular and widely considered benchmark algorithms. Figure 4c gives the trend of approximation techniques used in manipulator modelling and controller design. The figures show Grünwald–Letnikov, Riemann–Liouville, Caputo, Oustaloup/refined Oustaloup approximations are the most frequently used techniques in the literature. More details regarding these approximation techniques can be found in [7]. A more detailed review of these control and optimization techniques stargates will be given in the following section.

Figure 5 shows the summary of validation type and type of toolbox, collected from Table 1. Figure 5a shows that about 65% of works, either modelling or validating controller, have been performed in the simulation environment. At the same time, the remaining 35% of results have validated the proposed approaches, practically. For these validations, approximately 90% of the researchers have used MATLAB, while others used LabVIEW, C++, and Solidworks. It is also worth highlighting that several researchers have used externally developed MATLAB-based toolboxes such as CRONE, Ninteger, and FOMCON to realize fractional-order systems and controllers [7].

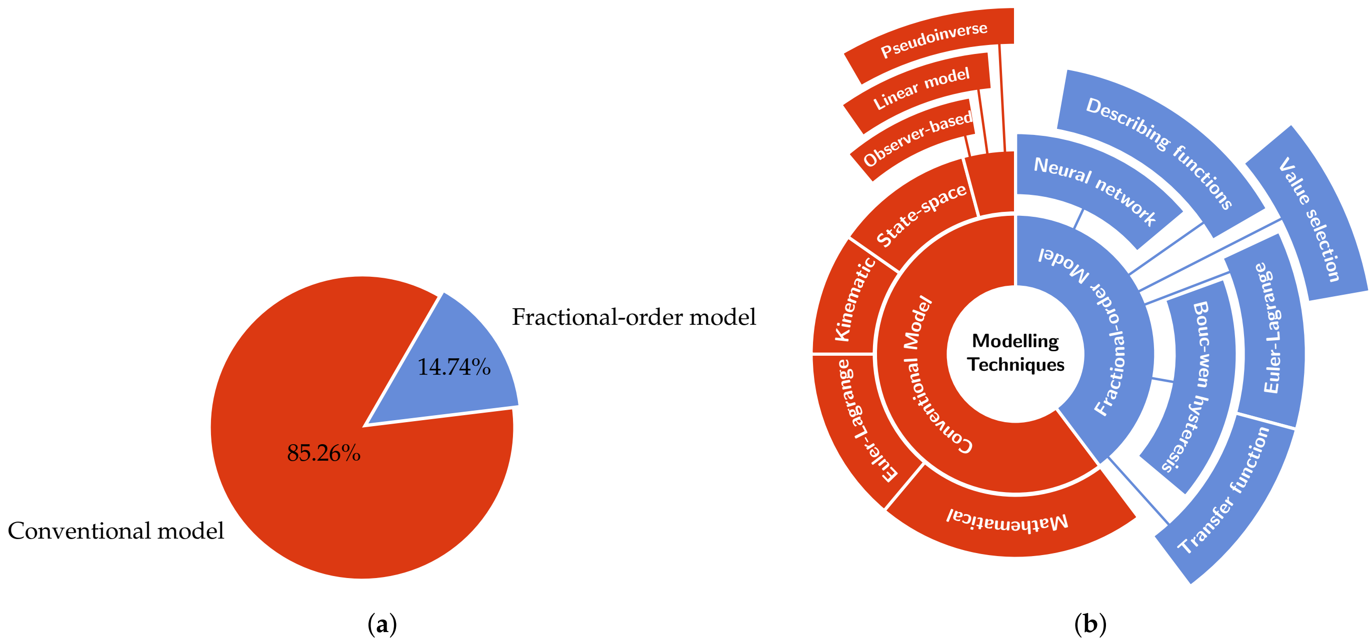


Figure 3. Summary of modelling details from Table 1. (**a**) Type of modelling approach. (**b**) Various types of modelling techniques.

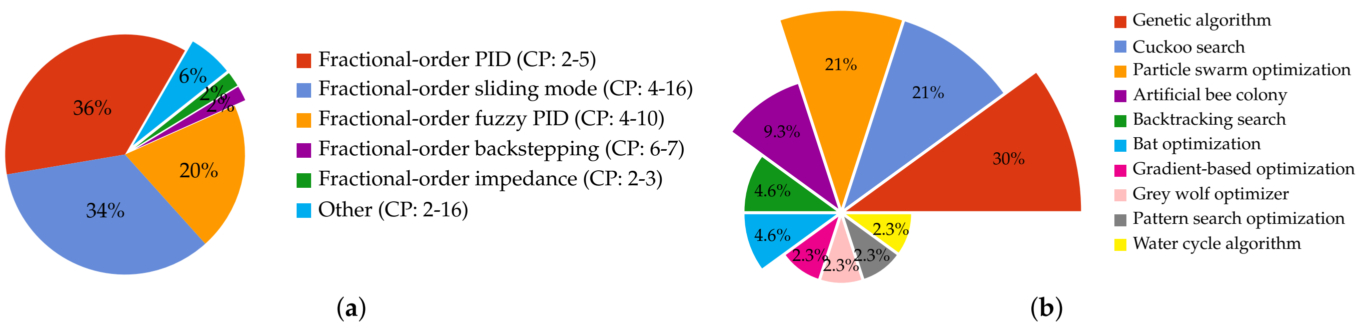
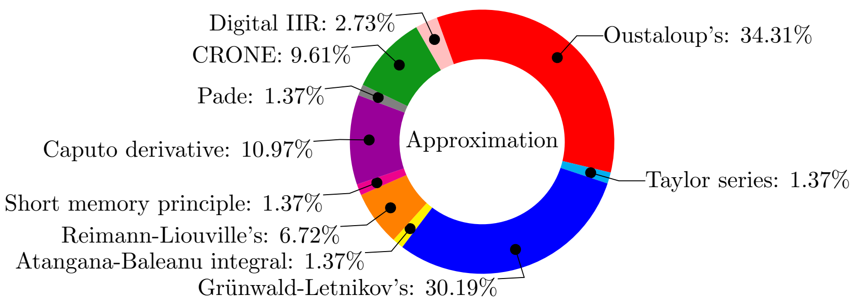
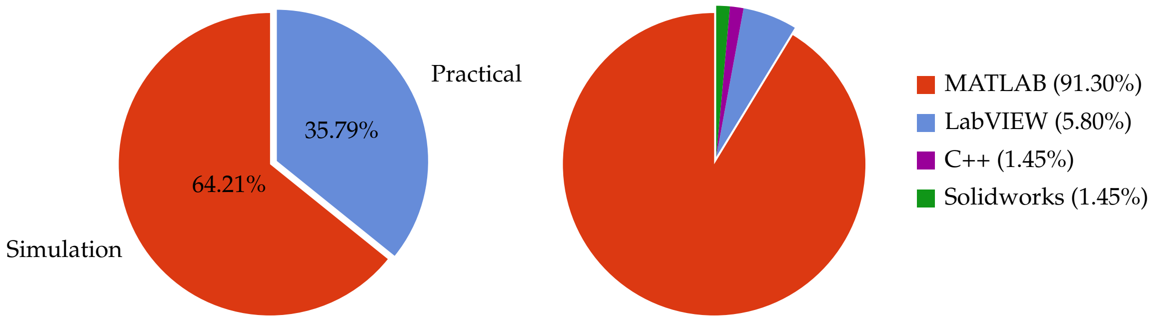


Figure 4. *Cont.*



(**c**)

Figure 4. Summary of controller, optimization and approximation technique details from Table 1. (**a**) Fractional-order controllers. (**b**) Optimization techniques. (**c**) Approximation techniques.



(**a**) (**b**)

Figure 5. Summary of implementation type from Table 1. (**a**) Validation type. (**b**) Software toolboxes.

4. Modelling of Robotic Manipulators

As mentioned in Section 3, the Newton–Euler equations and Lagrange-assumed modes methods are most widely used for obtaining the mathematical model of robotic manipulators [103–105]. The Newton–Euler equations are based on Newton's second law of motion, while the Lagrange method derives the motion equations by eliminating interaction forces between adjacent links. In other words, Newton–Euler is a force balance approach, whereas the Lagrange method is an energy-based approach to manipulators' dynamics. Moreover, the Euler–Lagrange relations will produce the same equations as Newton's, which help analyze complicated systems. Additionally, these relations have the advantage of taking the same form in any system of generalized coordinates and are better suited for generalizations. Therefore, for developing the dynamic models of single-, two- and three-link robotic manipulators, Euler–Lagrangian relations are used as explained underneath. Further, the generalized model for the N number of rigid and n number of elastic degrees of freedom using the same technique is also given underneath.

4.1. Single-Link Rigid and Flexible Robotic Manipulators

An ideal single-link planar rigid robotic manipulator is shown in Figure 6. The mathematical relationship between torque τ and position θ using Euler–Lagrangian formulation is given as [66,103,105],

$$ml^2\ddot{\theta} + gml\sin(\theta) + v\dot{\theta} = \tau, \tag{2}$$

where v is the friction coefficient.

Let us assume $x_1 = \theta$ and $x_2 = \dot{\theta}$, then (2) can be rewritten as,

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -\frac{g}{l}\sin(x_1) - \frac{v}{ml^2}x_2 + \frac{1}{ml^2}\tau. \end{aligned} \tag{3}$$

The nominal values of robotic manipulator parameters considered in most of the research works are $m = 2$ kg, $v = 6$ kgms, $l = 1$ m and $g = 9.81$ m/s^2. Thus, substituting these nominal values, (3) can be rewritten as,

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -9.81\sin(x_1) - 3x_2 + 0.5\tau. \end{aligned} \tag{4}$$

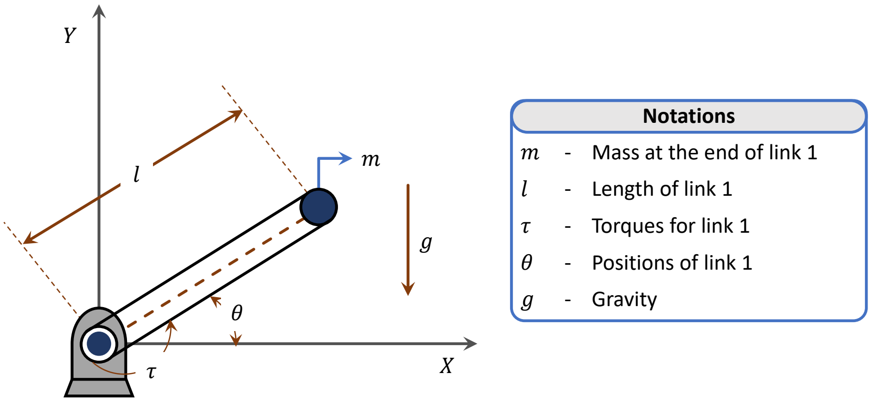


Figure 6. Single-link planar rigid robotic manipulator.

Similarly, the state space representation of an ideal single-link flexible robotic manipulator using Euler–Lagrangian formulation is given as [25,27,70],

$$\begin{aligned} \ddot{\theta} &= -k_1\dot{\theta} + k_2\alpha + k_3V_m, \\ \ddot{\alpha} &= k_1\dot{\theta} - k_4\alpha - k_3V_m, \end{aligned} \tag{5}$$

where α is the tip deflection, θ is the motor shaft position, V_m is the motor input voltage and $k_i, i \in (1,4)$ are constants.

Let us assume $x_1 = \theta$, $x_2 = \alpha$, $x_3 = \dot{\theta}$, $x_4 = \dot{\alpha}$ and $V_m = u$, then (5) can be rewritten as,

$$\begin{aligned} \dot{x}_1 &= x_3, \\ \dot{x}_2 &= x_4, \\ \dot{x}_3 &= p_2x_2 - p_1x_3 + p_3u, \\ \dot{x}_4 &= p_4x_2 + p_1x_3 - p_3u. \end{aligned} \tag{6}$$

From (6), the fractional-order model of a single-link flexible robotic manipulator in non-commensurate order is given as,

$$\begin{aligned} \dot{x}_1^{\beta} &= x_3, \\ \dot{x}_2^{\beta} &= x_4, \\ \dot{x}_3^{\alpha} &= p_2x_2 - p_1x_3 + p_3u, \\ \dot{x}_4^{\alpha} &= p_4x_2 + p_1x_3 - p_3u, \end{aligned} \tag{7}$$

where α and β are the fractional-orders.

4.2. Two-Link Planar Rigid Robotic Manipulator

An ideal two-link planar rigid robotic manipulator or a SCARA-type manipulator with a payload of mass m_p at the tip is shown in Figure 7. The mathematical relationship between torques (τ_1, τ_2) and positions (θ_1, θ_2) of both the links (1, 2) using Euler–Lagrangian formulation is given as [4,5,28,31,39,44,51,64,103,106,107],

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} -(m_2 l_1 l_{c2} \sin(\theta_2))\dot{\theta}_2 & -(m_2 l_1 l_{c2} \sin(\theta_2))(\dot{\theta}_1 + \dot{\theta}_2) \\ (m_2 l_1 l_{c2} \sin(\theta_2))\dot{\theta}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \\ + \begin{bmatrix} m_1 l_{c1} g \cos(\theta_1) + m_2 g (l_{c2} \cos(\theta_1 + \theta_2) + l_1 \cos(\theta_1)) \\ m_2 l_{c2} g \cos(\theta_1 + \theta_2) \end{bmatrix} + \begin{bmatrix} v_1 \dot{\theta}_1 \\ v_2 \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} p_1 sgn(\dot{\theta}_1) \\ p_2 sgn(\dot{\theta}_2) \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}, \quad (8)$$

where

$$M_{11} = m_1 + l_{c1}^2 + m_2(l_1^2 + l_{c2}^2 + 2 l_1 l_{c2} \cos(\theta_2)) + m_p(l_1^2 + l_2^2 + 2 l_1 l_2 \cos(\theta_2)) + I_1 + I_2,$$
$$M_{12} = m_2(l_{c2}^2 + l_1 l_{c2} \cos(\theta_2)) + m_p(l_2^2 + l_1 l_2 \cos(\theta 2)) + I_2,$$
$$M_{21} = m_2(l_{c2}^2 + l_1 l_{c2} \cos(\theta_2)) + m_p(l_2^2 + l_1 l_2 \cos(\theta 2)) + I_2,$$
$$M_{22} = m_2 l_{c2}^2 + m_p l_2^2 + I_2.$$

In (8), v_1, v_2 are the coefficients of viscous friction and p_1, p_2 are the coefficients of dynamic friction of links 1 and 2, respectively. The nominal values of robotic manipulator parameters considered in most of the research works are $m_1 = m_2 = 1.0$ kg, $l_1 = l_2 = 1.0$ m, $l_{c1} = l_{c2} = 0.5$ m, $I_1 = I_2 = 0.2$ kgm^2, $v_1 = v_2 = 0.1$, $p_1 = p_2 = 0.1$, $m_p = 0.5$ kg and $g = 9.81$ m/s^2.

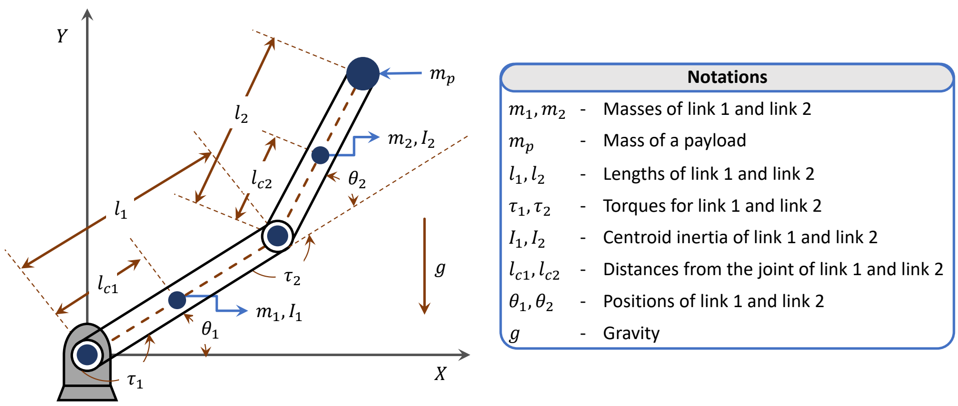


Figure 7. Two-link planar rigid robotic manipulator with a payload.

4.3. Three-Link Planar Rigid Robotic Manipulator

An ideal three-link planar rigid robotic manipulator with no friction, as shown in Figure 8, is where all the masses m_1, m_2 and m_3 exist as a point mass at the end point of each link. The mathematical relationship between torques (τ_1, τ_2, τ_3) and positions (θ_1, θ_2, θ_3) of all the links (1, 2, 3) using Euler–Lagrangian formulation is given as [56,65],

$$\begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} + \\ \begin{bmatrix} -l_1(m_3 l_3 \sin(\theta_2 + \theta_3) + m_2 l_2 \sin(\theta_2) + m_3 l_2 \sin(\theta_2))\dot{\theta}_2^2 - m_3 l_3 (l_1 \sin(\theta_2 + \theta_3) + l_2 \sin(\theta_3))\dot{\theta}_3^2 \\ l_1(m_3 l_3 \sin(\theta_2 + \theta_3) + m_2 l_2 \sin(\theta_2) + m_3 l_2 \sin(\theta_2))\dot{\theta}_1^2 - m_3 l_2 l_3 \sin(\theta_3)\dot{\theta}_3^2 \\ m_3 l_3 (l_1 \sin(\theta_2 + \theta_3) + l_2 \sin(\theta_3))\dot{\theta}_1^2 + m_3 l_2 l_3 \sin(\theta_3)\dot{\theta}_2^2 \end{bmatrix} + \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} +, \quad (9) \\ \begin{bmatrix} (m_1 + m_2 + m_3) g l_1 \cos(\theta_1) + (m_2 + m_3) g l_2 \cos(\theta_1 + \theta_2) + m_3 g l_3 \cos(\theta_1 + \theta_2 + \theta_3) \\ (m_2 + m_3) g l_2 \cos(\theta_1 + \theta_2) + m_3 g l_3 \cos(\theta_1 + \theta_2 + \theta_3) \\ m_3 g l_3 \cos(\theta_1 + \theta_2 + \theta_3) \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}$$

where

$$M_{11} = (m_1 + m_2 + m_3) l_1^2 + (m_2 + m_3) l_2^2 + m_3 l_3^2 + 2 m_3 l_1 l_3 \cos(\theta_2 + \theta_3) + 2(m_2 + m_3) l_1 l_2 \cos(\theta_2) + 2 m_3 l_2 l_3 \cos(\theta_3),$$
$$M_{12} = (m_2 + m_3) l_2^2 + m_3 l_3^2 + m_3 l_1 l_3 \cos(\theta_2 + \theta_3) + (m_2 + m_3) l_1 l_2 \cos(\theta_2) + 2 m_3 l_2 l_3 \cos(\theta_3),$$

$M_{13} = m_3 l_3^2 + m_3 l_1 l_3 \cos(\theta_2 + \theta_3) + m_3 l_2 l_3 \cos(\theta_3)$,
$M_{21} = m_2 l_2^2 + m_3 l_2^2 + m_3 l_3^2 + m_3 l_1 l_3 \cos(\theta_2 + \theta_3) + m_2 l_1 l_2 \cos(\theta_2) + m_3 l_1 l_2 \cos(\theta_2) + 2 m_3 l_2 l_3 \cos(\theta_3)$,
$M_{22} = m_2 l_2^2 + m_3 l_2^2 + m_3 l_3^2 + 2 m_3 l_2 l_3 \cos(\theta_3)$,
$M_{23} = m_3 l_3^2 + m_3 l_2 l_3 \cos(\theta_3)$,
$M_{31} = m_3 l_3^2 + m_3 l_1 l_3 \cos(\theta_2 + \theta_3) + m_3 l_2 l_3 \cos(\theta_3)$,
$M_{32} = m_3 l_3^2 + m_3 l_2 l_3 \cos(\theta_3)$,
$M_{33} = m_3 l_3^2$,
$R_1 = -2 l_1 (m_3 l_3 \sin(\theta_2 + \theta_3) + (m_2 + m_3) l_2 \sin(\theta_2)) \dot{\theta}_1 \dot{\theta}_2 - 2 m_3 l_3 (l_1 \sin(\theta_2 + \theta_3) + l_2 \sin(\theta_3)) \dot{\theta}_2 \dot{\theta}_3 - 2 m_3 l_3 (l_1 \sin(\theta_2 + \theta_3) + l_2 \sin(\theta_3)) \dot{\theta}_1 \dot{\theta}_3$,
$R_2 = -2 m_3 l_2 l_3 \sin(\theta_3) \dot{\theta}_1 \dot{\theta}_3 - 2 m_3 l_2 l_3 \sin(\theta_3) \dot{\theta}_3 \dot{\theta}_2$,
$R_3 = 2 m_3 l_2 l_3 \sin(\theta_3) \dot{\theta}_1 \dot{\theta}_2$.

In (9), it can be observed that the first, second (i.e., centrifugal), third (i.e., Coriolis) and fourth (i.e., potential energy) terms consist of $\ddot{\theta}_i$, $\dot{\theta}_i^2$, $\dot{\theta}_i \dot{\theta}_j$ and θ_i, respectively, where $i = 1, 2, 3$ and $i \neq j$. The nominal values of robotic manipulator parameters considered in most research works are $m_1 = 0.2$ kg, $m_2 = 0.3$ kg, $m_3 = 0.4$ kg, $l_1 = 0.4$ m, $l_2 = 0.6$ m, $l_3 = 0.8$ m and $g = 9.81$ m/s^2. The payload mass is added to the mass m_3.

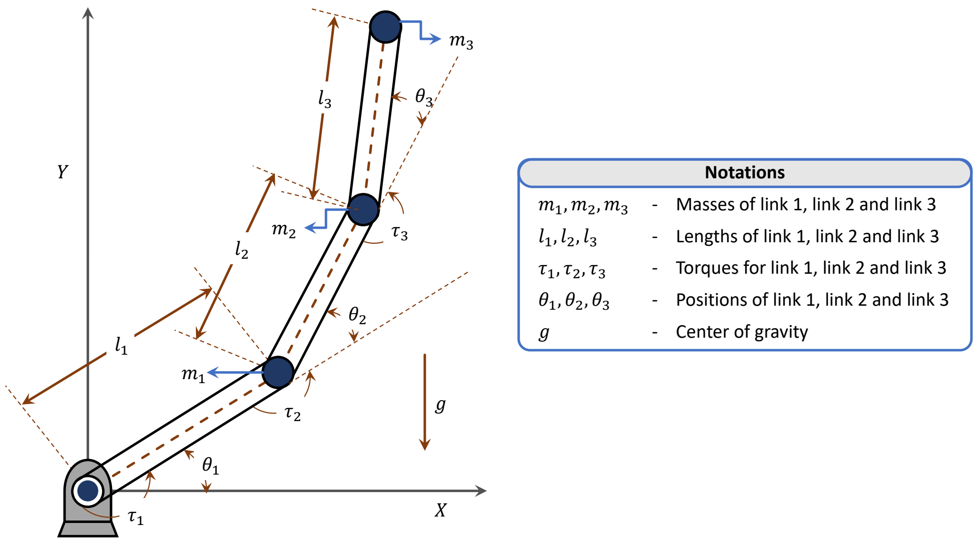


Figure 8. Three-link planar rigid robotic manipulator with a payload.

4.4. Generalized Model of Serial Link Planar Rigid Robotic Manipulator

The mathematical relationship between torques and positions of a robotic manipulator with N number of rigid and n number of elastic degrees of freedom using Euler–Lagrangian formulation is given as [104],

$$\begin{bmatrix} (M_{rr})_{N\times N} & (M_{rf})_{N\times n} \\ (M_{fr})_{n\times N} & (M_{ff})_{n\times n} \end{bmatrix}_{(N+n)\times(N+n)} \begin{bmatrix} (\ddot{q}_r)_{N\times 1} \\ (\ddot{q}_f)_{n\times 1} \end{bmatrix}_{(N+n)\times 1} + \begin{bmatrix} (H_r)_{N\times 1} \\ (H_f)_{n\times 1} \end{bmatrix}_{(N+n)\times 1} + \begin{bmatrix} (G_r)_{N\times 1} \\ (G_f)_{n\times 1} \end{bmatrix}_{(N+n)\times 1} = \begin{bmatrix} \tau_{N\times 1} \\ 0_{(n)\times 1} \end{bmatrix}_{(N+n)\times 1}, \quad (10)$$

where the matrices are defined as,

- M_{rr} and M_{ff} are the mass matrices related to rigid and flexible degrees of freedom, respectively,
- M_{rf} row matrix that defines the coupling between manipulators' rigid and flexible motions,
- M_{fr} row matrix that defines the coupling between manipulators' flexible and rigid motions,
- q_r and q_f are the manipulators' rigid and flexible degrees of freedom representing the motions of joints and elastic motions of flexible links, respectively,

- H_r and H_f are the centrifugal and Coriolis matrix related to rigid and flexible motion, respectively,
- G_r and G_f are the gravity matrix related to rigid and flexible motion, respectively,
- τ is the torque vector.

4.5. Other Robotic Manipulators

The modelling strategies of other robotic manipulators of various degrees of freedom are shown in Figure 9. The figure depicts that the most widely used Euler–Lagrangian formulation has been used to model lower and higher DOF manipulators such as inchworm/caterpillar [34,37], serial/joint manipulators, KUKA youBot [1], and Stewart platforms [89]. Similarly, the kinematic and inverse kinematic modelling approach has also been used for Delta robots [59], parallel manipulators, the Stewart platform [97], KUKA LWR IV [101], and Mitsubishi RV-4FL [2]. The next most widely used is a mathematical model developed for PUMA 560 [50], Quanser manipulators [83,88], Staubli RX-60 [54], Polaris-I [2], and UMD manipulators [21]. On the other hand, the fractional-order models have been developed for only Quanser [83], PUMA 560 [75], and Robotino-XT [94]. Thus, there is broad scope for exploring the concept of fractional-order modelling for various lower DOF manipulators such as inchworm/caterpillar and higher DOF manipulators such as Delta robot, KUKA youBot, Staubli RX-60, Robotino-XT, etc.

Figure 9. Modelling strategies used for various lower and higher DOF robotic manipulators [1–3,20,21,30,34,37,50,54,59,69,70,73,75,76,76,83,83,85,88,89,94,97,101].

5. Fractional-Order Control of Robotic Manipulators

This section presents a broad overview of fractional-order control strategies developed for various rigid, flexible, and joint robotic manipulators. These control strategies aim to achieve robust and stable performance despite uncertainties, external disturbances, and actual faults. As mentioned in Section 3, the developed fractional-order control strategies for various robotic manipulators are evolved versions of PID, sliding mode, backstepping, fuzzy, active disturbance rejection [82], and impedance control [91,97,98]. A more detailed review of these control strategies will be explained underneath.

5.1. Fractional-Order PID Controllers

The fractional-order PID controller with five parameters is an extension of the PID where the conventional integrator and differentiator are replaced with fractional ones. The serial rigid, flexible, and joint manipulators with DOF varying from 1 to 2 have been effectively controlled in simulation, and practice, using fractional-order PD/PID compared to PI/PD/PID and achieved better tracking accuracy and stability, practically [11,52,70,88,99,100,102,108]. However, the trial and error method has often been used to achieve the controller parameters. However, in the case of a two-link planar rigid robotic manipulator, the optimally tuned fractional-order PID and two-degree of freedom fractional-order PID controllers using the cuckoo search algorithm [4], particle swarm optimization [17,19], genetic algorithm [14,46] have performed better than the conventional and two-degree of freedom PID controllers [29,45]. A similar case has also been seen in a three-link planar rigid robotic manipulator, where fractional-order PID tuned using an evaporation rate-based water cycle algorithm has achieved better performance than the PID [56]. The best fractional-order PI/PD/PID performance is also true for higher DOF robotic manipulators, including Staubli RX-60 [54], UMD manipulator [21], PUMA 560 [75], Fanuc [20,24], Delta robot [59], KUKA LWR IV [101], and 3-RRR planar parallel robots [76]. Moreover, for these higher DOF robotic manipulators, the controller parameters are tuned using rule-based methods including Bode tuning [24] and decentralized tuning [20]. More details regarding the control actions of the fractional-order PID controller family, including two-degree of freedom configuration, can be found in [6,7,109,110].

5.2. Fractional-Order Fuzzy PID Controllers

It is widely known that PID is most often used in industry due to the advantages of simplicity and easy tuning and implementation [111]. As mentioned earlier, the performance of this controller is enhanced using fractional calculus. Moreover, the performance of this fractional-order PID is further enhanced using intelligent fuzzy techniques to achieve better servo and regulatory responses. Therefore, various combinations of fractional-order PID and fuzzy logic are proposed in the literature to form fractional-order fuzzy PID controller for two-link [4,39,43,44,51,62,63,67,79,92,95], three-link manipulators [48,65], SCARA [31,53], PUMA 560 [30], and Stewart platforms [89]. In addition, the authors of [64] have proposed a hybrid two-degree-of-freedom fractional-order fuzzy PID controller by combining two-degree-of-freedom PID, fractional-order concept, and fuzzy logic. These combinations have achieved better performance than the conventional and integer-order ones. Further, to incorporate the self-tuning of controller parameters rather than designing using precise mathematics, researchers have used several optimization techniques where the non-linear controller gains are updated in real-time using error and fractional rate of error. The optimization techniques used in the literature are artificial bee colony [39,43,53,95], genetic algorithm [30,39,43,64,79], cuckoo search [4,31,48,51,63], backtracking search [44,65], dragonfly [79], ant lion optimizer [79], particle swarm optimization [67,89] and grey wolf optimizer [95]. The robustness testing of these self-tuned fractional-order fuzzy PID controllers has shown superior tracking results in comparison to the conventional counterparts. However, in most of the works, the analytical stability analysis of these controllers has yet to be attempted. Thus, the research gap in the analytical proof of stability is noteworthy.

5.3. Fractional-Order Sliding Mode Controllers

Among the non-linear control methods such as an adaptive, fuzzy, neural network, sliding mode, H_∞, and model predictive controllers, the sliding mode control has been widely utilized due to its advantages of being computational simplicity, less sensitive to parameter uncertainties, highly robust to disturbances, and fast dynamic response [2,42]. However, the sliding mode controller has three significant problems: singularity, uncertainties, and chattering effect [78]. The singularity problem in the sliding mode control signal exists because of differentiating the exponential term in the controller equation. Thus, nonsingular sliding mode controllers have been developed to deal with this issue [69]. Moreover, various intelligent and optimization algorithms are hybridized with sliding mode controllers to compensate for the uncertainties issue, which also helps reduce the switching gains [58]. However, the problem of the chattering effect is still a drawback for the sliding mode controller. Therefore, researchers have recently developed fractional-order sliding mode controllers, which help reduce the chattering impact due to their memory and hereditary properties [81]. The two types of sliding mode controllers are given as linear sliding mode and terminal sliding mode controllers. The application of the fractional-order form of these two sliding mode controllers for various robotic manipulators will be explained underneath.

The linear fractional-order sliding mode controller has been developed for a single-link flexible manipulator for DOF varying from 1 to 2, achieving better performance than the conventional sliding mode controller and PID [22,25,42,66,68]. Even though the controller has no chattering effect, the singularity and uncertainties issues still exist. Thus, fuzzy and adaptive sliding mode controllers have been proposed for single-link, two-link, Mitsubishi RV-4FL, polar, and Inchworm/Caterpillar robotic manipulators. In [15,16,37,57,58], the authors have developed fuzzy and adaptive sliding mode controllers using bat optimization, genetic, and cuckoo search algorithms. The adaptive part of the controller will help reduce the uncertainties issue, and the fractional part of the controller will help reduce the chattering effect. On the other hand, the authors of [18] have proposed a fractional variable structure that helps minimize switching actions. However, the singularity problem still exists in these control techniques. Thus, the interest has been shifted towards using nonsingular sliding mode controller configurations.

Various configurations of terminal fractional-order sliding mode controllers have recently been developed for robotic manipulators to deal with singularity, uncertainties, and chattering effects. The authors of [26,55,69] have developed a fractional-order nonsingular terminal sliding mode controller for hydraulic and cable-driven manipulators, where the controller parameters are obtained using the trial and error method. This controller configuration has performed better than the integer-order nonsingular terminal sliding mode controller in both practical and simulation analysis. Even though the chattering and singularity issues have been solved, the controller still has uncertainty issues. Thus, in [1,28,34,61,73,74,78], an adaptive fractional-order nonsingular terminal sliding mode controller has been proposed for serial robotic manipulators, exoskeleton robot, KUKA youBot, and inchworm/caterpillar robotic manipulators. The controller has performed better than all its counterparts, including sliding mode controller, integer-order terminal sliding mode controller, fractional-order terminal sliding mode controller, and fractional-order nonsingular terminal sliding mode controller in solving the singularity issues, uncertainties, and chattering effect. However, this controller configuration is complex and needs more controller parameters to be tuned. Moreover, this controller configuration is further improved using time delay estimation, which forms the time delay estimation-based adaptive fractional-order nonsingular terminal sliding mode controller. In [3,36,81,86], the time delay estimation-based adaptive fractional-order nonsingular terminal sliding mode controller has been proposed for rigid hydraulic manipulators which have performed better than all of its counterparts and solved singularity, uncertainties, and chattering issues. At the same time, the controller configuration is very complex, and around 15 controller parameters need to be tuned. Thus, developing simple evolved versions of fractional-order sliding mode controllers to deal with singularity, uncertainties, and chattering effects are inevitable.

5.4. Fractional-Order Adaptive Backstepping Controller

The adaptive backstepping controller provides an improved tracking performance in the presence of uncertainties and faults, thanks to the controllers' adaptation law. In addition, the controller guarantees closed-loop system stability, which the conventional one failed to achieve. As finite-time convergence is crucial in robotic manipulators, thus, an adaptive backstepping controller is the perfect choice to achieve stable operation even in the presence of uncertainties and external disturbances. Further, to provide better steady-state and transient performances, the authors of [32,85] have proposed a fractional-order adaptive backstepping controller in the presence of actuators' faults and disturbances. The controller achieved adequate performance for PUMA 560 and a rotary manipulator under uncertainties, external load disturbances, and actuator faults. The controller also attained finite-time convergence and asymptotic stability. However, in both works, the controller parameters are chosen using the trial and error method. Thus, there is scope to develop a tuning approach for controller parameters of the fractional-order adaptive backstepping controller.

6. Conclusions

6.1. Findings

A comprehensive review of the application of the fractional-order concept in modelling and control techniques for various robotic manipulators has been discussed, as proposed by previous researchers. This comprehensive review summarizes the research outcomes published from 1998 until 2022 of around 100 works. Firstly, the study includes the conventional and fractional-order modelling strategies for robotic manipulators. Then, a review of developed fractional-order controllers for various robotic manipulators, which evolved from PID, sliding mode, fuzzy, backstepping, active disturbance rejection control, and impedance control, are presented. The graphical trend for existing research has been broadly presented in both cases. Thus, this review is expected to draw the attention of the investigators, experts, and researchers, allowing them to understand the most recent trends and work to advance in this field.

6.2. Future Perspectives

- There is broad scope for exploring the fractional-order modelling concept for various industrial robots, including Delta robot, KUKA youBot, Staubli RX-60, Robotino-XT, etc.
- The performance of fractional-order PID controllers can be further improved using the fractional-order form of predictive PI controllers for achieving robust servo and regulatory responses. Additionally, the performance of fractional-order PID controllers needs to be improved in the presence of uncertainties and faults.
- Even though fractional-order fuzzy PID controllers have achieved better servo and regulatory responses for proper industrial applications, the proof for analytical stability is a considerable research gap.
- The fractional-order nonsingular terminal sliding mode controller has achieved better response and surpassed the issues of singularity, uncertainties, and chattering effects. However, the controller configuration is very complex, and more parameters must be tuned. Thus, research on developing simple, evolved versions of controllers is inevitable.
- The adaptive backstepping controller provided an improved tracking performance in the presence of uncertainties and faults, thanks to the controllers' adaptation law. However, the controller parameters are chosen using the trial and error method. Thus, there is scope to develop a tuning approach for controller parameters.

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Abbreviations

The following abbreviations are used in this manuscript:

DOF	Degrees of freedom
FOMCON	Fractional-order modeling and control
IACCO	Integral of absolute change in controller output
IAE	Integral absolute error
ISCCO	Integral square of change in controller output
ISE	Integral square error
ISV	Integral of the square value
ITACO	Integral of time absolute change in controller output
ITAE	Integral time absolute error
ITSE	Integral time square error
LQR	Linear-quadratic regulator
MAD	Mean absolute deviation
MAE	Mean absolute error
MSE	Mean square error
MMFAE	Mean minimum fuel and absolute error
RMSE	Root mean squared error
STD	Standard deviation

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