



Article An Analysis of the Planar Vault under the Choir Loft of the Monastery of El Escorial

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Abstract: Arches and vaults are typical elements of ancient buildings. They are formed by voussoirs that resist the pressure they receive and transmit them through compression forces. The transmission of these forces justifies their curved shape. For this reason, arches and vaults are omnipresent elements in ancient constructions, all of them masonry structures. However, when visitors enter the Basilica of the Monastery of El Escorial, they find a narthex with a flat or planar vault. This vault is located under the floor of the choir loft. Its geometric characteristics and its shape, with no curvature, make it an architectural anomaly and a brilliant solution within masonry structures. Therefore, this article tries to analyse the construction process and structural behaviour of this vault, to understand its operation and how it remains standing five hundred years after its construction.

Keywords: Monastery of El Escorial; masonry structure; flat vault; planar vault; Juan de Herrera

1. Introduction

The Monastery of San Lorenzo de El Escorial was declared a UNESCO World Heritage Site on 2 November 1984 [1]. It was built during the second half of the 16th century by Philip II, King of Spain, to fulfil the promise of building a temple in gratitude for a war victory (the battle of San Quentin, won on 10 August 1557, the feast of San Lorenzo) and to build a mausoleum to bury his parents, Charles I of Spain and Isabel of Portugal [1,2].

The construction of the monastery began in 1563 [1] and ended in 1584 [2]. Juan Bautista de Toledo was the architect who began this work, but he died in 1567, before finishing the construction of the monastery, and the work was continued by the architect Juan de Herrera. His incorporation as architect coincided with the appearance of new needs for the monastery, forcing Herrera to adapt the initial project [2,3].

Philip II participated in the execution of the work by supervising the plans and frequently visiting to check its progress [2].

The final result was a building on a rectangular plan measuring 207 m by 161 m (an area of 33,327 square metres), located on the flank of the Sierra del Guadarrama, near Madrid. At each corner we discover a tower 55 m high, with a slate roof in the form of a pointed spire topped by a metal ball, a weathervane and a cross (Figure 1). In truth, the floor plan of the complex is not a rectangle but a grid, evoking the gridiron, which was the instrument of martyrdom of Saint Lawrence, to whom the monastery is dedicated. A figure of the saint also presides over the main entrance to the monastery (Figure 2).

The Basilica is located in the centre of the grid. The most important rooms of the monument are arranged around it (Figure 3). Visitors enter the Basilica through the Patio de Los Reyes. Figure 3 shows the floor plan of the Basilica, at ground level, with the position of the flat vault (element number 4). The open porch on the façade of the Basilica towards the Patio de Los Reyes is also indicated with the same number, although its position on the plan is lower. The side aisles (elements numbers 41 and 42) and the towers (element number 50, DD) also appear. The thick sections of the walls around the vault serve, among other things, to resist the horizontal pressure of the vault.



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Figure 1. East façade of Monastery of San Lorenzo de El Escorial (photo by author).



Figure 2. Main entrance to Monastery of El Escorial on its west façade (photo by author).



Figure 3. Floor plan of Basilica, at ground level, with position of flat vault (4) (graphic by author from a plan by Juan de Herrera [4]).

2. Materials and Methods

At the same time as when Juan de Herrera became the chief architect of the project, the number of Hieronymite monks who would live in the monastery doubled [2,3]. With this increase in the monastic community, the work took a new direction and many modifications had to be made. One of these modifications affected the choir loft, located over the entrance of the Basilica (Figure 3). Figure 4 shows the plan of the same area of the Basilica (Figure 3) at the level of the choir (first floor), with the position of the choir (room C) over the flat vault located between the two courtyards (room E). We can see that only the central area of

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the choir rests on the vault, while the rest is over the entrance to the nave from the narthex and the middle of the lower-floor atrium (room F) opening onto the Patio de Los Reyes.

Figure 4. Plan of the same area of Basilica (Figure 3) at the level of the choir (first floor), with the position of the choir (C) over the flat vault (graphic by author from a plan by Juan de Herrera [4]).

As there were more friars, a larger choir than projected was necessary. Furthermore, all of the monks had to be able to see the high altar from the choir during religious ceremonies, forcing the architect to rethink the dimensions of the temple [5].

Thus, it was necessary to raise the level of the high altar and lower the level of the choir loft. On the one hand, increasing the height of the high altar was relatively simple; on the other hand, lowering the choir was very complicated. The choir is just above the entrance to the Basilica. Figure 5 shows a photograph taken from the interior crossing of the Basilica of El Escorial, where we can see the choir loft and the narthex under the choir where the planar vault is located. Figure 6 shows a longitudinal section of the Monastery, enlarged to show the detail of the narthex (space between the letters L and M) and the planar vault under the choir.



Figure 5. Photograph taken from the interior crossing of the Basilica of El Escorial (photograph by author).



Figure 6. Longitudinal section of Monastery of El Escorial (graphic by author from a plan by Juan de Herrera [4]).

Lowering the level of the choir meant reducing the free height of the lower space, the narthex, so the thickness of the choir floor had to be reduced. In fact, as we can see in preliminary designs, the initially planned vault was of greater height and thickness. However, it had to be modified to minimise its thickness as much as possible. Considering these circumstances, Juan de Herrera adopted a solution in accordance with the design of the rest of the Basilica [6]. Herrera opted for the construction of a flat vault with a circular plan (Figures 7 and 8), achieving the maximum possible height in the narthex thanks to a thickness of the vault of just 1 ft (24 cm). The arch on which the vault is modelled is so low that, in practical terms, it is considered a flat or jack arch. Fray José de Sigüenza, the personal advisor to Philip II and librarian of the Monastery of El Escorial, spent the last

years of his life in the monastery. Having witnessed the construction of the monastery from the ground up, he recognised the architectural beauty of this unique construction element. Sigüenza wrote that, despite being made of stone and spanning such a large space between the widely set columns, the vault is as flat as the ground. The support of this vault was admirable, according to Sigüenza, being as firm and secure as an embankment. Despite his limited architectural and engineering knowledge, Sigüenza ventured to explain the phenomenon that allowed this vault to be built and remain standing: by properly cutting the stones, they lock together, forming arches along their courses, until they are closed by the keystone [2].



Figure 7. Narthex of the Basilica of El Escorial, under choir loft: floor plan (**left**) and section (**right**), with the flat vault analysed in this article marked in green [7].



Figure 8. Plan and sections of narthex of the Basilica of El Escorial [8].

His admiration was justified. The final result was a circular planar vault that remains in perfect balance supported on lowered arches (Figure 8). Its constructive and structural importance and originality deserve a much deeper analysis. On the one hand, Figure 9 shows the planar vault in a photograph taken from the ground. In it we can see the voussoirs that make up the keystone, the voussoirs arranged in a ring. We can also see that the planar vault rests on four lowered arches of greater thickness, and smooth pendentives that spring from the pillars to the outer edge of the seventh ring of voussoirs. But on the other hand, Figure 10 shows a partial view of the hall under the choir, surmounted by the planar vault, with the first pillars on which the flat vault rests and the small depressed vaults that, among other things, transmit their horizontal pressures to the thicker walls that enclose the complex from the narthex.



Figure 9. Photograph of planar vault taken from the ground (photograph by author).



Figure 10. Partial view of the hall under the choir (photograph by author).

3. Results

We are analysing a vault that covers a span slightly greater than six-and-a-half metres. It was built in a busy place and was intended to support a significant weight.

Current technology has made it possible to verify old measurements and planimetry. Through laser and CRP measurements, we have been able to verify the veracity of the plans drawn up 20 years ago and, for this reason and after this verification, we have used them as a basis in this investigation (Figures 7, 8 and 11).



Figure 11. Plan with exploded views of voussoirs and section of planar vault (graph by author).

The section shows that the central part of the vault has dropped a few centimeters with respect to the level of the outermost complete circular course (Figures 7, 8 and 10).

Viewing the vault from below (Figure 9), we see that the vault under the choir is formed by seven rings composed of granite voussoirs with a flat cross section. The centre of the voussoirs, and of the vault, is a circle made of two identical semicircular ashlars separated by a diametral joint. Figure 12 shows a detail of the centre of the vault, with its keystone formed by two symmetrical voussoirs. We can observe the detachment of the mortar between the joints of the ashlars around the central voussoirs that make up the keystone.



Figure 12. Detail of vault centre, with keystone formed by two symmetrical voussoirs (photograph by author).

This circle is surrounded by a first ring constructed with seven ashlars, this ring by another, and so on to the edge. Each ring is 42 cm wide, and the diameter of the central circle is approximately twice the width of the ashlars (80 cm).

To begin the mechanical analysis of this vault, we are going to resort to the simplest structural option: considering the vault as a monolithic element. According to Professor Félix Escrig Pallarés [9], we can compare the vault to a single piece or slab. In such a case, the flat part of the circular vault made of granite (density of the granite used 2.7 g/cm³), with a diameter of 6.72 m and thickness of 24 cm, would weigh almost 23 tons (225.38 kN to be exact). As this fictitious slab would have to rest on the edges, its diameter would increase to around 9.00 m, hence its weight would increase to over 40 tons (404.27 kN). Despite its size and weight, its placement would have been possible: larger and heavier construction elements had already been placed in other monuments, even at a greater height, before the Monastery of El Escorial was built [10].

If the vault is a monolith, this means that there are no cracks in the element. This would be ideal as it would mean that the vault is equiresistant, admitting compression stresses and tensile stresses, despite being a masonry structural element. This flat vault is a bidirectional flat element. Therefore, for calculation purposes, it is equivalent to a circular slab that rests on the outer circumference. This circular slab would support its own weight, the overload of the flooring and the overload of use. Therefore, for a diameter of 6.72 m and a weight of 10.00 kN/m^2 (resulting from screed and overloads), the tensile stresses (which, by assuming the equiresistance of the material, we can consider equal to the tensile stresses until failure) would reach nearly 2000 kN/m². This value, although it may be accepted because it is a good granite, is risky, and could cause the vault to collapse due to a minor concentration of stress. It is interesting to note that if we doubled the thickness of the vault, the weight of the vault would also double, but the resistance would quadruple. Therefore, the tensions would be reduced by half and, if Juan de Herrera had built that monolithic slab, the slab would have split. Therefore, before risking the vault breaking, it was better to build it broken (in pieces) which would facilitate the transportation of the ashlars and their placement.

If we chose lintel cuts in a single direction, we would be looking at a square vault 6.72 m on each side, built like a lintel. This lintel is made with voussoirs 24 cm thick and 6.72 m wide. Being a masonry structural element, the lintel will work only in compression by pushing on the edges. Let us remember that, as it is a masonry structural element, Professor Jacques Heyman's postulates are fulfilled. According to these [11], we assume that masonry has no tensile strength, but high compressive and friction strength. Therefore, this lintel will not be able to withstand tensile forces due to the characteristics of the granite and because the joints between ashlars prevent it.

If we look closely at the vault of the Monastery of El Escorial, we see that the mortar from the joints between the ashlars in the central area has been falling away (Figure 12). Figure 13 shows a detail of some of the voussoirs that make up the circular rings of the planar vault, where we can also see the detachment of the mortar between the joints. This indicates that no stress can pass through that lower part of the central area of the vault.



Figure 13. Detail of voussoirs that make up circular rings of planar vault (photograph by author).

We know that the construction of this vault caused many problems for the construction manager with the stonemasons [12]. Geometrically, the stone cut for this planar vault implies that each course of ashlars is different from the others, although symmetrical. Under these conditions, the calculation reference is a jack arch that, if we assume it without appreciable deflection so as not to lose mechanical edge and if we want to avoid cracks on the underside of the joints, will produce compression stresses of the order of 5.9 N/mm^2 (5900 kN/m²). This value is perfectly bearable for compressed granite but unaffordable for mortar joints, especially if we consider that these must set for a long time. This forces us to take the joints into account as they are also decisive, like the cutting of the ashlars.

Building the vault with full-width voussoirs was very difficult. Furthermore, any defect in the material, any highly concentrated load or differential movement could break a segment. The division of the central voussoir is not mechanically significant. For this reason, it is convenient to cut it into pieces.

However, the slow setting of the mortar causes a variable reduction in the thickness of the joint that deforms, turning into a wedge due to eccentric compression.

Given the limited maneuverability that Juan de Herrera had in these spaces, there were few options to reduce these efforts that affect the joints. The best alternative was to make the material work in two orthogonal directions. To do this, all that is needed, starting from the previous jack vault, to cut each course with escaped joints also in the transverse direction. Thus, the mechanical problem is divided in half, since the same load will be transferred to the edges through two paths of equal rigidity, and not through just one. However, the geometric problem is complicated because each ashlar is different from the others (they are only equal four by four, as can be seen in Figure 9) and because the angles of the ashlars become excessively acute as they move away from the centre. This means frequent breakages and apparently creates new problems.

For it to fall, the lintel would have to have a strong edge that would allow for possible pressure lines with angles less orthogonal to the planes of the joints. Therefore, even though the work is inverted, friction prevents it from falling thanks to its slenderness.

In a more complex way, we could calculate the vault as a set of segments or jack arches, each formed by two triangles joined in the keystone by a line. In this case, each weight is moved to the edge from the key. This segment analysis would be accurate if the vault were cracked radially. This occurs, for example, in the lower area of many hemispherical domes (approximately from the 45th parallel to the equator). We can consider the plane vault of the Monastery of El Escorial to be a polar cap extracted from a hemispherical dome [9,11]. In other words, to explain the absence of curvature, we can say that this plane vault is the pole of a dome of infinite diameter. This allows us to resort to a structural typology based on membrane theory [13]. In this typology, the forces are compressive in any direction, which is why cracks or radial cracks do not appear. Furthermore, the value of these efforts would be practically identical in all cases, producing stresses of 2000 kN/m², the same as the stresses that we indicated at the beginning, when we studied the monolithic option. Now, we achieve that value using small pieces.

The value of these stresses can be excessive for mortars over time. This is observed in the shortening of the thickness of the joints, in the subsequent slight descent of the plane and in the increase in the depth of the discharging arch (although perhaps it would be more correct to speak of a compression surface). All this has led it to occupy a greater space in the thickness of the vault to lower the tensions to an equilibrium level. This increase in the mechanical edge supposes the departure of the central core of inertia and the appearance of traction forces on the lower face. We can see the appearance of these forces in the openings of the joints that we already mentioned above, and that we can observe from below (Figures 12 and 13). The appearance of these small cracks forces us to investigate the vibration of the vault when, for example, we jump on it. If a person climbed up to the choir and jumped, the people on top of the planar vault would vibrate, and they would vibrate at the same time as the vault. This is because the load generated is lower than the critical load and because the material is in the elastic phase.

The central circle, formed by two ashlars, was possibly planned this way to function as a key. In other words, the two ashlars, wedged together, would transmit force to the whole, putting it under load. Unfortunately, it was impossible to achieve pressure through wedges applied only in the centre that would adjust the entire vault with the means available in the 16th century. To achieve this, all the ashlars must be slightly moved, and therefore their friction against the lower formwork must be overcome. To do this, forces of approximately 400 kN were needed as we mentioned at the beginning (we would accept a friction angle of 45°). These forces can only be achieved with mechanical jacks, which were not available in the 16th century [12]. This impossibility likely produced the sagitta that we see in the vault today during the removal of the formwork. If so, this sagitta would be necessary to achieve this loading because when all the ashlars are slightly rotated, their projected length in the horizontal plane is greater than the original one, thus achieving sufficient compression to achieve balance.

As mentioned above, the construction of this vault caused the construction manager many problems with the stonemasons. The main cause of these problems was the absence of a traditional formwork, as would be required by a normal vault [12].

Herrera was surely aware that each of the rings, once completed, makes the resulting partial vault stable, from the edge, due to the general state of compression of the structure. Thus, it is enough to wedge each ring into the radial joints, to close it, and against the previous ring, to put the assembly under load step by step (Figure 12). Figure 14 shows a virtual simulation of the construction of the planar vault voussoirs. By building it from the outside in, each time one of the seven rings was completed and its ashlars came under load, the whole was balanced, remaining stable without the need for the interior rings to be made.



Figure 14. Virtual simulation of construction of planar vault voussoirs (graph courtesy by Telemadrid [14]).

In the vault under the choir loft, the considerable horizontal pressures recorded are counteracted by sufficient sizing of the walls on which it rests (Figure 15). For this reason, flat vaults are always found boxed in areas with large section walls or walls that resist higher weights and are capable of centering the load lines transmitted by them while maintaining balance. The small depressed vaults, among other things, transmit their horizontal pressures from the narthex to the thicker walls that enclose the complex (Figure 10). All of this occurs with tensions that are perfectly acceptable due to the material used.



Figure 15. Schematic representation of the pressure transmission from the planar vault to the lateral curved vaults and the lateral walls (graph by author).

Enrique Rabasa, in the Stonework Workshop of the School of Architecture of the Universidad Politécnica de Madrid (Polytechnic University of Madrid, Madrid, Spain), led a simulation of a similar vault [15]. To do this, students carved the voussoirs of a similar smaller planar vault. Its design was as simple as possible. To do this, the voussoirs formed simply conical strata. It is important to note that the voussoirs were imperfect, and the fit of the voussoirs were equally imperfect. The stone that made up the voussoirs was softer than granite. The total weight of this replica was approximately 6.00 kN. Furthermore, mortar was not placed in the joints between segments, however, this simulation tried to verify whether this imperfect vault would remain in these disadvantageous conditions.

Once the vault was built, it was raised to analyse its behaviour suspended in the air. To raise the vault, they used perimeter metal plates, anchored to the perimeter segments (the last row). These plates made possible to lift the entire vault using a hoist (Figure 16). With the help of chains, the hoist raised the plates and the replica of the vault was raised [16].



Figure 16. Graphics of reduced flat vault model tested at Stonework Workshop of the School of Architecture (Polytechnic University of Madrid), with lifting mechanism used (graph courtesy by Enrique Rabasa [15]).

Once raised, the vault would be supported by pushing to the side, as we have seen here, with the plate receiving this lateral push (Figure 15) [16].

When the elevation began, some voussoirs descended slightly. The pieces that went down were the ones that had the most space between each other. To solve this, the perimeter joint of these voussoirs was filled with mortar. Afterwards, the vault was raised again, and it was possible to verify that the vault remained standing, registering only a small seat in the central area, as occurred with the original vault.

4. Discussion

Before the construction of the Monastery of El Escorial, there is hardly any evidence of the construction of a structural element of these characteristics. Some authors have found two planar vaults without ribs, prior to the construction of the monastery: an Etruscan vault in Rome and another in Syria [7], but neither of them had the precision of this solution designed by Juan de Herrera.

Several authors have pointed out that the vault analysed here was a success preceded by two experiments within the monastery [5,7,8,10]—one of them failed and another was smaller and imperfect.

Today we can see the failed vault in the basement of the Monastery of El Escorial. This vault is rougher and larger $(8.36 \times 8.60 \text{ m} [7,8])$ than the one analysed in this article. However, this planar vault does not stand on its own. The vault is supported by four arches resting on the perimeter walls of the room. These arches in turn rest on a central pillar. The deflection of the vault above the head of the pillar is evident. This deflection justifies the need to install a support in the middle of the room, as must have been conducted after its construction, seeing that it was not supported [10]. It is possible that this vault had these deformation problems because it was carved with less care and had a greater number of rows of ashlars, and therefore, more joints between ashlars. Thus, once the formwork was removed, the levelling mortar used was unable to withstand the compression stress required, and the vault deformed, sinking in its centre until it found equilibrium. It is true that we have no evidence that it collapsed. In any case, it is clear that this vault was wedged and lowered for security reasons, meaning that we can call it a failed experiment.

We find the other planar vault in the hallway of the Monastery convent. Its horizontal surface is smaller and has a greater inclination than the narthex vault (it is a vault with less flatness). Therefore, this is not a flat vault like the one built under the choir loft. This vault was built after the basement vault [2,7], and is still standing.

Both vaults were built more than a decade before the vault built under the choir [7]. This indicates that Juan de Herrera learned from the mistakes of his predecessor as architect of the Monastery [5]. For this reason, the vault under the choir loft has a square plan, with dimensions slightly smaller than those of the basement vault. Thus, analysing the section of the vault designed by Juan de Herrera (Figure 6), we see that in the slab there is a central horizontal zone and a slight rear cant at each end. This horizontal zone includes the keystone and the following five courses of voussoirs. By measuring under the vault, we verified that this descent exists (9.60 cm) and corresponds to the descent of the vault indicated by Juan de Herrera in its longitudinal section. This smooth descending transition, with its corresponding increase in thickness (Figure 6) and the use of 42 cm circular courses, was essential for this planar vault to remain standing.

Therefore, the chronology of the construction of the Monastery of El Escorial [2,3] and the construction of the three planar vaults built in it tell us about a first failed attempt, a second approach and the definitive solution under the choir space. This definitive solution was achieved thanks to the lowering of the vault at its ends, in the diagonal section, and the provision of a smaller number of courses of voussoirs, using voussoirs of less thickness.

Looking for construction elements similar to the planar vault of Juan de Herrera, we find the crypt of the Cathedral of Cádiz in southern Spain [10]. This vault is flat, larger and lacks central ashlars. Figure 17 shows the planar vault in the crypt of the Cathedral of Cadiz. This vault was built later, inspired by Juan de Herrera's vault, and is larger than

the Escorial vault and is located under the geometric centre of the Cathedral building. In fact, this is not a flat vault, but a hollow vault. This vault was built 150 years later, inspired by Herrera's planar vault. The work was executed by the architect Vicente Acero y Arebo, director of works at the Cathedral [17].



Figure 17. Planar vault in crypt of Cathedral of Cadiz (photograph courtesy by ArtiSplendore [18]).

In this case, we could affirm that the solution was perfected, not only because of its dimensions (it is three times larger than the vault of El Escorial) but because of its location in the Cathedral. This planar vault is located in the centre of the floor of the Cathedral [17]. Therefore, this vault supports the weight of the entire presbytery and its heavy building. We must point out that this crypt is located below sea level.

The absence of a central voussoir shows that Juan de Herrera was not wrong when cutting up the central voussoir of his flat vault.

Almost at the same time as Cádiz Cathedral was being built, the Cathedral of Lugo was being built 900 km away in northwest Spain. Two twin planar vaults were built there, to resolve the first floor into separate towers on the main façade [19]. On this occasion, the design was by the architect Julián Sánchez Bort. However, these planar vaults imitated a design devised by Joseph Abeille. We have no evidence that this French engineer executed his design in practice [20].

Unlike the planar vault of El Escorial, we cannot say that the Abeille vault is composed of voussoirs. We cannot break this vault into converging strata towards the same point or centre, forming a wedge, in order to guarantee its support. Abeille's vault is different, as it is a polyhedral-shaped piece with a square base (the intrados), limited in its upper part by a rectangle (the extrados). On its sides, the vault is limited by four inclined planes that link the intrados with the extrados. Thus, lintels, formed by ashlars with a trapezoidal cross section, are crossed until the entire surface is covered. Each ashlar is carried by two neighbouring ones, and at the same provides support for two other ashlars in the other direction.

Therefore, the two vaults of the Lugo Cathedral are bidirectional flat constructions. In them the forces are around 2950 kN/m^2 . For this, it is considered that both directions work equally and with a reasonable construction procedure. This procedure was solved with equal pieces of simple flat cuts, without delicate acute angles. This makes it different from the planar vault of the Monasterio de El Escorial. Its circular shape is a variant of Abeille's bidirectional shape. In Herrera's vault, one of the orthogonal directions is radial [10], which

forces the other direction to describe circles. If the stiffness is homogeneous, the bidirectional analysis is applicable. This depends less on the family of cuts chosen (orthogonal or radial) than on the uniformity of size of the ashlars. In this aspect, both vaults are the same.

5. Conclusions

A thin planar vault has been analysed. This structural element was pioneering, since hardly any similar elements had been built before and some previous experiments failed.

In this structural element, the pressure line that allows balance to be maintained is practically tangential to the extrados of the central section of the vault. In the upper area of the central area of the vault, forces with a large horizontal component are concentrated.

Likewise, in the rings furthest from the centre, the pressure line approaches its intrados with eminently horizontal component stresses. A jack arch works in an analogous way. This is manifested in part by discovering that the filling mortar is detached from the lower joints in the centre of the vault, since no stress can pass through that area of the plane vault.

To understand and justify the balance of this masonry structure, we must consider the maximum thrust and the minimum thrust at the beginning of the vault.

Since it is a vault, it has one more dimension than if we were studying an arch: it would be a flat arch extended through 360°. Juan de Herrera used the rings as a wedge for each of the voussoirs that make up this hypothetical jack arch. Thus, each ring is capable of maintaining the tension necessary for the vault to remain balanced.

To do this, the cutting angles of the stone had to be calculated so that, if part of the inner rings were removed, the voussoirs of the first circular ring that remained towards the inner oculus would not move upward, by maintaining the radial pressures that balanced the rest of the vault.

These radial pressures were absorbed by the first ring inside. These radial pressures were transformed, within the first ring, into pressure lines that, following the section of the ring (perpendicular to the radial resistance pressure), would be resolved like the pressure lines of any other semicircular arch. This would make the set stable. The only difference would be that this arch would be in a horizontal position, not vertical, and extended through 360°, not 180°.

To put it another way, the vault could have been built without using falsework—each of the rings that is completed keeps the entire set balanced. Given this condition, it would be enough to have held the voussoirs that make up each of the rings while the vault was built from the outside in (Figure 13). Once the ring is closed, and despite the centre not being completed, the planar vault fraction executed would already be stable.

Therefore, the balance of the vault is based on the exposed system. There is no need to look for especially complex cuts or strange shapes. For the scheme to be effective, it is logical to assume that the size of each of the voussoirs must be very well executed, with perfectly smooth meeting faces that prevent, on the one hand, clearances that would imply a lowering of the structure or irregularities that give rise to points of concentration of stress that can end up fracturing the stone.

Finally, the considerable horizontal pressures generated by the vault are counteracted thanks to the sufficient dimensioning of the walls on which it rests. For this reason, plane vaults are always boxed in areas with large section walls or walls that resist greater weights and that are capable of centering the load lines transmitted by the vaults, maintaining balance. Naturally, all this occurs with tensions that are perfectly acceptable due to the material used.

The vault is made up of circular courses of voussoirs, like a hemispherical (or halforange) vault, although it is supported by triangles like pendentives. Therefore, we could say that it is a low vault. In fact, its intrados, in the area in which the courses are complete, rather than flat, is convex. Its profile shows that the central part has dropped a few centimetres with respect to the level of the outermost complete circular course. This was not intended that way. It is due to a seat in the vault, since the same deformation exists in the marble pavement that covers the back wall. There has been speculation about the internal connection of the pieces. Between the circular courses there may be some type of step that prevents sliding, but it is not necessary. The beds that separate the courses can simply be truncated conical, as in the case of hemispherical vaults.

In cross section, this vault would resemble a lintel arch. The layers of this type of arches are convergent and the pieces are wedge-shaped, although in some cases, to prevent slippage due to the movement of the supports, the voussoirs are carved with beds whose recesses and protrusions fit together. Similarly, in the El Escorial vault it would be enough to have conical beds, but it is difficult to know if any other precautions were taken. As we have seen, in other rooms of the Monastery there were previous tests that failed.

The vault analysed is more than 500 years old. Its age demonstrates that Herrera's solution was mechanically optimal. Herrera achieved a reduced stress level to be accepted by the mortars and an optimal structural typology constructively. The number of joints is small, its thickness is thin and the good workmanship fits the ashlars excellently. Therefore, Juan de Herrera demonstrated great knowledge and marvelous assurance by uniting, in this small work, novelty with perfection.

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