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Calibration and Evaluation of the SIMPLE Crop Growth Model Applied to the Common Bean under Irrigation

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Abstract: Bean production is at risk due to climate change, declining water resources, and inadequate crop management. To address these challenges, dynamic models that predict crop growth and development can be used as fundamental tools to generate basic and applied knowledge such as production management and decision support. This study aimed to calibrate and evaluate the SIMPLE model under irrigation conditions for a semi-arid region in north-central Mexico and to simulate thermal time, biomass (Bio), and grain yield (GY) of common beans cv. 'Pinto Saltillo' using experimental data from four crop evapotranspiration treatments (ETct) (I_{50}, I_{75}, I_{100}, and I_{125}) applied during the 2020 and 2021 growing seasons. Both experiments were conducted in a randomized complete block design with three replicates. Model calibration was carried out by posing and solving an optimization problem with the differential-evolution algorithm with 2020 experimental data, while the evaluation was performed with 2021 experimental data. For Bio, calibration values had a root-mean-square error and Nash and Sutcliffe's efficiency of <0.58 t ha^{-1} and >0.93, respectively, while the corresponding evaluation values were <1.80 t ha^{-1} and >0.89, respectively. The I_{50} and I_{100} ETct had better fit for calibration, while I_{50} and I_{75} had better fit in the evaluation. On average, the model fitted for the predicted GY values had estimation errors of 37% and 22% for the calibration and evaluation procedures, respectively. Therefore, an empirical model was proposed to estimate the harvest index (HI), which produced, on average, a relative error of 6.9% for the bean-GY estimation. The SIMPLE model was able to predict bean biomass under irrigated conditions for these semi-arid regions of Mexico. Also, the use of both crop Bio and transpiration simulated by the SIMPLE model to calculate the HI significantly improved GY prediction under ETct. However, the harvest index needs to be validated under other irrigation levels and field experiments in different locations to strengthen the proposed model and design different GY scenarios under water restrictions for irrigation due to climate change.

Keywords: *Phaseolus vulgaris* L.; differential-evolution algorithm; dynamic model; evapotranspiration; grain yield; harvest index; drought stress

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1. Introduction

Due to its high protein content, the common bean (*Phaseolus vulgaris* L.) is the most widely produced and consumed legume in the world. Mexico ranks ninth in world production [1]. Annually, its per capita consumption is 8 kg [2]. Zacatecas state, located in north-central Mexico is the main producer of this legume grown in semi-arid and arid agricultural lands [3]. This climate presents droughts, irregular episodes of rainfall, and high evapotranspiration demand during the crop growing season, where the water supply, via irrigation, for common bean cultivation is mandatory. [4]. In Zacatecas, more than 64,000 ha are annually bean-cultivated under irrigation yielding between 1.7 and 2.4 t ha^{-1} [5], some of the lowest yields worldwide. In addition to adverse climatic conditions, low yields are also accompanied by degraded soils, inadequate crop water management, and aquifer overexploitation. However, yields can be improved by understanding crop management within the soil-plant-atmosphere continuum, where crop growth and development models become relevant. Mathematical models are effective tools for describing and understanding complex systems, but they are also used for crop management, generation of government policies, evaluation, and adaptation to climate change impacts [6]. Dynamic mathematical models are composed of a set of first-order ordinary differential equations or difference equations [7]. These equations have a set of physiological coefficients (e.g., radiation use efficiency), which are difficult to measure directly [8] and estimate [9].

On the other hand, the models used to simulate bean growth and development of the DSSAT family [10] highlight the BEANGRO model [11], GenoGro [12], CROPGRO [13], and the Wageningen family models such as the models WOFOST [14], SODCOM [15], SUCROS [16], and DIACROS [17]. There are other models, such as the Cereal–Legume model, which simulates the growth of beans intercropped with corn, the SSM–Legumes model [18], and the Daisy model [19]. Nevertheless, Yuan et al. [20] and Zhao et al. [21] have developed bean models that require only a few parameters, and other models for irrigation management in beans (SWB model [22]; SALTMED [23]; AquaCrop [24].

Also, most of the models used to simulate crop growth and development require a large number of parameters which are difficult to obtain as a result of the wide diversity of agroecological environments, crops, and varieties. It is here that the importance of models using few parameters is highlighted [25]. The Simple Simulation Model (SSM) developed by Soltani and Sinclair [18] involves five state variables and twenty-seven parameters. The HORTSYST model includes seven state variables and twenty-four parameters [26]. Recently, Zhao et al. [21] proposed the SIMPLE model with two state variables and fourteen parameters for field crops. The SIMPLE model was developed as a generic model based on intercepted radiation and simulates thermal time (TT °C d) and biomass (Bio t ha^{-1}), but also calculates grain yield (GY t ha^{-1}) [21].

The SIMPLE model was adopted for soybean biomass and yield estimations under climate change scenarios of air temperature and atmospheric CO_2 [27]. This model has been applied to simulate maize biomass and yield for Vietnam's autumn-winter and winter-spring growing seasons [28] and to predict flax biomass and yield under four arid and semi-arid scenarios in China. So, the SIMPLE model can be extended to many crop species by adding variable modules such as nutrient dynamics, water stress, temperature stress, or pests [29]. Therefore, the low GY, the economic importance of bean crops, and the lack of growth and development models applied to this species, motivated this research. This study aimed to calibrate and evaluate the SIMPLE model under irrigation conditions for a semi-arid region in north-central Mexico and to simulate the TT, Bio, and GY of common beans cv. 'Pinto Saltillo' using experimental data from the 2020 and 2021 growing seasons. So, in the face of the global warming impact on the annual crops, we hypothesized that the SIMPLE model would be useful for simulating bean biomass and yield for the semi-arid agroecological scenarios of north-central Mexico. These kinds of models are particularly important for these growing areas because they only use a few parameters for crop modeling.

2. Materials and Methods

2.1. Description of the Study Area

The experiments were conducted from April to August for the 2020 and 2021 growing seasons at the Zacatecas Experimental Station of the National Institute of Forestry, Agriculture and Livestock Research (INIFAP) located in Calera de Víctor Rosales, Zacatecas, Mexico (22°54′ N; 102°39′ W, elevation 2197 m). The experimental site has a mean annual temperature of 14.6 °C and a mean annual rainfall of 416 mm, 75% of which occurs between July and October [30]. The mean annual pan evaporation is 1609 mm. Before setting up the study, the experimental site was bleached with oatmeal during the winter of 2019–2020 (northern hemisphere). Afterwards, a physicochemical soil analysis was carried out in the soil-water laboratory of INIFAP. The experimental site's soil is clay loam with a saturation point, field capacity (FC), permanent wilting point (PWP), and average bulk density of 0.48 m^3 m^{-3}, 0.29 m^3 m^{-3}, 0.14 m^3 m^{-3}, and 1.1 g cm^{-3}, respectively. The grain size distribution is 35%, 28%, and 37% for sand, silt, and clay, respectively. The content of N is 3.5 mg kg^{-1}, P is 7.8 mg kg^{-1} and K is 31 mg kg^{-1} in the first 40 cm of soil depth. The organic matter content of the soil is low (1.3%) and has a pH of 8.2.

2.2. Genetic Material and Crop Management

'Pinto Saltillo' is a common bean variety with indeterminate growth. Seeds were sown on April 16 and April 21 in 2020 and 2021, respectively. The distance between furrows was 0.76 m, with 0.1 m between plants. The planting density was 131, 578 plants ha^{-1}. The planting depth was between 0.06 and 0.07 m. Mineral fertilization consisted of N and P only. The sources of N and P were urea (40 units of N) and mono ammonium phosphate (60 units of P), respectively, which were fractionated during the crop cycle in 20–20, 10–20, and 10–20 units of N and P, respectively. Fertilization doses were applied at 26, 36, and 47 days after sowing (DAS) in 2020 and at 17, 29, and 44 DAS in 2021. Weed, disease, and pest controls were performed as required.

2.3. Irrigation Treatments and Experimental Design

The experiments were conducted in two consecutive growing seasons. They consisted of four irrigation treatments according to the atmospheric demand: 50, 75, 100 (as control), and 125% of crop evapotranspiration (ETc).

Previous to sowing, a gravimetric soil sampling was carried out to determine the residual soil moisture content (θ_s), to apply the initial irrigation depth (IID) at FC in all irrigation treatments (Equation (1)) [31]. Subsequently, the irrigation schedule consisted of water supply twice a week based on the daily data of the reference evapotranspiration (ETo) obtained from an automated weather station placed 1.5 km from the experiment, and 75% of the effective rainfall (Er > 5 mm) was subtracted from the accumulated ETc (Equation (2)) [32].

$$IID = (FC - \theta_s)/100 \cdot Da \cdot Pd \quad (1)$$

where IID, FC, and θ_s were already defined, Da is the bulk density (g cm^{-3}), and Pd is the profile depth (cm).

From here, irrigation treatments were applied based on a climatic water balance proposed by Servin-Palestina et al. [33], using Equation (2):

$$Lr_i = \sum_{i=1}^{n} ETc_{\ i-1} - \sum_{i=1}^{n} Er_{i-1} \quad (2)$$

where Lr_i is the irrigation depth (mm), ETc is the daily crop evapotranspiration in (mm), Er was already defined, and n is the number of days between irrigation events (twice a week).

To estimate ETc (Equation (3)), the local crop coefficient (Kc) for beans was estimated with Equation (4) [33]:

$$ETc = Kc \cdot ETo \quad (3)$$

$$Kc = -3.4829x^3 + 4.5973x^2 - 0.8725x + 0.3786 \quad (4)$$

where ETo was already defined and estimated by the Penman–Monteith method [34], and $x, \in |0, 1|$ is the phenological-stage fraction of the crop calculated by TT, where zero value means planting and one is the physiological maturity of the crop.

The experiments were conducted in a randomized complete block design based on plot soil slope, where the irrigation treatments were randomized within every three blocks.

The experimental unit comprised 12 furrows, each 11 m in length. To avoid horizontal movement of irrigation water among plots, there was a 1.52 and 0.8 m separation among blocks and treatments, respectively.

Irrigation water application per treatment was by self-compensated 6 mil-gauge irrigation tape with emitters spaced at 20 cm and with 0.94 L h^{-1} flow per emitter. Pre-sowing and establishment of irrigation were the same for all treatments. A volumetric meter was used to determine the amount of water (irrigation depth) applied to each treatment. The cumulative biomass curve was obtained with destructive sampling by collecting plants contained in a 1 m furrow section. From planting to harvest, seven plant samplings were carried out in both experiments. The sampled plants were divided into leaves, stems, and pods. Plant organ samples were oven-dried at 65 °C for 72 h to constant mass. Then, Bio was estimated with the sum of each organ. GY was obtained in triplicate (sub-sampling) for each experimental unit. The plants contained in a 1.5 m linear section of the central rows were harvested for GY determinations at 121 and 122 DAS in 2020 and 2021, respectively. The grain was dried at room temperature and weighed when 12% moisture was reached. Bio data were analyzed with a randomized complete block model and treatment means were grouped by Fisher's least significant difference test at $p \leq 0.05$. All calculations were carried out using the general linear model procedure of Statistical Analysis System software [35].

2.4. Climate Information

Daily climatic data for maximum temperature (Tmax), minimum temperature (Tmin), solar radiation (Rs), rainfall, and ETo for both growing seasons were obtained from the Adcon® automated weather station, located at 22.909° N–102.659° W at a distance of 1.5 km from the experimental site. The Adcon® platform estimates ETo by the Penman–Monteith method, using grass as a reference crop [34].

2.5. Description of the SIMPLE Model

The SIMPLE discrete-time dynamic model, proposed by Zhao et al. [21], simulates water-limited growth, development, and yield of crops using a daily time step, with functions or equations that explain the effect of daily temperature, heat stress, soil water availability, and atmospheric CO_2 concentration (Table 1). TT (°C d) and Bio (t ha^{-1}) were the state variables and GY (t ha^{-1}) was an output variable. The SIMPLE model had thirteen parameters (Table 2), nine related to the crop type and four that specify differences among varieties (Figure 1). TT was used for crop growth modeling.

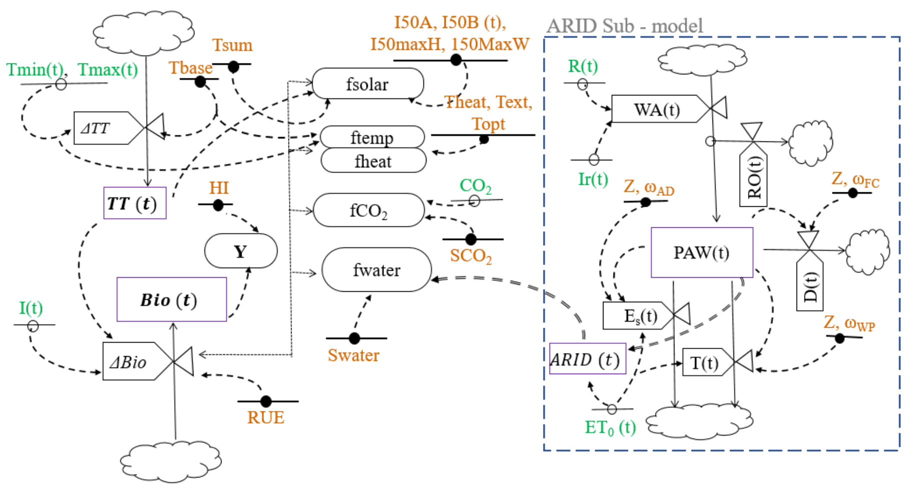

Figure 1. Relational diagram of the SIMPLE crop growth model. State variables are represented by rectangles, rates of change by valves, parameters with a horizontal line and a circle with black filling, input variables with a circle and a horizontal line, and auxiliary variables with circles. Material flows are represented by normal arrows and information flows with dashed lines.

Table 1. SIMPLE modal auxiliary equations.

Description	Equation
$fSolar$ for leaf growth and senescence period is based on the Beer–Lambert law, $fsolar\_max = 0.96$	$fSolar = \begin{cases} \frac{fsolar\_max}{1+e^{-0.01(TT-I_{50A})}}, & leaf\ growth\ period \\ \frac{fsolar\_max}{1+e^{-0.01(TT-(Tsum-I_{50B}))}}, & leaf\ senescence\ period \end{cases}$
The impact of temperature on the growth rate of biomass	$f(Temp) = \begin{cases} 0 & T < T_{base} \\ \frac{T-T_{base}}{T_{opt}-T_{base}} & T_{base} \le T < T_{opt} \\ 1 & T \ge T_{opt} \end{cases}$
The impact of heat stress on the biomass growth rate	$f(heat) = \begin{cases} 1 & T_{max} \le T_{heat} \\ 1 - \frac{T_{max}-T_{heat}}{T_{ext}-T_{heat}} & T_{heat} < T_{max} \le T_{extreme} \\ 0 & T_{max} > T_{extreme} \end{cases}$
The cumulative temperature required to achieve 50% radiation interception during canopy senescence (I_{50B}) is increased by heat stress	$I_{50Bi+1} = I_{50Bi} + I_{max,\ heat}\ (1 - f(heat))$
The impact of CO_2 on the RUE	$f(CO_2) = \begin{cases} 1 + SCO_2\ (CO_2 - 350) & 350\ \text{ppm} \le CO_2 < 700\ \text{ppm} \\ 1 + SCO_2\ (350) & CO_2 > 700\ \text{ppm} \end{cases}$
Drought stress based on water retention.	$f(water) = 1 + S_{water}\ (ARID)$
Standardized Drought Stress Index	$ARID = 1 - \frac{\min(ET_o\ ,\ 0.096*PAW)}{ET_o}$
Drought stress reduces RUE	$I_{50Bi+1} = I_{50Bi} + I_{max,\ water}\ (1 - f(water))$
Radiation interception affected by drought stress	$fSolar\_water = \begin{cases} 0.9 + f(water) & f(water) < 0.1 \\ 1 & f(water) \ge 0.1 \end{cases}$

The SIMPLE model uses cumulative temperature to determine the rates of phenological development [36] as follows:

$$TT_{i+1} = TT_i + \Delta TT \quad i = 1,\ 2,\ 3, \ldots n \tag{5}$$

$$\Delta TT = \begin{cases} T - T_b; & T > T_b \\ 0 & ; T \le T_b \end{cases} \tag{6}$$

where TT_i (°C d) is the thermal time in the i-th day, ΔTT (°C) is the daily increase in TT, T (°C) is the mean daily temperature, T_b (°C) is the base temperature for phenological crop development and n is the number of simulation days.

Biomass growth is based on radiation use efficiency [37], e.g., a fraction of the daily photosynthetically active radiation is intercepted by the plant and transformed into crop biomass.

$$Bio_{i+1} = Bio_i + \Delta Bio \quad i = 1,\ 2,\ 3, \ldots n \tag{7}$$

$$\Delta Bio = Rs \cdot fSolar \cdot RUE \cdot f(CO_2) \cdot f(Temp) \cdot \min[f(Heat), f(Water)] \tag{8}$$

where Bio_i is the cumulative biomass on the i-th day, Bio_{i+1} (t ha^{-1}) is the daily cumulative biomass to physiological maturity, ΔBio (t ha^{-1} day^{-1}) is the daily biomass growth rate, n is the number of simulation days and *fSolar* is the fraction of solar radiation intercepted by crop canopy; RUE (g MJ^{-1} m^{-2}) is the radiation use efficiency, and f(heat) is the heat stress factor; $f(CO_2)$ is the impact of CO_2 on RUE, $f(Temp)$ is the temperature impact on biomass growth rate, and f(Water) is the simple water-budget routine to estimated drought stress, factors that may or may not favor biomass accumulation. The $f(CO_2)$ effect is expressed through SCO_2 (SIMPLE modal auxiliary equations are given in Table 1). This parameter is used for estimating the stress factor due to CO_2 contraction.

GY is calculated as the product of total cumulative biomass (Bio_{cum}) and the harvest index (HI) [38] as follows:

$$\hat{GY} = Bio_{cum} \cdot HI \tag{9}$$

The SIMPLE model performs a soil–water balance based on the runoff-curve-number methodology for estimated surface runoff [39]. For this balance, four parameters are used to characterize of the experimental soil plot: (1) available water-holding capacity (0.123 m^3 m^{-3}), (2) deep drainage coefficient (0.5), (3) runoff number curve (0.81), and (4) root zone depth (600 mm). In addition, the model uses the standardized agricultural reference index for drought (ARID, Table 1) [40] to relate the effect of soil water content with the cumulative biomass, where Swater is a water stress parameter. The parameters obtained consider that the experiment was carried out on agricultural land with a clay loam texture with little development and a slope > 3. Many of the SIMPLE functions have been used in other crop models, described extensively by Zhao et al. [21].

2.6. Model Calibration

According to the dynamic systems modeling procedure [41–43], before calibration, a global sensitivity analysis (GSA) was conducted to identify the most influential parameters in the model [44,45]. The GSA was performed using the Sobol method [46] where ± 20% uncertainty was applied to most of the parameters, avoiding cardinal temperature overlap and using a uniform distribution, except for T_{sum} and S_{water}, where a normal function was used (Table 2). The most influential candidate parameters for calibration were I_{50A}, T_{opt}, S_{water}, T_b, I_{50maxW}, and T_{sum}.

Table 2. SIMPLE model parameters with their nominal values and exploration limits used for global sensitivity analysis and calibration in 'Pinto Saltillo' beans.

Parameter	Description	Nominal	Threshold *	Units	Cite
T_{sum}	Cumulative temperature from sowing to maturity	1200	1047–1356 [&]	°C d **	B-G
HI	Harvest index	0.36	0.29–0.43	-	B-G
I_{50A}	The cumulative temperature required for leaf area development to intercept 50% of radiation	450	360–540	°C d	Z
I_{50B}	Cumulative temperature till maturity to reach 50% radiation interception due to leaf senescence	200	160–240	°C d	Z
T_b	Baseline temperature for phenology development and growth	8	6.4–9.6	°C	B-G
T_{opt}	The optimal temperature for biomass growth	30	22–30	°C	B-G
RUE	Radiation use efficiency (above ground only and no respiration)	3.21	2.57–3.85	g MJ^{-1} m^{-2}	K
I_{50maxH}	Maximum daily reduction in I_{50B} due to heat stress	90	72–108	°C d	Z
I_{50maxW}	Maximum daily reduction in I_{50B} due to drought stress	20	16–24	°C d	Z
T_{max}	Threshold temperature to start accelerating heat-stress senescence	35	32.1–42	°C	O
T_{ext}	Extreme temperature threshold when RUE becomes 0 due to heat stress	45	42.1–52.5	°C	Z
SCO_2	The relative increase in RUE per ppm of CO_2 after 350 ppm	0.07	0.06–0.08	ppm	Z
S_{water}	Sensitivity of RUE to drought stress	0.9	0.48–1.28 [&]	-	Z

* Minimum and maximum limits, [&] parameters to which normal function was applied, ** degree days. Highlighted parameters are the most influential ones; B-G = Baez-Gonzalez et al. [47], Z = Zhao et al. [21], K = Karimzadeh et al. [48], O = Omae et al. [49].

2.6.1. Differential-Evolution Algorithms

Model calibration was performed with a differential evolution (DE) algorithm, which is evolutionary algorithm for solving global optimization problems. The DE algorithm is considered a simple, effective, and efficient heuristic search method inspired by natural evolution [50]. The DE algorithm includes a population of potential solutions and explores the search space using mutation, crossover, and selection operators. This algorithm has only three parameters that must be specified to optimize a problem. The parameters are population size (PS), mutation factor (MF), and crossover probability (CP).

The DE algorithm characteristics applied in the simulation consisted of six estimated parameters (D), a PS of 60 (D · 10), and the number of generations equal to 1000; the minimum values were taken from the mean of 25 runs and the DE/rand/1/bin algorithm strategy. This DE algorithm was programmed in the MATLAB®environment version 2020b. The parameter values of the DE algorithm were PS = 60, MF = 0.6, and CP = 0.9. The exploration threshold of the calibrated parameters is specified in Table 2.

2.6.2. Objective Function

An objective function to optimize a problem is defined as:

$$\hat{p} = \text{arg}\mathit{min}\ J(p) \tag{10}$$

$$J(p) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\hat{y}(t_i, p) - y(t_i))^2} \tag{11}$$

where $\hat{y}(t_i, p)$ is the biomass predicted by the SIMPLE model at a time t_i and $y(t_i)$ is the measured variable at the time t_i, N is the number of samples during the growth period, p is the parameter vector established for calibration and $\hat{p}$ is the parameter vector that yields the minimum value of $J(p)$.

2.7. Model Evaluation

After performing the SIMPLE model calibration, the average estimated values and nominal values of the parameters, given in Table 2, were used to perform a new simulation for estimating the TT, Bio, and GY values using the climatic and crop data obtained during the growing season 2021.

2.8. Measures for the Degree of Fit

To measure the calibration and evaluation quality in the biomass simulation by the SIMPLE model, the following measures of agreement [51] between the observed and simulated values were used: (1) the bias (BIAS), (2) the mean absolute error (MAE), (3) the root-mean-square error (RMSE), and (4) the efficiency (EF) proposed by Nash and Sutcliffe (1970), which characterizes the behavior of the simulation model. The perfect model should have an efficiency close to 1 [52].

3. Results

3.1. Climate and Irrigation Schedule

The average multi-year rainfall (2002–2019) from April to August was 276.7 mm. The cumulative precipitation during the experimental period was 352 mm and 387 mm, respectively, for the 2020 and 2021 growing seasons. The maximum temperature occurred in June and May, and the minimum temperature in April for 2020 and 2021, respectively. The average temperature for the growing season 2020 was 4% higher than in 2021 (Figure 2A). However, air temperatures recorded throughout the growing season determined crop development, with flowering starting at 54 and 50 DAS for the 2020 and 2021 growing seasons, respectively. In addition, the average solar radiation was 28.1 MJ m^2 day^{-1} for both experimental years, 3% lower than the historical average (Figure 2B).

Climatic conditions, mainly rainfall and reference evapotranspiration, influenced the irrigation depths applied in each treatment (Table 3). In 2020, low Pp and high ETo were recorded, resulting in a high crop water demand. The I_{50} (82%) and I_{125} (66%) irrigation treatments were applied mostly during the reproductive phenological stage in 2020 respectively. The corresponding values for these irrigation treatments at the same phenological stage were 100% and 77%, respectively, in 2021. While the total water applied increased, water use efficiency decreased, and vice versa, in both growing seasons. Water productivity (WP) values were not clear in the 2020 data, but in the 2021 data WP increased as water irrigation inputs decreased (Table 3).

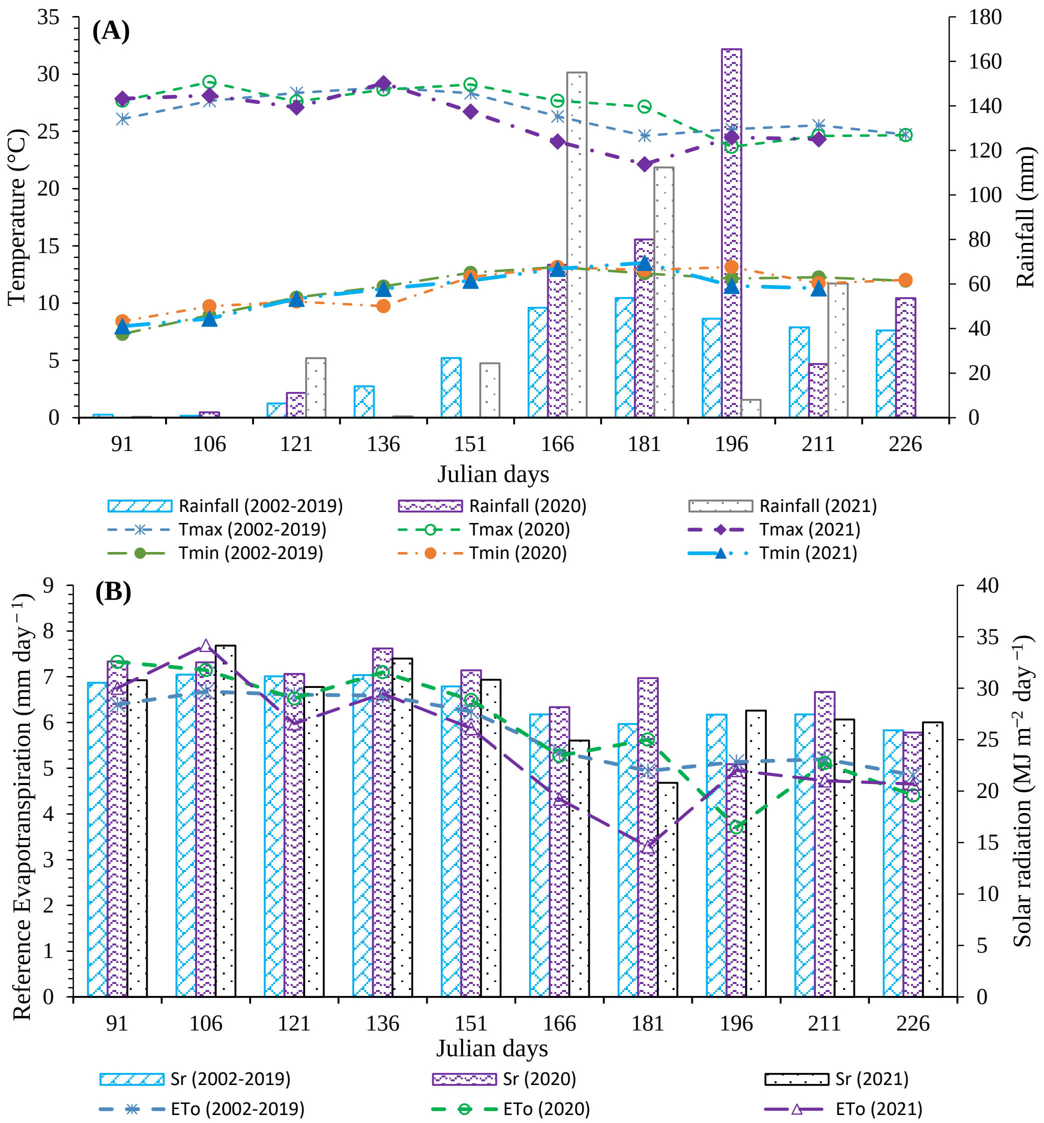

Figure 2. Climatic variables recorded during the growing season with historical averages (17 years) for the CEZAC INIFAP-Zacatecas station ((**A**)Temperature and (**B**) Reference Evapotranspiration). Averages of every fifteen days; Tmax and Tmin are the maximum and minimum temperature, respectively, Sr is the solar radiation, and ETo is the reference evapotranspiration.

Table 3. Applied irrigation depth (AID), effective rainfall (ER), total water applied (TWA), water use efficiency (WUE), and water productivity (WP) of bean grain yield (Y) as influenced by irrigation treatments (IT) in Zacatecas, Mexico.

Year/IT	AID (mm)	ER (mm)	TWA (mm)	Y (t ha^{-1})	WUE (kg m^{-3})	WP (kg m^{-3})
2020						
I_{125}	349		593.1	4.88	1.4	0.8
I_{100}	293	244.1	537.1	4.64	1.6	0.9
I_{75}	234		478.2	3.12	1.3	0.7
I_{50}	183		427.1	3.27	1.8	0.8
2021						
I_{125}	276		546.5	4.30	1.6	0.8
I_{100}	219	270.4	489.5	4.75	2.2	1.1
I_{75}	174		444.5	3.95	2.3	0.9
I_{50}	133		403.5	3.74	2.8	0.9

WUE is the relation between Y (kg ha^{-1}) and AID (m^3 ha^{-1}); WP is the relation between Y (kg ha^{-1}) and TWA (m^3 ha^{-1}).

3.2. Biomass Accumulation Curve

During the first 25 DAS, cumulative biomass was very slow and there were no measurable readings among treatments, since emergence occurred at 8 and 7 DAS for the 2020 and 2021 growing seasons, respectively. When irrigation treatments were applied, a dynamic growth proportional to the amount of water applied was observed at the start of flowering at 54 and 50 DAS for 2020 and 2021, respectively. Subsequently, in grain filling, the variation in biomass increase among treatments can be attributed to the presence of rainfall. The most intense rainfall events were recorded in the middle of the reproductive cycle and at the beginning of flowering for the 2020 and 2021 growing seasons, respectively (Tables 4 and 5). Data generated in 2020 and 2021 were used for the calibration and evaluation procedures, respectively.

Table 4. Mean values of cumulative biomass (t ha^{-1} ± standard deviation) of 'Pinto Saltillo' bean under irrigation treatments (IT) in Zacatecas, Mexico in the 2020 growing season.

	Days after Sowing						
IT	**13**	**25**	**36**	**48**	**62**	**77**	**89**
I_{125}	0.021 ± 0.002	0.082 ± 0.006	0.222 ± 0.016 b	1.714 ± 0.028 a	5.06 ± 1.1 a	9.27 ± 1.48 a	8.01 ± 0.33 a
I_{100}	0.021 ± 0.002	0.082 ± 0.006	0.277 ± 0.011 a	0.972 ± 0.135 b	2.96 ± 0.99 ab	8.74 ± 0.38 a	9.18 ± 1.35 a
I_{75}	0.021 ± 0.002	0.082 ± 0.006	0.239 ± 0.01 ab	1.119 ± 0.175 b	1.91 ± 0.12 b	4.02 ± 1.02 b	6.62 ± 1.93 ab
I_{50}	0.021 ± 0.002	0.082 ± 0.006	0.22 ± 0.028 b	0.989 ± 0.223 b	2.07 ± 0.47 b	--	3.41 ± 0.95 b
CV			7.987	11.703	28.865	17.449	21.315
RMSE			0.019	0.140	0.866	1.035	1.450
LSD			0.049	0.360	2.222	2.655	3.718

CV is the coefficient of variation, RMSE is the root-mean-square error, and LSD is the least significant difference ($p \leq 0.05$). Mean values with different lowercase letters are statistically different at $p \leq 0.05$.

Table 5. Mean values of cumulative biomass (t ha^{-1} ± standard deviation) of '*Pinto Saltillo*' bean under irrigation treatments (IT) in Zacatecas, Mexico, in the 2021 growing season.

	Days after Sowing						
IT	**17**	**31**	**43**	**57**	**75**	**85**	**100**
I_{125}	0.029 ± 0.009	0.287 ± 0.015 a	1.19 ± 0.29 a	3.16 ± 0.52 a	6.03 ± 0.65 ab	11.92 ± 0.27 a	12.01 ± 0.66 b
I_{100}	0.029 ± 0.009	0.273 ± 0.015 a	0.10 ± 0.16 a	2.70 ± 0.48 ab	7.91 ± −1.21 a	7.63 ± 0.19 c	11.55 ± 0.45 b
I_{75}	0.029 ± 0.009	0.286 ± 0.054 a	1.04 ± 0.10 a	2.45 ± 0.41 ab	6.52 ± 0.68 ab	7.67 ± 0.79 c	15.23 ± 0.23 a
I_{50}	0.029 ± 0.009	0.285 ± 0.027 a	0.863 ± 0.02 a	1.91 ± 0.15 b	5.28 ± 0.83 b	9.20 ± 0.30 b	--
CV		12.33	18.133	18.283	14.126	5.589	3.698
RMSE		0.035	0.185	0.466	0.909	0.509	0.507
LSD		0.090	0.48	1.196	2.331	1.305	1.300

CV is the coefficient of variation, RMSE is the root-mean-square error, and LSD is the least significant difference ($p < 0.05$). Mean values with different lowercase letters are statistically different $p \leq 0.05$.

3.3. Calibration and Evaluation

Using the differential evolution method, 20 optimizations were carried out. The mean of each parameter per treatment is indicated in Table 6. The largest standard deviation in the parameter set was observed at 1.17×10^{-16}. This value is indicative of the fact that the calibration process converged to the global maximum; therefore, the results were considered reliable. On the other hand, for the evaluation of the SIMPLE model, the average of the calibrated parameter values (Table 6) with data from the year 2020 and the nominal values (Table 2) were used, as appropriate. With this, the new parameter vector was generated; subsequently, the biomass was simulated, and then the performance measures were obtained in the evaluation stage for the observed 2021 data.

Table 6. Vector of parameters resulting from the SIMPLE model calibration using the differential-evolution algorithm.

Calibration Treatment	Optimal Parameters						Statistics	
	T_{sum}	I_{50A}	T_b	T_{opt}	I_{50maxW}	S_{water}	RMSE	S
I_{125}	1356	540	8.7	24.0	16	1.00	0.51	3×10^{-17}
I_{100}	1356	540	9.6	27.1	16	0.74	0.43	0.0
I_{75}	1354	540	9.6	29.8	16	0.79	0.59	1×10^{-16}
I_{50}	1356	526	9.5	30.0	16	0.85	0.09	0.0
Average	1356	536	9.3	27.8	16	0.84	0.40	4×10^{-17}

Value of the minimum objective function achieved by the DE (RMSE is the root-mean-square error) algorithm, standard deviation (S). T_{sum} is the cumulative temperature from sowing to maturity, I_{50A} is the cumulative temperature required for leaf area development to intercept 50% of radiation, T_b is the base temperature for phenological growth and development, T_{opt} is the optimal temperature for biomass growth, I_{50maxW} is the maximum daily reduction in I_{50B} due to drought stress, and S_{water} is the sensitivity of radiation use efficiency to drought stress.

Using the exploration thresholds in Table 2 for the six parameters in the calibration process, the T_{sum}, I_{50A}, and I_{50maxW} values were found to be close to the extreme values (Table 6). T_{sum} shifted to the upper end with a value from 1354 to 1356 °C d. It should be noted that it was tested with other values higher than 1356 °C d (unpublished data). However, overfitting was observed. For instance, the statistics in the calibration are improved for some treatments, but in the evaluation stage the statisticians indicate the low reliability of the SIMPLE model for simulating bean biomass. T_{opt} was inversely proportional to the irrigation treatment, e.g., the treatment with the highest water availability had the lowest T_{opt} value. S_{water} was the parameter that correlated with water availability within the SIMPLE model. However, no linear trend was observed concerning the treatments. Otherwise, for I125 and I75 irrigation treatments, the T_b was 8.7 °C and 9.6 °C, respectively (Table 6). However, T_{opt} is a cultivar parameter and there should be no variation between irrigation treatments. Also, a relationship between I_{50A} and Swater was observed, with the I_{50} irrigation treatment having the highest water stress with the lowest I_{50A} value and S_{water} increasing according to the trend of the parameters reported in the calibration stage. However, T_{sum} is expected to be the parameter with the greatest variability, since the crop cycle decreases with water stress. The results can be attributed to the method used by the SIMPLE model to estimate crop development.

In the calibration process, the cumulative biomass simulated did not follow a linear trend concerning the irrigation treatment. Values fluctuated between 3.47 t ha^{-1} and 10.2 t ha^{-1} for the I_{50} and I_{100} irrigation treatments, respectively. The I_{75} irrigation treatment had the maximum RMSE value (0.59 t ha^{-1}) of biomass. In the evaluation process, the predicted bean biomass adequately fitted the observed data for the four irrigation treatments with RMSE < 1.80 t ha^{-1} and EF > 0.89 values, with average values of 1.48 t ha^{-1} and 0.92 for RMSE and EF, respectively. In addition, the I_{50} irrigation treatment had the best fit in calibration and evaluation processes according to the statistics used to measure model performance (Table 7).

Table 7. Evaluation statistics of the SIMPLE model for simulating the biomass of beans subjected to different moisture levels.

Statistics	Calibration					Evaluation				
	I_{125}	I_{100}	I_{75}	I_{50}	Average	I_{125}	I_{100}	I_{75}	I_{50}	Average
Bias	−0.06	−0.11	−0.11	−0.02	−0.08	−0.91	−1	−0.28	−0.46	−0.66
MAE	0.36	0.34	0.43	0.07	0.30	1.01	1.04	0.99	0.47	0.88
RMSE	0.51	0.43	0.59	0.09	0.41	1.74	1.8	1.49	0.89	1.48
EF	0.98	0.99	0.93	0.99	0.97	0.91	0.89	0.91	0.95	0.92

MAE is the mean absolute error, RMSE is the root-mean-square error, and EF is the efficiency.

The simulated biomass for each irrigation treatment responded to a sigmoidal trend curve in both the calibration and evaluation process (Figure 3). Bean plants had slow growth before flowering (51 DAS) until pod formation (59 DAS), then an exponential growth was observed between 60 and 80 DAS and, finally, crop growth had an asymptotic pattern from grain filling (85 DAS) until maturity (120 DAS). In the evaluation process, the simulated biomass had a positive linear trend with respect to the irrigation treatments, e.g., the more available soil moisture, the greater the biomass accumulation. In addition, biomass values were higher than those values obtained during calibration for all irrigation treatments. This condition can be attributed to the fact that during the 2020 growing season (calibration data), rainfall was < 35.4 mm and reference evapotranspiration was > 62.6 mm higher than in 2021 (evaluation data). Also, as expected, the Bias, MAE, RMSE, and EF statistics showed better fit in calibration than in evaluation. In the model evaluation, I_{100} had the lowest values, with 1.8 t ha^{-1} and 0.89 for RMSE and EF, respectively. However, the overall efficiency for all irrigation treatments was 0.92, which is acceptable for simulating crop biomass.

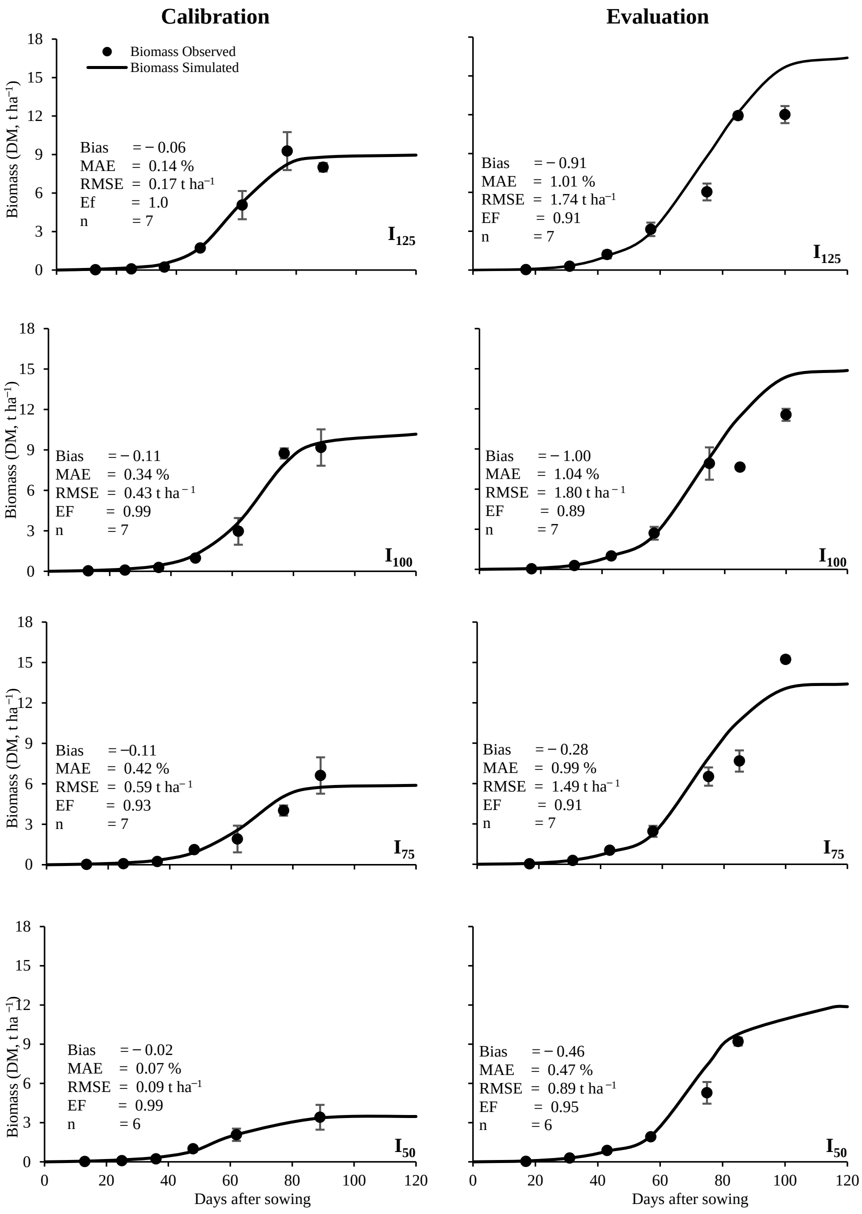

Figure 3. Observed vs. simulated biomass (DM = dry matter) for the four irrigation treatments. In calibration (2020 data) and evaluation (2021 data), MAE is the mean absolute error, RMSE is the root-mean-square error, and EF is the efficiency.

The overall data performance for the calibration and evaluation processes, with the relationship between observed and simulated biomass, is given in Figure 4. The evidence confirmed that the SIMPLE model calibration had a better fit between observed and simulated dry biomass compared with the evaluation procedure (Figures 4A and 4B, respectively). In the calibration process, the values furthest from the 1:1 line corresponded to the data obtained at 77 and 89 DAS from the I125 and I75 irrigation treatment, respectively (Figure 4A). That is, the general statistics evidenced that the model simulated bean biomass adequately under all irrigation treatments tested here. In the evaluation process, values less than 4 t ha^{-1} overlapped on the 1:1 line (Figure 4B). That is, it presented a good fit for the vegetative stage, while in the reproductive stage the estimated values were underestimated. Nevertheless, in the evaluation stage the SIMPLE model's statistics pointed to a good performance in predicting bean biomass under all irrigation treatments studied here.

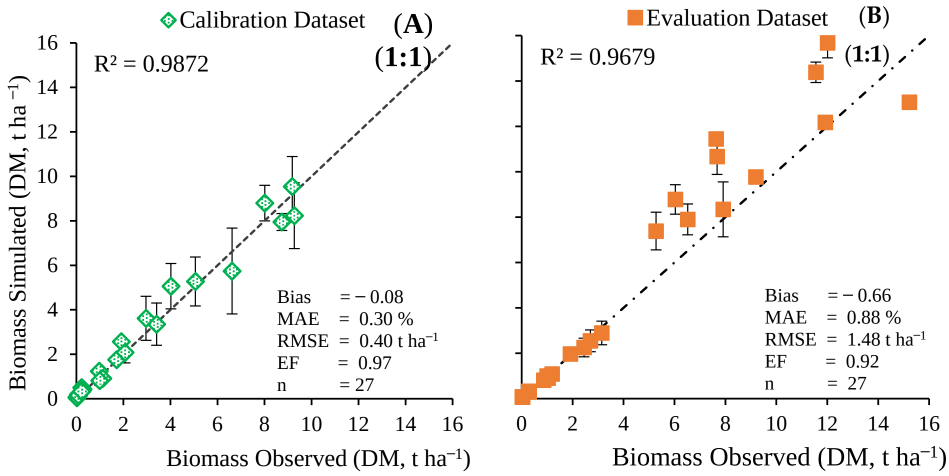

Figure 4. Calibration (**A**) and evaluation (**B**) processes of observed and simulated biomass. At each point, the vertical bars indicate the standard deviation, MAE is the mean absolute error, RMSE is the root-mean-square error, and EF is the efficiency.

The grain yield estimation (Y; t ha^{-1}) of the 'Pinto Saltillo' bean by the SIMPLE model using the HI parameter of 0.36 was unsatisfactory (unreported data). The average errors for all irrigation treatments in the calibration and evaluation procedures were 37.1% and 21.6% for the 2020 and 2021 data, respectively. They ranged between 12.7% and 61.8%, corresponding to the I_{100} and I_{50} irrigation treatments for the growing seasons of 2020 and 2021, respectively. For this reason, a multiple regression model involving biomass and transpiration was proposed to estimate the harvest index (HI) and reduce estimation errors.

3.4. Harvest Index

The regression model developed for estimating the harvest index (HI) considered two steps: (1) the selection of the regression model and (2) the calibration process.

In the first step, a regression model was generated using the observed Bio and Y, using data obtained at 100 DAS in 2021. Triplicate data from each irrigation treatment and HI were calculated by relating Y to Bio. Also, the sum of transpiration (ST_j), resulting from the last 100 days of the SIMPLE model simulation, was used. Subsequently, relative values were obtained by relating Bio_j, Y_j, and HI_j to their corresponding maximum values. The first-order multiple regression model (Equation (12)) was performed in the statistical analysis system (SAS, 2011). The model was significant ($p < 0.05$) and explained 50% (R^2) of the dataset variability, with a coefficient of variation of 18.3% and 0.13 RMSE.

$$\frac{HI_j}{HI_{j\_max}} = \left[\beta_0 - \beta_1\left(\frac{Bio_j}{Bio_{j\_max}}\right) + \beta_2\left(\frac{Bio_j}{Bio_{j\_max}} \cdot \frac{ST_j}{ST_{j\_max}}\right)\right] \quad \therefore$$
$$HI_j = \left[0.64 - 2.34\left(\frac{Bio_j}{Bio_{j\_max}}\right) + 2.66\left(\frac{Bio_j}{Bio_{j\_max}} \cdot \frac{ST_j}{ST_{j\_max}}\right)\right] \cdot HI_{j\_max} \tag{12}$$

where HI_{j_max} is the maximum harvest index observed (0.29), Bio_{j_max} is the maximum cumulative biomass observed (16.7 t ha^{-1}), and ST_{j_max} is the sum of simulated transpiration (419 mm).

In the second step, the simulated cumulative biomass (Bio_{Sim}) and the sum of simulated transpiration (ST_{Sim}) at the end of the growing season were used for each of the treatments for 2020 and 2021. Bio_{Sim} and ST_{Sim} were the results of simulation for all treatments with the SIMPLE model, using the calibrated parameters in Table 7.

Consecutively, the values of β_0, β_2, Bio_{j_max}, and ST_{j_max} were optimized until reaching a minimum RMSE between observed and simulated yield. For the optimization analysis, the lsqnonlin.m function of the MATLAB® optimization tool was used. HI_{j_max} remained fixed; however, there was a better fit when using the base harvest index HI_o of 0.36 obtained for the 'Pinto Saltillo' variety under climatic conditions similar to those in this study. The model for estimating the resulting harvest index was the following:

$$\hat{HI} = \left[2.29 - 2.34\left(\frac{Bio_{Sim}}{9.04}\right) + 1.31\left(\frac{Bio_{Sim}}{9.04}\cdot\frac{ST_{Sim}}{423.2}\right)\right]\cdot 0.36 \quad (13)$$

To determine the model's performance, corrected relative error (CRE) was used to avoid negative values. CRE measures the quality of the estimate, where values >10% are considered 'excellent' and 10 > REC < 20 are 'good' for estimating grain yield.

$$CRE = \left[\frac{\max(Y_{sim},\ Y_{obs}) - \min(Y_{sim},\ Y_{obs})}{\max(Y_{sim},\ Y_{obs})}\right]\cdot 100 \quad (14)$$

where Y_{obs} represents the observed yield and Y_{sim} the value predicted by the model as a function of Bio_{Sim} and $\hat{HI}$.

The proposed regression model simulated the harvest index reliably for most treatments because a CRE of less than 10% was observed in almost all treatments, except for the I_{50} and I_{100} irrigation treatments in the 2020 and 2021 growing seasons, respectively (Table 8).

Table 8. Simulated and observed grain yield of 'Pinto Saltillo' bean under irrigation treatments (IT) in Zacatecas, Mexico.

Year	IT	Y_{obs} (t ha^{-1})	ST_{sim} (mm)	Bio_{Sim} (t ha^{-1})	$\hat{HI}$	Y_{Sim} (t ha^{-1})	CRE (%)
2020	I_{125}	4.88 ± 0.82 a *	483.0	10.2	0.48	4.89	0.08
	I_{100}	4.64 ± 0.79 ab	456.3	8.1	0.52	4.24	8.69
	I_{75}	3.12 ± 0.66 b	423.4	5.8	0.59	3.40	8.34
	I_{50}	3.27 ± 0.69 ab	391.5	4.1	0.64	2.62	19.90
	CV	16.39					
	RMSE	0.65					0.17 t ha^{-1}
2021	I_{125}	4.30 ± 0.69 ab	479.9	19.0	0.26	4.30	0.00
	I_{100}	4.75 ± 0.51 a	457.0	17.7	0.27	4.06	14.57
	I_{75}	3.95 ± 0.53 ab	437.2	16.4	0.29	3.95	0.04
	I_{50}	3.74 ± 0.48 b	417.7	15.2	0.33	3.88	3.58
	CV	13.00					
	RMSE	0.12					0.12 t ha^{-1}

Y_{obs} is the observed grain yield at maturity, ST is the sum of transpiration during the growing season, and HI is the harvest index. Bio is the cumulative biomass at maturity, CV is the coefficient of variation, RMSE is the root-mean-square error, and CRE is the corrected relative error. * Mean values with different lowercase letters are statistically different $p \leq 0.05$.

4. Discussion

4.1. Climate and Irrigation Schedule

Temperature is one of the environmental factors that influence crop growth and development [53]. Barrios et al. [54] pointed out that beans can grow with average temperatures ranging between 15 and 27 °C, with an optimal temperature of 25 °C. In contrast, Beebe et al. [55] indicated that the optimal temperature for this crop is between 17.5 and 23.1 °C. The air temperature during the two growing seasons was in good agreement with those observed in other studies (Figure 2).

Late rainfed-bean varieties planted in northern Mexico require more than 240 mm of rainfall during the production cycle with yields of 0.6 t ha^{-1}, but in rainy years (323 mm of rainfall) high yields could be achieved (1.4 t ha^{-1}) [56]. High rainfall was recorded atypically in both growing seasons studied during the vegetative stage, with 96% and 87% for 2020 and 2021, respectively (Figure 2). Lynch and van Beem [57] reported that the bean crop can experience water stress due to erratic rainfall, with the reproductive stage being the most sensitive [58]; this did not occur during the experimental period.

Beebe et al. [55] state that the bean crop requires from 363 to 450 mm of irrigation depth throughout the growing season. Rai et al. [59] applied 375 mm in the largest irrigated treatment under semi-arid conditions in Wyoming, USA. In the experiments described here, the irrigation depths applied were lower than that indicated by Rai et al. [59]. In this study, considering the ER plus the applied irrigation depths, the total water applied for I_{50} was 535 mm and 520 mm, respectively, for the 2020 and 2021 growing seasons (Table 3). These rainfalls may have temporally masked the effect of the I_{50} and I_{75} irrigation treatments.

4.2. Cumulative Biomass Curve

In the 2020 growing season, the irrigation treatments accumulated less total biomass compared with their counterparts in the 2021 growing season (Tables 3 and 4). This may be attributed to lower water availability at the flowering stage in the 2020 growing season due to the *Kc* value used, which could underestimate the water requirements of bean crops [58]. Water stress at the flowering stage reduces leaf number and stem length, and therefore, this is reflected in grain mass per plant [58]. Simulated biomass increased proportionally to the irrigation treatments in both growing seasons (Table 8). This was clearer in 2020 than in 2021. In the latter year, 53% of the rainfall occurred at the pre-flowering stage. This could then, temporally, mask the irrigation treatment effect. The opposite occurred in the former year, where 70% of rainfall occurred at the grain-filling stage in favor of beans plants under I_{100} and I_{125}. Therefore, based on Equations (7) and (8), bean plants experiencing I_{100} and I_{125} irrigation treatments suggest a greater canopy for intercepting solar radiation and, therefore, more photo assimilates were available and distributed to growing organs (sink), mainly to the grains (Table 8) [60]. Therefore, the SIMPLE model may be suggested to simulate bean Bio and Y for different scenarios in the semi-arid agricultural lands of north-central Mexico.

4.3. Calibrated Parameters

The value of standard deviation close to zero and the value of the objective function > 0.6 are reliable indicators for determining the effectiveness of search algorithms, according to Trejo-Zúñiga et al. [61]. In addition, the DE algorithm offers better approximations of the global optimum compared to other algorithms [61,62]. On the other hand, T_{sum} is the parameter that indicates the cumulative temperature from sowing to physiological maturity. The Tsum values reported for beans ranged between 950 and 2700 °C d [21,47,59,63,64]. However, the physiological maturity of the bean crop is mainly for beans with determined growth habits, variety, and photoperiod. Some studies indicate that it is mainly irrigation management which modifies the development cycle, although the SIMPLE model did not adequately represent this effect. I_{50A} is the cumulative temperature requirement for leaf area development to intercept 50% of radiation; in beans, values of 450 °C d have been reported [21]. T_b is the temperature at which crop development stops or starts. For beans,

T_b values of 5 °C [59], 8 °C [47], and 12 °C [64] have been reported. Topt is the optimal temperature at which growth reaches its maximum expression; thus, values of 25 °C [65], 27 °C [21], and 30 °C [47] have been reported. Furthermore, beans can withstand extreme temperatures of between 5 and 40 °C [66]. The T_{sum}, T_{opt} and T_b values reported as a result of calibration in this study are within the ranges mentioned by the authors cited above (Table 6).

The S_{water} is the parameter of RUE sensitivity to the ARID index [21]. That is, it incorporates the water availability effect on the crop, where an ARID equal to zero indicates that the crop is at its maximum water demand (no stress) [40]. However, the average value for the treatment with the highest irrigation depth applied (I_{125} in 2020) between vegetative growth and flowering initiation stages was 0.39 (moderate stress). This was indicative that Kc underestimated the ETc compared with the 0.09 (no stress) observed between flowering initiation and physiological maturity, which was coincident with an atypically rainy season for both growing seasons. In addition, the linear behavior of Swater values and the applied irrigation depth was not observed. That is, Swater alone did not reflect the water effect in the model because there is an interaction between parameters. Besides, as pointed out by Bulatewicz et al. [67], here it was not possible to explore a wider range of irrigation depths due to the establishment of the rainy season which equalized water storage in the soil, thereby masking the effect of the irrigation treatments. Teweldebrhan et al. [68] argue that there is a multiplicative interaction between the parameters of a model; that is, the effect of one parameter is reflected in the output variables, as long as another parameter intervenes.

4.4. Calibration and Evaluation

In individual performance (Figure 3), the biomass curve obtained was similar to that established by other authors [69–71]. The calibration and evaluation statistics were within an acceptable range for all irrigation treatments [72,73]. Also, the observed RMSE was slightly higher than those recorded for beans subjected to different moisture levels evaluated with the SALTMED model, specifically for water balance [74]. In general, the statistics observed in this study (Figure 4) were considered acceptable in the crop biomass simulation, considering that the estimated values of the parameters are only approximations of the real values and probably have a fairly substantial error [9], taking into consideration the fact that the bean crop has a significant genotype x environment interaction [54] and that the SIMPLE model does not consider genetic parameters.

In the calibration stage, the observed biomass values were lower than those reported in the evaluation; these results are attributable to the climatic conditions of 2021, which favored bean development. In this respect, Emam et al. [75] point out that common bean biomass decreased, as did the water supply. However, the biomass values found in this study agreed with other studies. For example, Morales-Rosales et al. [76] and Dewedar et al. [74] reported values of 13.0 t ha^{-1} and 14.6 t ha^{-1}, respectively, for irrigated beans, while mean values between 4.0 and 8.0 t ha^{-1} for different bean varieties were reported by Acosta-Gallegos and Rosales-Serna [77].

4.5. Harvest Index

The estimate of the performance with the SIMPLE model raises a linear relationship with the estimated biomass. That is, it calculates the yield as a percentage of the cumulative simulated biomass at the end of the cycle without considering any other environmental or management factor. Nevertheless, here, HI was not linearly proportionally to the cumulative biomass at the maturity stage. Attributes such as crop management and genotypic adaptation may play a part in the components of yield productivity under water limitation [78]. In addition, several methods have been proposed to calculate HI, which consider cumulative biomass after anthesis, and whose behaviors were nonlinear [79]. Bean HI is related to the availability of photoassimilates which are exported mainly to sink organs (grains) [76]. These are, in turn, related directly to the intercepted solar radiation by the crop and the soil water availability to satisfy bean crop evapotranspiration demand.

The yield data generated in both experiments agreed with those of Ranjan et al. [80]. These authors, evaluating moisture and tillage levels in 'Pinto Saltillo' beans, obtained grain yields between 3.0 and 5.8 t ha^{-1}. Thus, the model proposed in this study to estimate grain yield (Y) gave excellent results compared with those reported with the CROPGRO Dry-bean model. Dallacort et al. [81] state that this model underestimated Y by between 19% and 29% for beans under water-deficit conditions. Monpremier et al. [82] also used CROPGRO Dry-bean to simulate Y of different bean varieties with relative errors of 32.1% and 15.6% in the calibration and evaluation procedures, respectively. The SSM-Legumes model produced an RMSE of 0.41 t ha^{-1} in the grain-yield simulation [83]. In another study, relative differences < 4.9% between observed and simulated bean yields under different irrigated conditions were reported by using the SALMED model [74]. Similarly, with the AquaCrop model, an RMSE > 0.28 t ha^{-1} was found [84].

5. Conclusions

The SIMPLE model acceptably simulates biomass dynamics in the common bean crop, since when calibrating the I_{50A}, T_{opt}, S_{water}, T_b, I_{50maxW}, and T_{sum} parameters a good fit between predictions and measurements was obtained. Despite having only a few parameters, the SIMPLE model had the potential to simulate bean biomass under different moisture conditions, and therefore, it can be a feasible tool for planning productive activities for bean cultivation. However, when changes in soil moisture are very small, the model cannot express these in the biomass simulation.

In the evaluation process under conditions other than rainfall and evapotranspiration, the SIMPLE model adequately simulates crop biomass. The results indicate that the SIMPLE model is a reliable and robust tool for simulating biomass. It can be used to calculate the daily water demand of common beans as a function of growing degree days. Therefore, irrigation can be programmed in real-time for arid and semi-arid conditions in northern Mexico.

The cumulative bean biomass concerning the applied irrigation depth responded to a linear model. While the cumulative biomass curve showed a sigmoid behavior, the grain yield had an expected behavior. Nevertheless, in the proposed multiple linear regression model, a good bean-yield estimation was possible. However, the harvest index needs to be validated under other irrigation levels and further field irrigation experiments in different locations are needed to strengthen the SIMPLE model to simulate accurate bean yield under climate change scenarios.

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